Use the Radon Transform for Describing Brachiopod Shape

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Abstract

In this paper we propose a novel method for describing fossils shape, we take the case of brachiopods which has Complex forms. Our approach based on the radon transform, the R-transform is our principal transform used, and that is invariant to common geometrical transformations like scale, translation and rotation. For the unicity of the approach, we compute the R-transform for different levels of the chamfer distance transform. We can deduce that the results obtained are optimized, in the sense that they allow the classification of Brachiopods.

Keywords: Fossils, Brachiopods, Radon transform, R- transform, Distance transform, Chamfer distance
1 Introduction

The analysis and quantification of the fossil morphology are of a great interest in palaeontological studies. On the one hand, they make it possible to understand the biodiversity in its morphological dimension. On the other hand, they highlight the morphological transformations undergone during the biological evolution. Historically, the form of was encircled by a purely descriptive approach based on the qualitative evaluations of the morphological change starting from simple images. This approach was replaced gradually by the biometric methods having leads to the populated design of the fossil species. The variables used in such methods are linear dimensions, angles, surfaces and ratio or combination of dimensions. But, these biometric descriptors are insufficiently informative since they give only one approximate quantitative representation of the form and its changes [1], [2], [3], [4]. Then we used the Fourier analysis which consists in approximating the shape by a goniometrical function defined by a sum of terms of sine and cosine [5], [6], [7], [8], [9], [10]. This function is broken up into a series of amplitude of harmonics and phases or into a series of coefficient of Fourier being useful like variables for the quantitative analyses. But this method is valid right for the forms regular, indeed when morphologies become complex, it is not more possible to use such descriptors. In this paper, we propose a new method to identify Brachiopods (Figure 1) based on the Radon transform [11], [12], [13], [14]. We use an adaptation of Radon transform called R-transform [15], [16], [17]. To improve the uniqueness of the approach, a binary shape is projected into the Radon space for different levels of the Chamfer distance transform. The advantages of the proposed method are robustness to noise, and invariant to common geometrical transformations scale, translation and rotation. This article is organized in the following way: In the first section we defined the concept or the Radon transform. The R-transform and 2D R-transform are described in section 3. The measure of similarity between two shapes (Brachiopod) is defined in section four.

Figure 1. Images of Brachiopod families

- Cyclothyris vespertilio
- Anathyry exquerai
- Cheirothyris fleuriausa
2 Radon transform

The Radon transform of an image \( f(x, y) \) is determined by a set of projections of the image along lines taken at different angles \( \theta \). The resulting projection is the sum of the intensities of the pixels in each direction. The result is a new image \( R_f(\rho, \theta) \). Therefore, the Radon transform can be written as [18]:

\[
R_f(\rho, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \delta(\rho - x \cos(\theta) - y \sin(\theta)) dx dy
\]

Where \( \delta(.) \) is the Dirac delta-function \( \delta(t) = 1 \) if \( t = 0 \) and 0 elsewhere, \( \theta \in [0, \pi] \) and \( \rho \in [-\infty, +\infty] \). In other words, \( R_f \) is the integral of \( f \) over the line \( L_{(\rho, \theta)} \) defined by \( \rho = x \cos(\theta) + y \sin(\theta) \). In shape recognition we consider \( f \) as a binary image that is:

\[
f(x, y) = \begin{cases} 
1 & \text{if } (x, y) \in D \\
0 & \text{otherwise}
\end{cases}
\]

Where \( D \) is the domain of binary shape as shown in the figure (2):

![Figure 2. Radon transform](image)

There are two distinct Radon transforms. The source can either be a single point or it can be an array of sources [19] as shown in (Figure 3). The method discussed in this paper uses an array of sources.
The source and sensor are rotated about the center of the object. For each angle \( \theta \) the density of the pixel the ray from the source passes through is accumulated at the sensor. This is repeated for a given set of angles, usually from \( \theta \in [0, 180] \). The angel 180 is not included since the result would be identical to the angel 0. The equation of the summation line is given as \( y = ax + b \). As can be seen by using trigonometry, the inclination is \( a = -\cos(\theta) / \sin(\theta) \) and the intersection with the y axis is \( b = \rho / \sin(\theta) \). These parameters are determined for each combination of \( \theta \) and \( \rho \). In order to reduce the number of calculations necessary the maximum and minimum of either x or y are determined. \( \rho_{\text{max}} \) is the size of the diagonal of the image. So the algorithm is:

For k from 1 to 180
\[
\theta = \frac{k\pi}{180}, \quad \text{Compute} \quad a = -\cos(\theta) / \sin(\theta)
\]

For \( \rho \) from 1 to \( \rho_{\text{max}} \)
Compute \( b = \rho / \sin(\theta) \), Determine \( y_{\text{min}} \) and \( y_{\text{max}} \)
For y from \( y_{\text{min}} \) to \( y_{\text{max}} \)
\[
\text{Compute} \quad x = (y - b) / a ,
\]
\[
R_j(\rho, \theta)^+ = f(x, y)
\]
End
End
End

The value of x in the algorithm can be real, to resolve the problem we used a linear interpolation. Applied to the Radon transform algorithm used an array of sources on binary images we begin with the step of binarization of the real image of the brachiopod see Figure (4). The original image is converted to binary image using Otsu binarization algorithm [20]. Figure (5) shows the results obtained.
The figure (6) presents the sinogram of Radon transform used an array of sources. We compared our implementation with the result obtained using the Radon transform found in MATLAB as shown in figure (7). This is nearly identical, the difference is presumably due to using a different interpolation.

