Optimum Order Quantity with Time Lag
and Truncated Distribution

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Abstract

This paper investigates the effect of inventory-level dependent demand of the displayed stock in the single item inventory model. It is supposed that the inventory-level dependent demand operates under the inventory received by the retailer in the beginning of the period and the time lag described between the display of the stock and customer’s response to the inventory. Using truncated probability distributions, we derive an optimum order quantity which should be ordered to maximize the expected profit.

Mathematics Subject Classification: 90B05, 60A05

Keywords: Inventory control, inventory-level dependent demand, truncated distribution, optimum order quantity

1. Introduction

In literature several researchers developed models for inventory-level dependent demand. B.N.Mandal and S.Phaudar(1989) developed an order level inventory model for deteriorating items with uniform rate of production and stock dependent demand. Jinn-Tsair Teng and Chun-Toa Chang [6] established an
Economic production quantity model for deteriorating items when the demand rate depends on the on-display stock level but also the selling price per unit. Ray J, Goswami A, Chaudhuri KS [13] developed an algorithm to obtain the optimal solution of an order-level inventory model with stock dependent demand and two storage levels. A.K. Pal, A.K. Bhunai and R.N. Mukherjee [11] determined the lot size of a single deteriorating item with stock dependent demand rate selling price of an item and frequency of advertisement. S. Pal, A. Goswami and K.S. Chaudhuri [10] developed a deterministic inventory model by assuming stock dependent rate and the deterioration of items at a constant rate. G. Padmanabhan, Prem Vrat [9] presented inventory models for perishable items with stock dependent selling rate. Datta T.K. and Paul.K [2] analyzed an inventory system with demand rate influenced by stock level and selling price. T.K. Datta and A.K. Pal [3] studied a situation where demand rate declines along the stock – level down to a certain level of the inventory and then the demand rate becomes constant for the rest of the cycle. In these articles they have assumed that the inventory-level dependent demand was effective throughout the period.

The inventory-level dependent demand concept is similar to the concept of lead time. The lead time is the time to respond by the Supplier whereas the inventory-level dependent demand effect is the time to respond by the customer. A customer’s attention towards the displayed stock is received by suitable advertisement by means of media. The advertisement plays a vital role in influencing customer’s purchase habit. The impact of the advertisement in customer’s purchase habit cannot be predicted because in some situations the customers react slowly to the advertisement. Some customers may respond quickly to the displayed stock and some others may postpone their visit. In some worst cases, customers may not turn up for the whole inventory cycle. Such cases are termed as “no inventory-level dependent demand effect”.

The effort of this paper is to determine the optimum order quantity of a single item inventory model with inventory-level dependent demand.

2. Assumptions and Notations:

- An order is placed and a lot size of Q units is received at time t=0. No new orders will be placed during the cycle.
- Initially normal demand takes place and the inventory-level dependent demand effect starts after a time lag.
- The time lag is denoted by a random variable “u” with a probability density function f(u),0< u ≤ U.
- Mean=E(u)=µ.
- U is the realization of inventory-level dependent demand effect.
- Lead time is zero.
- Shortages are not allowed.
- A=Ordering cost/order
3. Mathematical Model:

Let us assume that the demand occurs at a constant rate initially and the inventory-level dependent demand effect starts after a time lag. Let us consider the demand function \( D(Q) \) such that \( D(Q) = \alpha, 0 \leq t \leq u \) and \( \alpha + \beta Q, u < t \leq T \) where \( \alpha \) is the initial constant demand rate at \((0, u)\) and \( \alpha + \beta Q \) is a linear form describing inventory-level dependent demand after the time lag \( u \). Here \( \beta \) is a constant indicating the inventory-level dependent demand factor. The on-hand inventory level is \( Q \) and the on-hand inventory level at \( u \) will be \( Q-\alpha u \). This stock will be consumed by time \((Q-\alpha u)/(\alpha + \beta Q)\).

