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Ratio Edge Antimagic Labeling for Subdivision Graphs

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Abstract

Recently J. Jayapriya and K. Thirusangu introduced max-min edge antimagic labeling and exhibited the same for path, cycles, star-$K_{1,n}$, P^+_n and C^+_n. In this paper it has been renamed as ratio edge antimagic labeling and extended the work on subdivision graphs.

Keywords: graph, function, labeling, antimagic, subdivision.

1 Introduction and Preliminaries

The graph labeling problem that appears in graph theory has a fast development recently. This problem was first introduced by Alex Rosa in 1967. Since Rosa's article, many different types of graph labeling problems have been defined around this. This is not only due to its mathematical importance but also because of the wide range of the applications arising from this area, for instance, x-rays, crystallography, coding theory, radar, astronomy, circuit design, and design of good Radar Type Codes, Missile Guidance Codes and Convolution Codes with optimal autocorrelation properties and communication design. An enormous body of literature has grown around the subject in about 1500 papers. They gave birth to families of graphs with attractive names such as graceful, Harmonious, felicitous, elegant, cordial, magic antimagic, bimagic and prime labeling etc. A useful survey to know about the numerous graph labeling methods is the one by J.A. Gallian recently [2012][3].

Definition 1.1: A graph G (p, q) is said to be (1,1) edge-magic with the common edge count k_0 if there exists a bijection f : V(G) ∪ E(G) → {1,2,…, p + q} such that f (u) + f (v) + f (e) = k_0 for all e = (u, v) ∈ E (G). It is said to be (1, 1) edge-antimagic if f (u) + f (v) + f (e) are distinct for all e = (u, v) ∈ E (G).

Definition 1.2: A graph G (p, q) is said to be (1,1) vertex-magic with the common vertex count k1 if there exists a bijection f : V (G) ∪ E (G) → {1,2,…,p + q} such that for each u ∈V (G), f (u) + $\sum_e f(e) = k_1$ for all e = (u, v) ∈ E (G) with v ∈V (G). It is said to be (1,1) vertex-antimagic if f (u)+$\sum_e$ f (e) are distinct for all e = (u, v) ∈ E (G) with v ∈V (G).

Definition 1.3: A graph G (p, q) is said to be (1,0) edge-magic with the common edge count k_2 if there exists a bijection f : V (G) → {1,2,…,p} such that for all e = (u, v) ∈ E (G), f (u) + f (v) = k_2. It is said to be (1,0) edge-antimagic if for all e = (u,v) ∈ E (G), f (u) + f (v) are distinct.

Definition 1.4: A graph G (p, q) is said to be (1,0) vertex-magic with the common vertex count k_3 if there exists a bijection f : V (G) → {1,2,…,p} such that for each u ∈ V (G), f (u) + f (v) = k_3 for all v ∈ V (G) such that (u, v) ∈ E(G) . It is said to be (1, 0) vertex-antimagic if for each u ∈ V (G), f (u) + f (v) are distinct for all v ∈V (G) such that (u, v) ∈ E (G).

Definition 1.5: A graph G (p, q) is said to be (0, 1) vertex-magic with the common vertex count k_4 if there exists a bijection f : E (G) → {1,2,…,q} such that for each u ∈ V (G), $\sum_e f(e) = k_4$ for all e = (u, v) ∈ E (G) with v ∈ V (G) . It is said to be (0, 1) vertex-antimagic if for each u ∈V (G), $\sum_e f(e)$ are distinct for all e = (u, v) ∈ E (G) with v ∈ V (G).

Definition 1.6: A graph G (p, q) is said to be (0,1) edge-magic with the common edge count k_5 if there exists a bijection f : E (G) → {1,2,…,q} such that for each

$e \in E(G)$, $f(e) + f(e_0) = k_5$ for all $e_0 \in E(G)$ such that e and e_0 are adjacent in G . If for each $e \in E(G)$, $f(e) + f(e_0)$ are distinct for all $e_0 \in E(G)$ such that e and e_0 are adjacent in G , then G is said to be (0,1) edge-antimagic.

