Verification of Mathematical Model of Development

Cooperation Programs between

Natural Monopoly and Regional Authorities

Sergey Vikharev

Ural Federal University
Office 607, Turgeneva str. 4, Ekaterinburg, Russia, 620075
sergey@viharev.com

Copyright © 2013 Sergey Vikharev. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Abstract

As it is shown in the [1], the interaction between the natural monopoly (next NM) and the region is expedient to be carried out on the basis of a joint development railway transport program in this region. The project configuration is an investment structure into ongoing project by stakeholders and, hence, this is a structure of incomes as consequences of project implementation. In this paper we tested a method agreement of cooperation programs. Target parameters are effectiveness ratio of the regional authorities and the NM.

Keywords: Developing a cooperation program, Mathematical modeling, Pareto efficiency.

1 Introduction

Participatory share of the parties in each project and efficiency and solvency depending on it, have to be reconciled by high-performance cooperation program configuration of a NM and certain subject of the RF. This is defined by the following parameters:
\[ x_i \] - the amount of funds allocated to the \( i \)-project NM, where \( i = 1, 2, \ldots, n \);

\[ y_i \] - the amount of funds allocated to the \( i \)-project be the regional administration, where \( i = 1, 2, \ldots, n \);

The overall effectiveness ratio of the NM program [1-3] in the case \((x, y)\) is:

\[ k(x, y) = \frac{D(x, y)}{D(x, y) + \frac{1}{T} \sum_{i=1}^{T} \left( \sum_{j=1}^{T} (B_{\text{bank}}^j) \right) + \frac{1}{T} \sum_{i=1}^{T} \left( \sum_{j=s+1}^{\infty} (R_{\text{loss}}^j) \right)} \]

In the same way the ratios for the second party of interaction in the case \((x, y)\) – regional administration - are:

\[ \Delta W(x, y) = \sum_{i=1}^{n} (\alpha_1 M_{\text{reg}}^i + \alpha_2 V_{\text{reg}}^i + \alpha_3 B_{\text{reg}}^i + \alpha_4 N_{\text{reg}}^i + \alpha_5 S_{\text{reg}}^i + \alpha_6 L_{\text{reg}}^i + \alpha_7 \Phi_{\text{reg}}^i) \]

Now there is a problem to find a situation \((x, y)\), where the efficiency \( k(x, y) \) satisfies the NM, and the solvency change \( \Delta W(x, y) \) meets the regional administration’s requirements, i.e. the common cooperation program in the configuration \((x, y)\) can be accepted by both sides.

To solve the problem of searching the suitable situation \((x, y)\) the best variant is game theory. Its mathematical apparatus allows choosing according to some criteria variants from all possible number of variants that satisfy both parties according to formulated interests of parties-players.

The function named as the Rating of the situation \((x, y)\) for the regional administration \( R_n(x, y) \) is defined analogically.

Among all the situations \( z_k = (x^{(k)}, y^{(k)}) \), where \( k = 1, \ldots, m \), obtained during solving the linear programming problem, let’s choose the situations that are optimal according to Pareto criterion [4—8], i.e. the situations \( z^* = (x^*, y^*) \) where there are no more preferred for both players situations \( z = (x, y) \), i.e. such that:

\[ k(x^*, y^*) \leq k(x, y), \quad \Delta W(x^*, y^*) \leq \Delta W(x, y), \]
\[ R_0(x^*, y^*) \geq R_0(x, y), \quad R_n(x^*, y^*) \geq R_n(x, y). \]

Here \( R_0(x, y) \) is the rating of situation \((x, y)\) for the NM,

\( R_n(x, y) \) is the rating of situation \((x, y)\) for the regional administration.

According to the results of problem solving there is a set of Pareto efficiency situations \( z^* = (x^*, y^*) \)

By means of negotiations and discussion on the basis of calculations made above, from the limited set one can choose such situation, i.e. the configuration of the common program, that satisfies both parties by the effectiveness ratio and solvency, and, hence, by the income level, expenditure level and its influence for the socio-economic performance of the region.

