The Influence of Hydrodynamics on the Spread of Pollutants in the Confluence of two Rivers

Basuki Widodo

Mathematics Department
Institut Teknologi Sepuluh Nopember
Surabaya-Indonesia
b_widodo@matematika.its.ac.id

Abstract

In a river system, confluence is a very important issue. This is due to the confluence can affect the morphology and hydraulics in upstream and downstream. The aim of this study is to determine the effect of hydrodynamics on the spread of pollutants in the confluence of two rivers. The model is built based on partial differential equations in two-dimensional depth-averaged surface-water flow and the Continuum Principle. Furthermore it is solved using an implicit finite difference methods namely Alternating Direction Implicit (ADI) method because it has the accuracy and numerical stability relatively good. These results are then simulated using MATLAB 7.1 software. The Parameter used are COD, BOD, DO, discharge and velocity water flow obtained from The public company of Jasa Tirta I. Running the model performed on location Karangpilang through Sepanjang bridge, with discharge from the paper industry PT. S.Wijaya and oil industry PT Sarimas Permai and stream flow velocity 0.5 m / sec at a distance of 15 m from the confluence of two rivers. The results showed that the spread of pollutants followed in the longitudinal direction and the spread of pollutant was changed in the value of the velocity and concentration because of differences in location (before and after confluence).

Keywords: Hydrodynamics, Spread Of Pollutants, Confluence, ADI method
1. Introduction

River has a very important role in human life. Therefore, to maintain the water quality is required monitoring in order to not be affected by pollutant. One of the serious problems that need to be considered is the pollution caused by industrial waste, because it is potentially became pollution in the water bodies. In general, Measurement of water quality in water bodies not relate hydrodynamics elements which can influence on the pattern of the spread of pollutants in the river (Karnaningroem, 2006; Widodo, 2012a; widodo, 2012b, Wibowo and Widodo, 2013; Hanif and Widodo, 2012), such as debit and velocity. Furthermore, to facilitate the monitoring of water quality and to identify the spread of pollutants, so the mathematical modeling is required, which it is built based on partial differential equations. This research was conducted in the form of numerical simulations on the computer and it is assumed that stream line is uniform and incompressible.

2. Mathematical Model

The basic equation is needed to explain the motion in the river is the continuity equation (conservation of mass) and conservation of momentum. The Profile of the river that will be examined are as follows:

![Figure 1: Location Map](image)

Below is a sketch of the profile along the river above and its control volume. Seen that the image resembles the shape of the Shazy-Shabayek’s river (Shabayek, 2002; Widodo and Siing, 2011):

![Figure 2. (a) The confluence of two river and (b) The control volume](image)
The basic equation that is needed to explain the motion in the river is the continuity equation (conservation of mass) and conservation of momentum.

2.1 Governing Equation

Governing equation of two-dimensional depth-averaged surface-water flow is as follows:

**a. Main Stream:**

**Continuity equation of flow:**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

where

\[ u = \text{velocity in x direction (m/sec)} \]
\[ v = \text{velocity in y direction (m/sec)} \]

**X-Momentum equation:**

\[
\rho \left[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2}
\]

**Y-Momentum equation:**

\[
\rho \left[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}
\]

where

\[ \rho = \text{density of water (kg/m}^3) \]
\[ p = \text{pressure (kg/m s}^2) \]
\[ \mu = \text{dynamic viscosity (kg/s m)} \]

**b. Lateral Stream:**

**Continuity equation of flow:**

\[
\frac{\partial u\cos \theta}{\partial x} + \frac{\partial v\cos \theta}{\partial y} = 0
\]

**X-Momentum equation:**

\[
\rho \left[ \frac{\partial u\cos \theta}{\partial t} + u \frac{\partial u\cos \theta}{\partial x} + v \frac{\partial u\cos \theta}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u\cos \theta}{\partial x^2} + \mu \frac{\partial^2 u\cos \theta}{\partial y^2}
\]

**Y-Momentum equation:**

\[
\rho \left[ \frac{\partial v\cos \theta}{\partial t} + u \frac{\partial v\cos \theta}{\partial x} + v \frac{\partial v\cos \theta}{\partial y} \right] = -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v\cos \theta}{\partial x^2} + \mu \frac{\partial^2 v\cos \theta}{\partial y^2}
\]
To calculate the amount of concentration that is diffused by a fluid flow, in this paper is chemical pollutants, obtained from the scalar transport equation (Apsley, 2013) are as follows:

a. **Laminar Flow**

If the Reynolds number \( Re < 2300 \)

\[
\frac{\partial C}{\partial t} = -u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} - R
\]

(7)

where

\( C \) = concentration (mg/liter)

\( R \) = transformation speed of reaction (kg/ m\(^3\) s)

b. **Turbulent Flow**

If the Reynolds number \( Re > 2300 \)

\[
\frac{\partial C}{\partial t} = -u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} + E_x \frac{\partial^2 C}{\partial x^2} + E_y \frac{\partial^2 C}{\partial y^2} - R
\]