Above definitions 1.1 to 1.6 are introduced and studied by V. Yegnanarayanan in [7]. In [5] J. Jayapriya and K. Thirusangu introduced a new labeling max-min edge antimagic labeling. As this is the ratio between the maximum and minimum of the labels of the end vertices of the edge, here after we call the max-min labeling as ratio labeling. Also it has been shown that path, cycles, star-$K_{1,n}$, C^+_n and P^+_n are admitting max-min edge antimagic labeling.

We now redefine the labeling bellow,

Definition 1.7: Let G (V, E) be a simple graph with p vertices and q edges. A bijection $f : V(G) \to \{1,2,\ldots p\}$, is said to be ratio edge antimagic labeling if for every edge uv in E, the real valued weight $\lambda(uv) = \dfrac{\max\{f(u), f(v)\}}{\min\{f(u), f(v)\}}$ is distinct.

Definition 1.8: A subdivision of a graph is a graph that can be obtained from G by a sequence of edge Subdivision. By once subdivision means including a vertex to each edge.

Definition 1.9: A Y - tree is a graph obtained by appending an edge to the vertex adjacent to one of the pendent vertices of the path P_n.

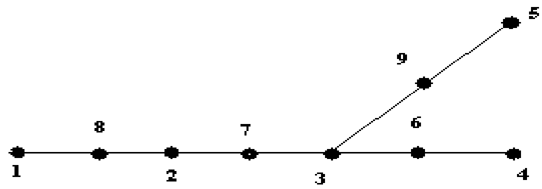

Fig 1. Subdivision of Y_5-tree

2 Main Results

Theorem 2.1: The graph obtained by once subdivision of the edges of Y-tree admits ratio edge antimagic labeling.

Proof: The Y-tree graph has n vertices and on subdivision it has 2n-1 vertices. Let the vertices be $V = \{v_1, v_2,\ldots, v_{n-1}\} \cup \{u\} \cup \{ v^1_1, v^1_2, \ldots, v^1_{n-1}\}$.
Let $f : V \to \{1,2,\ldots 2n-1\}$ such that $f(v_i) = i$; $1 \le i \le n-1$, $f(u) = n$, $f(v^1_i) = (2n-1)-i$; $1 \le i \le n-2$ and $f(v^1_{n-1}) = 2n-1$ the edge set $E = \{v_i v^1_i;\ 1 \le i \le n-2\} \cup \{ v^1_i v_{i+1};\ 1 \le i \le n-2\} \cup \{v_{n-2} v^1_{n-1}\} \cup \{v^1_{n-1}u\}$.

For $1 \le i \le n-2$, $\lambda (v_i v^1_i) = \frac{\max\{f(v_i), f(v^1_i)\}}{\min\{f(v_i), f(v^1_i)\}} = \frac{(2n-1)-i}{i}$.

For $1 \le i, j \le n-2$, when $i \neq j$, clearly $\lambda(v_i v^1_i) \neq \lambda (v_j v^1_j)$.

If $\lambda (v_i v^1_i) = \lambda (v_j v^1_j)$, then this implies $2n = 1$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n-2$, $\lambda (v^1_i v_{i+1}) = \frac{\max\{f(v^1_i), f(v_{i+1})\}}{\min\{f(v^1_i) f(v_{i+1})\}} = \frac{(2n-1)-i}{i+1}$.

For $1 \le i, j \le n-2$, $i \neq j$, clearly, $\lambda (v^1_i v_{i+1}) \neq \lambda (v^1_j v_{j+1})$.

If $\lambda (v^1_i v_{i+1}) = \lambda (v^1_j v_{j+1})$, then this implies $i = j$, which is a contradiction therefore all edge labels are distinct.

Also $\lambda (v^1_{n-1}\ u) = \frac{(2n-1)}{i}$. Thus all edge labels are distinct. Hence Graph obtained by Subdivision of the edges of Y-tree admits ratio edge antimagic.

Theorem 2.2: The graph obtained by once subdivision of the edges of ladder graph $P_n \times P_2$ admits ratio edge antimagic labeling.