The expected cycle length will be \( E(T) = E(u) + E[(Q-\alpha u)/(\alpha + \beta Q)] \)

\[ E(T) = \frac{(1+\beta \mu)}{(\alpha + \beta Q)} \] 

Let \( A_1 \) be the stock on-hand during \((0,u)\). Then the expected holding cost will be

\[ E(A_1) = H\left(\frac{Q \mu - (\alpha (\mu^2 + \sigma^2)/2)}{\alpha + \beta Q}\right) \]

Let \( A_2 \) be the stock on-hand during \((u, T)\). Then the expected holding cost will be

\[ E(A_2) = HE[(Q-\alpha u)^2/(\alpha + \beta Q)] \]

Therefore the expected holding cost during the entire cycle \((0,T)\) will be

\[ E(A_1+A_2) = H(Q^2(1+2\beta \mu)-\alpha \beta Q(\mu^2 + \sigma^2))/ 2(\alpha + \beta Q) \]

Then the expected holding cost per unit will be

\[ E(A_1+A_2)/E(T) = H(Q^2(1+2\beta \mu)-\alpha \beta Q(\mu^2 + \sigma^2))/ 2Q(1+\beta \mu) \]

The expected ordering cost per unit time will be

\[ E(A)/E(T) = A(\alpha + \beta Q)/ Q(1+\beta \mu) \]

The time lag \( u \) is considered as a random variable with probability density function \( f(u), 0 \leq u \leq U \). Since the on-hand inventory received at time \( t=0 \) is \( Q \) and normal demand rate is \( \alpha \), \( U \) takes values between 0 and \( Q/\alpha \). If \( u=0 \), then entire cycle runs with inventory-level dependent demand effect and if \( u=Q/\alpha \), then there is no inventory-level dependent demand effect throughout the cycle. The normal cycle length without inventory-level dependent demand would be \( Q/\alpha \) and it is reduced to \( E(T) \) given in (1). The difference \( (Q - \alpha E(T)) \) represents the increase in the demand
rate due to the effect of inventory-level dependent demand during the cycle.

The gain due to the inventory-level dependent demand effect is studied as

\[ G = (P-C)(Q-\alpha E(T)) \]

The expected gain per unit time due to inventory-level dependent demand effect will be

\[ E(G)/E(T) = ((P-C)\beta(Q-\alpha \mu))/(1+\beta \mu). \]

Thus the expected total cost of inventory will be

\[ TC(Q) = (A\alpha/(Q+Q\beta \mu)) + ((HQ+2HQ\beta \mu)/(2+2\beta \mu)) - ((P-C)Q\beta/(1+\beta \mu)) + \text{terms independent of } Q \]

Differentiating (2) with respect to "Q", we get

\[ Q^2 = 2A\alpha/(H(1+2\beta \mu)-2(P-C)\beta) \]

Therefore the optimum order quantity is given by

\[ Q^* = \sqrt{2A\alpha/(H(1+2\beta \mu)-2(P-C)\beta)} \]

which is called as Barabbas EOQ model or Barabbas formula.

In the following models, the nature of optimal order quantity is examined when the time lag \( u \) follows truncated probability distributions.

4. Model Formulations and Solution:

4.1 When \( u \) follows Truncated Exponential Distribution:

Suppose that the time lag \( u \) follows exponential distribution with probability density function in \( 0 \leq u \leq Q/\alpha \). Then we derive truncated exponential distribution with \( f(u) = g(u)/P(u>U) \) where \( g(u) = \theta \exp(-\theta u), \theta > 0 \) and \( 0<u<Q/\alpha \).

Thus the truncated exponential distribution is given by \( f(u) = \theta \exp(-\theta u)/\exp(-\theta U) \).

Case: (i)

If \( u \) take values between \( (0, Q/4\alpha) \), then the average delay in inventory-level dependent demand effect is taken as \( U = Q/4\alpha \) of the cycle length. Then the parameter \( \theta \) can be taken as the reciprocal of the average as \( \theta = 4\alpha/Q \) from which the mean \( \mu = 3.607Q/\alpha \).