(8)

where

\( E_x \) = coefficient diffusivity in x direction

\( E_y \) = coefficient diffusivity in y direction

3. **Numerical Method**

The conservation of momentum and scalar transport models are applied to calculate the flow fields and concentration distributions of the pollutants injected into the river. Furthermore, to get the numerical solution is used an implicit finite difference method namely alternating direct implicit method (ADI) based on Lam (1994): .

a. **Water Flow of Main Stream**

X-Momentum equation:

Discretization in the first step is:

\[
\rho \left[ \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} \right] + u_{i,j}^{n} \left( \frac{u_{i+1,j}^{n+1} - u_{i-1,j}^{n+1}}{2(\Delta x)} \right) + v_{i,j}^{n} \left( \frac{u_{i,j+1}^{n+1} - u_{i,j-1}^{n+1}}{2(\Delta y)} \right)
\]

\[
= -p_{i+1,j}^{n+1} + p_{i-1,j}^{n+1} + \mu \left( \frac{u_{i+1,j}^{n+1} - 2u_{i,j}^{n+1} + u_{i-1,j}^{n+1}}{\Delta x^2} \right) + \mu \left( \frac{u_{i,j+1}^{n+1} - 2u_{i,j}^{n+1} + u_{i,j-1}^{n+1}}{\Delta y^2} \right), \quad n = 0, 2, ...
\]

(9)

Discretization in the second step is:
\[ \rho \left[ \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} \right] + u_{i,j}^n \left( \frac{u_{i+1,j}^{n} - u_{i-1,j}^n}{2(\Delta x)} + v_{i,j}^n \left( \frac{u_{i,j+1}^{n+1} - u_{i,j-1}^n}{2(\Delta y)} \right) \right) \\
= -p_{t,j}^{n+1} + \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j-1}^n}{\Delta x^2} + \mu \left( \frac{u_{i,j+1}^n + 2u_{i,j}^n + u_{i,j-1}^n}{\Delta y^2} \right) + \frac{2(\Delta y)}{\Delta x} \right), n = 1,3, \ldots \quad (10) \]

**Y-Momentum equation:**

Discretization in the first step is:
\[ \rho \left[ \frac{v_{i,j}^{n+1} - v_{i,j}^n}{\Delta t} \right] + u_{i,j}^n \left( \frac{v_{i+1,j}^{n+1} - v_{i-1,j}^n}{2(\Delta x)} + v_{i,j}^n \left( \frac{v_{i,j+1}^{n+1} - v_{i,j-1}^n}{2(\Delta y)} \right) \right) \\
= -p_{t,j}^{n+1} + \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{\Delta x^2} + \mu \left( \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{\Delta y^2} \right), n = 0,2, \ldots \quad (11) \]

Discretization in the second step is:
\[ \rho \left[ \frac{v_{i,j}^{n+1} - v_{i,j}^n}{\Delta t} \right] + u_{i,j}^n \left( \frac{v_{i+1,j}^{n+1} - v_{i-1,j}^n}{2(\Delta x)} + v_{i,j}^n \left( \frac{v_{i,j+1}^{n+1} - v_{i,j-1}^n}{2(\Delta y)} \right) \right) \\
= -p_{t,j}^{n+1} + \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{\Delta x^2} + \mu \left( \frac{v_{i,j+1}^n + 2v_{i,j}^n + v_{i,j-1}^n}{\Delta y^2} \right), n = 1,3, \ldots \quad (12) \]

**b. Water Flow of Lateral Stream**

**X-Momentum equation:**

Discretization in the first step is:
\[ \rho \left[ \frac{u_{i,j}^{n+1} - u_{i,j}^n}{\Delta t} \right] \cos \theta + u_{i,j}^n \cos^2 \theta \left( \frac{u_{i+1,j}^{n+1} - u_{i+1,j-1}^n}{2(\Delta x)} \right) \\
+ v_{i,j}^n \cos^2 \theta \left( \frac{u_{i,j+1}^{n+1} - u_{i,j-1}^n}{2(\Delta y)} \right) \\
= -p_{i,j}^{n+1} + \frac{u_{i,j}^n + 2u_{i,j}^n + u_{i,j}^n}{\Delta x^2} + \mu \cos \theta \left( \frac{u_{i+1,j}^n - 2u_{i,j}^n + u_{i,j}^n}{\Delta x^2} \right) \\
+ \mu \cos \theta \left( \frac{u_{i,j+1}^n - 2u_{i,j}^n + u_{i,j}^n}{\Delta y^2} \right), n = 0,2, \ldots \quad (13) \]

Discretization in the second step is:
\[
\rho \left[ \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} \right] \cos \theta + u_{i,j}^{n} \cos^{2}\theta \left( \frac{u_{i+1,j}^{n} - u_{i-1,j}^{n}}{2(\Delta x)} \right) \\
+ v_{i,j}^{n} \cos^{2}\theta \left( \frac{v_{i,j+1}^{n} - v_{i,j-1}^{n}}{2(\Delta y)} \right) \\
= -\frac{p_{i,j+1}^{n} + p_{i,j-1}^{n}}{2(\Delta y)} + \mu \cos\theta \left( \frac{v_{i+1,j}^{n} - 2v_{i,j}^{n} + v_{i-1,j}^{n}}{\Delta x^2} \right) \\
+ \mu \cos\theta \left( \frac{v_{i+1,j}^{n} - 2v_{i,j}^{n} + v_{i-1,j}^{n}}{\Delta y^2} \right), \quad n = 0, 2, \ldots \quad (15)
\]