![Figure 4. Original image of Brachiopod](image1)

![Figure 5. Binary image of Brachiopod](image2)

![Figure 6. Result obtained with our implementation](image3)

![Figure 7. Result obtained with Radon transform found in MATLAB](image4)

### 3 R-transform

R-transform has been developed by Tabbone in [15], [16], [17]. Its principle is simple: It consists to do for a given value of theta (i.e. within the same of column the Radon matrix), the sum of squared elements. Can be written as:

$$R_T = \int_{-\infty}^{+\infty} R_f^2(\rho, \theta) d\rho$$

(3)
Where $R_f$ is the Radon transform of the function $f$. We applied here a Radon transform used an array of sources. The study of this R-transform allows us to define the following properties:

- **Periodicity:**

  \[ R_f (\theta \pm \pi) = \int_{-\infty}^{\infty} R_f^2 (\rho, \theta \pm \pi) d\rho = R_f (\theta) \tag{4} \]

  The R-transform is periodic and the period is $\pi$.

- **Rotation:**

  For a rotation of the shape by an angle $\theta_0$, the R-transform is:

  \[ \int_{-\infty}^{\infty} R_f^2 (\rho, \theta + \theta_0) d\rho = R_f (\theta + \theta_0) \tag{5} \]

  So a rotation of the image by an angle $\theta_0$ implies a translation of the R-transform of $\theta_0$.

- **Translation:**

  For a translation of vector $u_0 = (x_0, y_0)$ the R-transform is:

  \[ \int_{-\infty}^{\infty} R_f^2 ((\rho - x_0 \cos(\theta) - y_0 \sin(\theta)), \theta) d\rho = \int_{-\infty}^{\infty} R_f^2 (\nu, \theta) d\nu \tag{6} \]

  We set $\nu = \rho - x_0 \cos(\theta) - y_0 \sin(\theta)$

  The R-transform is invariant under a translation of $f$ by a vector $u_0 = (x_0, y_0)$

- **Scaling:**

  For a scaling $\alpha > 0$ the R-transform is:

  \[ \frac{1}{\alpha} \int_{-\infty}^{\infty} R_f^2 (\alpha \rho, \theta) d\rho = \frac{1}{\alpha} \int_{-\infty}^{\infty} R_f^2 (\nu, \theta) d\rho \tag{7} \]

  \[ = \frac{1}{\alpha^3} R_f (\theta) \]

  we set $\nu = \alpha \rho$

  A zoom (before or back) of $\alpha$ generates a zoom of $\alpha^3$ for R-transform.
The R-transform is invariant under translation and scaling but only a rotation of the shape implies a translation of the R-transform. For the normalization we used the element number of R-transform. We observed that the R-transform compressed the form, so one R-transform per shape is not efficient to describe the shape. To overcome this problem, we used 2D R-transform. Each shape is projected in the Radon space for different segmentation levels of the distance transform.

A distance transform of a binary image specifies the distance from each pixel to the nearest non-zero pixel. The result of the distance transform is a gray level image. There are different families of distance transformation see [21], [22] for more information. In our study we used the distance transform called Chamfer distance transform, is fast and simple to implement, and provides a good approximation of the Euclidean distance. To compute the Chamfer distance transform, two additive masks (see Figure 9) are applied in two passes on the image. In the forward pass the first mask starts in the upper left corner of the image moving from left
to right and from top to bottom. The opposite operations are performed for the application of the backward mask.

![Figure 9. The masks used for computing the Chamfer distance transform](image)

One the Chamfer distance transform is computed, we applied to distance image a successive thresholds at regular intervals, we obtained various cuts of the shape as show in figure (11). After for each cut the R-transform is computed, we got 2D R-transform see figure (12). The X-axis present the angle $\theta$ in the Radon transform and the Y-axis reports the number of cuts in the distance transform. This 2D R-transform is invariant under translation, scaling and rotation through Heritage of property from R-transform.

![Figure 10. Binary shape and his Chamfer distance transform](image)

![Figure 11. Ten different cuts of Chamfer distance](image)
4 Measure of distance

Each shape (Brachiopod) is described by 2D R-transform. For two given shapes S and S’ the Euclidean distance between their two Fourier feature vectors $X^S$ and $X^S'$ is selected in this paper:

$$X^S = \left( \frac{FR^0_S (1)}{FR^0_S (0)}, \frac{FR^\pi_S (1)}{FR^\pi_S (0)}, \frac{FR^m_S (1)}{FR^m_S (0)}, \frac{FR^m\pi_S (1)}{FR^m\pi_S (0)} \right)$$

$$X^S' = \left( \frac{FR^0_{S'} (1)}{FR^0_{S'} (0)}, \frac{FR^\pi_{S'} (1)}{FR^\pi_{S'} (0)}, \frac{FR^m_{S'} (1)}{FR^m_{S'} (0)}, \frac{FR^m\pi_{S'} (1)}{FR^m\pi_{S'} (0)} \right)$$

Where $FR$ is a one-dimensional Fourier transform applied to discrete R-transform of two shapes S and S’ and m is the number of cuts of the Chamfer distance transform. Obviously, similar shapes will have a smaller distance while bigger values mean shape S and S’ differ.

5 Conclusion

In this work we presented an approach for classification of the Brachiopods based on the Radon transform. We used an adaptation of Radon transform called R-transform, to improve the uniqueness of the approach, a binary shape is projected into the Radon space for different levels of the Chamfer distance transform the proposed approach is invariant to common
geometrical transformations. The proofs of transformation invariance including translation, scaling and rotation are provided. Our method gives satisfactory and encouraging results.

References


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