Case: (ii)

If \( u \) take values between \( (0, Q/2\alpha) \), then the average delay in inventory-level dependent demand effect is taken as \( U = 2\alpha/Q \) of the cycle length. Then the parameter \( \theta \) can be taken as the reciprocal of the average as \( \theta = 2\alpha/Q \) from which the mean \( \mu = 0.976Q/\alpha \).

Case: (iii)

If \( u \) takes values between \( (0, 3Q/4\alpha) \), then the average delay of inventory-level dependent demand effect is taken as \( U = 3Q/4\alpha \) of the cycle length. Then the parameter \( \theta \) can be taken as the reciprocal of the average as \( \theta = 4\alpha/3Q \) from which the mean \( \mu = 0.7518Q/\alpha \).
Optimum order quantity

The delay in inventory-level dependent demand cannot be taken as \( Q/\alpha \) as it means no inventory-level dependent demand in the entire cycle.

4.2 When \( u \) follows truncated uniform distribution:

Suppose that the random lag \( u \) follows uniform distribution with probability density function in \( 0 \leq u \leq Q/\alpha \). Then we derive the truncated uniform distribution with \( f(u) = g(u)/P(u \geq U) \) where \( g(u) = \alpha/Q, 0 \leq u \leq Q/\alpha \). Thus the truncated uniform distribution is given by \( f(u) = (\alpha/Q)/(1-(\alpha U/Q)) \).

Case: i

If the truncation point is at \( Q/4\alpha \) (after which the inventory-level dependent demand effect starts), then \( U = Q/4\alpha \). Its mean is given by \( \mu = 5Q/8\alpha \).

Case: ii

If the truncation point is at \( Q/2\alpha \) (after which the inventory-level dependent demand effect starts), then \( U = Q/2\alpha \). Its mean is given by \( \mu = 3Q/4\alpha \).

Case: iii

If the truncation point is at \( 3Q/4\alpha \) (after which the inventory-level dependent demand effect starts), then \( U = 3Q/4\alpha \). Its mean is given by \( \mu = 7Q/8\alpha \).

Numerical Illustration:

Let \( A = 500, \alpha = 100, H = 0.4, P = 30 \) and \( C = 25 \) be fixed. When \( u \) follows truncated exponential distribution and truncated uniform distribution, for the different values of \( \mu \) and \( \beta \), the optimum order quantity are calculated and tabulated as given in Table: 1 and Table: 2 respectively.

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \beta = 0.01 )</th>
<th>( \beta = 0.02 )</th>
<th>( \beta = 0.03 )</th>
<th>( \beta = 0.04 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0,Q/4\alpha) )</td>
<td>551.48</td>
<td>623.05</td>
<td>732.44</td>
<td>931.69</td>
</tr>
<tr>
<td>( (0,Q/2\alpha) )</td>
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<td>681.01</td>
<td>900.11</td>
<td>1789.36</td>
</tr>
<tr>
<td>( (0,3Q/4\alpha) )</td>
<td>571.64</td>
<td>686.75</td>
<td>920.40</td>
<td>2038.79</td>
</tr>
</tbody>
</table>

Table: 1

<table>
<thead>
<tr>
<th>( \mu )</th>
<th>( \beta = 0.01 )</th>
<th>( \beta = 0.02 )</th>
<th>( \beta = 0.03 )</th>
<th>( \beta = 0.04 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (0,Q/4\alpha) )</td>
<td>572.59</td>
<td>690.06</td>
<td>932.50</td>
<td>2236.06</td>
</tr>
<tr>
<td>( (0,Q/2\alpha) )</td>
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<td>686.80</td>
<td>920.57</td>
<td>2041.24</td>
</tr>
<tr>
<td>( (0,3Q/4\alpha) )</td>
<td>570.73</td>
<td>683.58</td>
<td>909.09</td>
<td>1889.82</td>
</tr>
</tbody>
</table>

Table: 2
Graphical Illustration for Table:1 and Table:2 is shown below.

5. Conclusion:

From both the models we conclude that the optimum order quantity increases as the inventory-level dependent demand increases. It is reasonable because as inventory-level dependent demand increases, the demand for the inventory also increases which obviously leads to an increase in optimum order quantity.

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References


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