2 Experiments

The proposed algorithm with a set of situations, acceptable for negotiations, provides objective arguments in favor of certain configurations of common cooperation program that allows simplifying and predetermining the process of negotiations about approval the program significantly. This paragraph is devoted to
the practical implementation of the developed in [1] algorithm of approval a cooperation program between PLC “Russian Railways” (next RR) and a subject of Russian Federation.

It is considered that a subject of Russian federation is interested in the following projects developing: The development of transport process and increase production at a “K” plant; The high-speed train to the airport; The building of the access road to the south terminal. And RR is interested in the following projects: The allocation of parts for rolling stock production in the region; The movement organization of a high-speed train to the airport. In the course of the algorithm operations and in the result of programs merger, the common program of interaction was developed. It contains:

A1. (Cost 70) The development of transport process and increase production at a “K” plant;
A2. (Cost 50) The high-speed train to the airport;
A3. (Cost 60) The allocation of parts for rolling stock production in the region;
A4. (Cost 300) The building of the access road to the south terminal.

Let’s consider carried out experiments. Let’s for the experiments \( \lambda_j = 1 \), \( p_j = 0,95 \).

<table>
<thead>
<tr>
<th>Part</th>
<th>Rate</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RR</td>
<td>Region</td>
</tr>
<tr>
<td>A1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>A2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>A3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>A4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Common experiment input data

**Experiment #1.**
For the algorithm operability illustration let’s illustrate the calculations in the case of absence opportunities to finance a cooperation program for both sides. As it was expected in the case of financial capability lack, the only possible scenario (it is Pareto efficiency) is located in the zero values.

**Experiment #2.**
Let’s consider the situation where one of the sides is ready to implement one of the projects by their own, while there is a lack of financing for all other projects (Table 2).
The results of the experiment are $\Delta W = 10736, k = 0,922$. There is the only one Pareto efficiency situation.

In this situation, indeed, there are only two possible versions: to finance or not to finance the fourth project. A computer in the course of solving the linear programming problem has identified the most optimal distribution of shares of funding, based on the obtaining effectiveness ratios and solvency. It is hard to imagine at random, i.e. without algorithm usage, that a responsible for negotiations employee of RR can identify the most advantageous ratio of 229.70 to 70.30 in financing of the fourth project (Fig.4).

<table>
<thead>
<tr>
<th>Experiment #2</th>
<th>Experiment #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>RR</td>
<td>Region</td>
</tr>
<tr>
<td>min</td>
<td>max</td>
</tr>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>300</td>
</tr>
</tbody>
</table>

Table 2. Experiments input data

Experiment #3.

The situation of excess funding for some projects is considered in this experiment, i.e. such projects which implementation requires a minimum investment amount (Table 2). The experimental results are shown in the Table 3.
The Pareto efficiency situations in plane efficiency-solvency are illustrated in the fig. 5. Meanings are especially close, because in fact the distribution of funds for the first and the third projects is fixed and it corresponds the minimal investments that parties are ready to make. The distribution of share of funding for each of five sceneries is shown in the Fig.6.

![Graphs of distribution of funding share](image)

**Fig.6.** The graphs of the distribution the funding share in the experiment #4 in respect of five Pareto efficiency scenarios

**Results**

The proposed algorithm of elaboration technique and approval a cooperation program has been tested on various baseline data and it has demonstrated its operability applying to different variants of interaction. The offered unimprovable configurations of cooperation program financing meet the theoretic-mathematical
requirements and they correspond the intuitive explanation about mutually beneficial interaction that is most appreciated. The algorithm of technique at boundary conditions has been checked. In the case when all parties have enough resources to implement projects, the meanings of Pareto efficiency situations are close enough. It corresponds to common sense (the available resources are enough for the all proposed projects). Variants of program financing, generated by the algorithm, are deliberated and well though-out because they are based on the proven interaction valuations (efficiency and solvency change). The detection of Pareto efficiency variants of possible situations is carried out on the basis of calculated forecasted efficiency of the all cooperation program.

References


Received: August 23, 2013