Proof: The ladder graph has 2n vertices and on subdivision it has 5n vertices. Let the vertices be $V = \{v_1, v_2, \ldots, v_n\} \cup \{u_1, u_2, \ldots, u_n\} \cup \{v^1_1, v^1_2, \ldots, v^1_n\} \cup \{u^1_1, u^1_2, \ldots, u^1_n\} \cup \{w_1, w_2, \ldots, w_n\}$. Let $f : V \rightarrow \{1, 2, \ldots, 5n-2\}$ such that $f(v_i) = 2i-1$; $1 \le i \le n$, $f(u_i) = 2i$; $1 \le i \le n$, $f(v^1_i) = 2(n+i)$; $1 \le i \le n-1$;
$f(u^1_i) = 2n+(2i-1)$; $1 \le i \le n-1$ and $f(w_i) = (4n-2)-i$; $1 \le i \le n$.

Let $E = \{v_i v^1_i ; 1 \le i \le n\} \cup \{v^1_i v_{i+1}; 1 \le i \le n-1\} \cup \{u_i u^1_i; 1 \le i \le n\} \cup \{u^1_i u_{i+1}; 1 \le i \le n-1\} \cup \{v_i w_i; 1 \le i \le n\} \cup \{w_i u_i ; 1 \le i \le n\}$.

For $1 \le i \le n-1$, $\lambda(v_i v^1_i) = \frac{\max\{f(v_i), f(v^1_i)\}}{\min\{f(v_i), f(v^1_i)\}} = \frac{2(n+i)}{2i+1}$.

For $1 \le i, j \le n-1$, when $i \neq j$, $\lambda (v_i v^1_i) \neq \lambda (v_j v^1_j)$.

If $\lambda (v_i v^1_i) = \lambda (v_j v^1_j)$, then this implies $2n = -1$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n-1$, $\lambda (v^1_i v_{i+1}) = \frac{\max\{f(v^1_i), f(v_{i+1})\}}{\min\{f(v^1_i) f(v_{i+1})\}} = \frac{2(n+i)}{2i+1}$.

For $1 \le i, j \le n-1$, $i \neq j$, $\lambda (v_{1i} v_{i+1}) \neq \lambda (v^1_j v_{j+1})$.

If $\lambda (v^1_i v_{i+1}) = \lambda (v^1_j v_{j+1})$, then this implies $2n = 1$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n-1$, $\lambda (u_i u^1_i) = \frac{\max\{f(u_i), f(u^1_i)\}}{\min\{f(u_i), f(u^1_i)\}} = \frac{2n+(2i-1)}{2i}$.

For $1 \le i, j \le n$, when $i \neq j$, clearly $\lambda (u_i u^1_i) \neq \lambda (u_j u^1_j)$.

If $\lambda (u_i u^1_i) = \lambda (u_j u^1_j)$, then this implies $2n = 1$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n-1$, $\lambda (u^1_i u_{i+1}) = \dfrac{\max\{f(u_{i+1}), f(u^1_i)\}}{\min\{f(u_{i+1}), f(u^1_i)\}} = \dfrac{2n+(2i-1)}{2i+2}$.

For $1 \le i, j \le n-1$, $i \ne j$, $\lambda (u^1_i u_{i+1}) \ne \lambda (u^1_j u_{j+1})$.

If $\lambda (u^1_i u_{i+1}) = \lambda (u^1_j u_{j+1})$, then this implies $2n = 3$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n$, $\lambda (v_i w_i) = \dfrac{\max\{f(v_i), f(w_i)\}}{\min\{f(v_i), f(w_i)\}} = \dfrac{(4n-2)-i}{2i-1}$.

For $1 \le i, j \le n$, when $i \ne j$, clearly $\lambda (v_i w_i) \ne \lambda (v_j w_j)$.

If $\lambda (v_i w_i) = \lambda (v_j w_j)$ then this implies $8n = -5$, which is a contradiction therefore all edge labels are distinct.

For $1 \le i \le n$, $\lambda (w_i u_i) = \dfrac{\max\{f(w_i), f(u_i)\}}{\min\{f(w_i), f(u_i)\}} = \dfrac{(4n-2)-i}{2i}$.

For $1 \le i, j \le n$, when $i \ne j$, $\lambda (v_i u_i) \ne \lambda (v_j u_j)$.

If $\lambda (w_i u_i) = \lambda (w_i u_i)$ then this implies $2n = 1$, which is a contradiction therefore all edge labels are distinct. Hence graph obtained by subdivision of the edges of ladder admits ratio edge antimagic.