Discretization in the second step is:
\[
\rho \left[ \frac{v_{i,j}^{n+1} - v_{i,j}^{n}}{\Delta t} \right] \cos \theta + u_{i,j}^{n} \cos^{2}\theta \left( \frac{v_{i+1,j}^{n} - v_{i-1,j}^{n}}{2(\Delta x)} \right) \\
+ v_{i,j}^{n} \cos^{2}\theta \left( \frac{v_{i,j+1}^{n} - v_{i,j-1}^{n}}{2(\Delta y)} \right) \\
= -\frac{p_{i+1,j}^{n} + p_{i-1,j}^{n}}{2(\Delta y)} + \mu \cos\theta \left( \frac{v_{i+1,j}^{n} - 2v_{i,j}^{n} + v_{i-1,j}^{n}}{\Delta x^2} \right) \\
+ \mu \cos\theta \left( \frac{v_{i+1,j}^{n} - 2v_{i,j}^{n} + v_{i-1,j}^{n}}{\Delta y^2} \right), \quad n = 1, 3, \ldots \quad (16)
\]

### c. Pollutants Transport Equation

#### Laminar Flow

Discretization in the first step is:
\[
\left( \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} \right) = -u_{i,j}^{n} \left( \frac{C_{i+1,j}^{n+1} - C_{i,j}^{n+1}}{2(\Delta x)} \right) - v_{i,j}^{n} \left( \frac{C_{i,j+1}^{n+1} - C_{i,j}^{n+1}}{2(\Delta y)} \right) - R , \quad n \\
= 0, 2, \ldots \quad (17)
\]

Discretization in the second step is:
\[
\left( \frac{C_{i,j}^{n+1} - C_{i,j}^{n}}{\Delta t} \right) = -u_{i,j}^{n} \left( \frac{C_{i+1,j}^{n+1} - C_{i,j}^{n+1}}{2(\Delta x)} \right) - v_{i,j}^{n} \left( \frac{C_{i,j+1}^{n+1} - C_{i,j}^{n+1}}{2(\Delta y)} \right) - R , \quad n \\
= 1, 3, \ldots \quad (18)
\]
Turbulent Flow
Discretization in the first step is:
\[
\frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t} = -u_{i,j}^n \left( \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^n}{2(\Delta x)} \right) \\
- v_{i,j}^n \left( \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^n}{2(\Delta y)} \right) \\
- E_x \left( \frac{C_{i+1,j}^{n+1} - 2C_{i,j}^{n+1} + C_{i-1,j}^n}{\Delta x^2} \right) \\
- E_y \left( \frac{C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^n}{\Delta y^2} \right) - R, \quad n = 0, 2, ...
\]  

Discretization in the second step is:
\[
\frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t} = -u_{i,j}^n \left( \frac{C_{i+1,j}^{n+1} - C_{i-1,j}^n}{2(\Delta x)} \right) \\
- v_{i,j}^n \left( \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^n}{2(\Delta y)} \right) \\
- E_x \left( \frac{C_{i+1,j}^{n+1} - 2C_{i,j}^{n+1} + C_{i-1,j}^n}{\Delta x^2} \right) \\
- E_y \left( \frac{C_{i,j+1}^{n+1} - 2C_{i,j}^{n+1} + C_{i,j-1}^n}{\Delta y^2} \right) - R, \quad n = 1, 3, ...
\]  

4. Numerical Result and Discussion
To get the dispersion pattern of pollutants in the river, it must first known the characteristics of pollutants, in order to predict how the behavior and spread in the river easily. There are two types of the pollutants, the conservative and non-conservative. Conservative pollutants are the pollutants that do not dissolve and transform into other substances when mixed with water while non conservative pollutants are otherwise. In this study, discussed about the non-conservative pollutants and parameters representing is COD, BOD, and DO.

The simulations were performed just on the main channel and taken from the part before the confluence (upstream) until the confluence of two rivers (downstream). The domain is selected about 45 m from the upstream to downstream and 45 m wide with a speed of 0.5 m/s and it is consisted of a cell with size of dx(longitudinal direction) = 5 m and dy(lateral direction) = 5 m.

The simulation results as Figure 1 and Figure 2 below:
It appears from the simulation result that the dispersion of pollutants at the beginning is high then spread following the longitudinal direction (X axis) and after the 15th grid (confluence of two river), it is smaller.

From the second simulation, it can be seen that the concentration of pollution at the beginning (before confluence) is high then after the confluence (the 15th grid) it is relatively constant because the effect of the velocity in X axis and Y axis. The greater velocity makes the concentration is smaller.
5. Conclusion

From the simulation results it can be seen that the spread of pollutants followed in the direction of the dominant stream elongated along the river and the spread of pollutant was changed in the value of the velocity and concentration because of differences in location. Concentration of pollutants is smaller in the confluence and the Velocity is greater.

References


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