Theorem 2.3: If G is ratio edge antimagic then G^+ also admits ratio edge antimagic labeling.

Proof: Let G(p, q) be a Ratio edge Antimagic labeling graph. Let $G^+(V^1, E^1)$ be a graph obtained by adding pendent edge to each vertices of G .The graph G^+ has 2p vertices and p+q edges. Let $f : V^1 \to \{1, 2, \ldots, 2p\}$ such that

$f (v^1_i) = \{p+ i: 1 \le i \le p\}$.

For $1 \le i \le p$, each edge $v_i v^1_i \in E_1$. The weight of each edge is calculated as follows

$$\lambda (v_i v^1_i) = \frac{\max\{f(v_i), f(v^1_i)\}}{\min\{f(v_i), f(v^1_i)\}} = \frac{(p+i)}{i}.$$

We have for i = 1,…, p the edge set as

$$\left\{1+\frac{p}{1}, 1+\frac{p}{2}, 1+\frac{p}{3}, \ldots, 1+\frac{p}{P}\right\} = \left\{1+1, 1+\frac{p}{p-1}, 1+\frac{p}{p-2}, 1+\frac{p}{p-3}, \ldots, 1+\frac{p}{1}\right\}$$

$$= \left\{2, 2+\frac{1}{p-1}, 2+\frac{2}{p-2}, 2+\frac{3}{p-3}, \ldots, 2+\frac{p}{1}\right\}.$$

The range of edge weights lie in [2, p+1]. Clearly all edge labels are distinct. Hence if G admits ratio edge antimagic then G^+ is also ratio edge antimagic.

Theorem 2.4: If G admits ratio edge antimagic then the graph G^1 obtained upon once subdivision of G also admits ratio edge antimagic labeling.

Proof: Let G (p, q) be a Ratio edge Antimagic Labeling graph. Let G^1 be the graph obtained on once subdivision of each of edge of G. The graph $G^1(V^1, E^1)$ has p+q vertices and 2q edges. Let x_{ij} be the vertices on subdivision of vertices v_i and v_j. Let $f : V^1 \rightarrow \{1,2,\ldots,p+q\}$ such that $f(x_{ij}) \in \{p+1, p+2, \ldots, p+q\}$. The edge weight is calculated as follows.

For, $1 \leq i,j \leq p+q$,

$$\lambda(v_i x_{ij}) = \frac{\max\{f(v_i), f(x_{ij})\}}{\min\{f(v_i), f(x_{ij})\}} = \frac{f(x_{ij})}{f(v_j)} = \frac{(p+i)}{f(v_i)} \text{ and}$$

$$\lambda(v_j x_{ij}) = \frac{\max\{f(v_j), f(x_{ij})\}}{\min\{f(v_j), f(x_{ij})\}} = \frac{f(x_{ij})}{f(v_j)} = \frac{p+j}{f(v_j)}.$$

Clearly $\lambda(v_i x_{ij}) \neq \lambda(v_j x_{ij})$ if $\lambda(v_i x_{ij}) = \lambda(v_j x_{ij})$ then

$$\frac{f(x_{ij})}{f(v_i)} = \frac{f(x_{ij})}{f(v_j)}, \text{ this implies } \frac{p+i}{f(v_i)} = \frac{p+j}{f(v_j)}.$$

Therefore, $\frac{p+i}{p+j} = \frac{f(v_i)}{f(v_j)}$ by using compendo divido method we have

$p = \left(\frac{f(v_i)+f(v_j)}{f(v_i)-f(v_j)}\right)\left(\frac{i+j}{2}\right)$, which leads to a contradiction. Clearly all edge labels are distinct. Hence if G^1 admits ratio edge antimagic labeling.

3 Conclusion

From Theorem 2.3 and 2.4 we conclude that the subdivision of path, cycles, star-$K_{1,n}$, P^+_n and C^+_n are ratio edge antimagic. We now state two open problems.

Open Problem 1: Any tree is ratio edge antimagic.

Open Problem 2: If G is (1, 0)-edge antimagic then it admits ratio edge antimagic labeling.

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