Mathematical Model of the Local Stability
of the Enterprise to its Vendors

II Research of the Node Stability

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Abstract

The paper presents a verification of the elementary model of interaction between the enterprise with suppliers. The influence of individual factors on the final evaluation of the external sustainability of the enterprise.

Keywords: supplier selection, supply chain management, risk management, supplier interaction, weighted sum.

1 Introduction

The method of calculation the $K^\text{out}_j$ factor is proposed and substantiated below [1-4], where $K^\text{out}_j$ is the instability factor of outside environment of the network model $G=(N, A)$ node $j$ using theory. Research in dependence on suppliers carried out for example in [5-7].

Let the node $P_0$ of the network model has $N$ adjacent vertices $P_{11}, P_{12}, \ldots, P_{1N}$, i.e. nodes located at a distance 1 from the node $P_0$.

Then $P_{11}, P_{12}, \ldots, P_{1N}$ enterprises make direct surrounding of the corporate network $P_0$ element, i.e. they are its suppliers.
2 The method of calculation the instability factor of a corporate network node

An elementary event $N (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N)$ will be named as critical if the sum of supplies by supplier-nodes side is less than the total demand of the node $P_0$:

$$\sum_{\varepsilon_j=1}^{\varepsilon_N} (V_{1j} + \Delta_{ij}) + \sum_{\varepsilon_j=0}^{\varepsilon_N} \left( V_{1j} \cdot \xi_{1j}(t) + 0 \cdot \Delta_{ij} \right) < A^m$$

Appearances of various $N (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_N)$ are independent events hence the required external environment instability factor is:

$$K_0^{\text{out}}(t) = \sum_{\text{critical}} N \left( \prod_{j=1}^{\varepsilon_N} \left( p_{1j}^{e_j} \cdot v_{1j}^{e_j} \right) \right)$$

Where the sum is taken over all possible critical $N$. It is emphasized one more time that the coefficient $K_0^{\text{out}}$ is a probability of appearance a critical situation with supplies of $A^m$ production.

The formula to the node $P_0$ external stability factor is:

$$p_0^{\text{out}}(t) = 1 - \sum_{\text{critical}} N \left( \prod_{j=1}^{\varepsilon_N} \left( p_{1j}^{e_j} \cdot v_{1j}^{e_j} \right) \right)$$

Analysis of this formula leads to conclusion that the stability of the node that we are interested in monotonically depends on the external environment stability and it is the higher the higher the integrated stability factor of the external environment is.

3 The research of the node stability factor in the corporate network structure and some regularities of its behavior

Let’s consider some examples of calculation the external stability factor of a corporate network certain node that are enough expository to identify regularities of defining stability factors behavior. All numerical experiments have been carried out according to the method described above.

The stability of one consumer-node will be considered in dependence of two factors: a) the statistically pre-defined stability of supplier-nodes and b) contract policy of a consumer-node (character and terms of contracts entered into supplier-nodes).

The resulting total external stability $p_0(t)$ is a random variable because the coefficients values of fulfilling a contract $\xi_{1j}(t)$ of adjacent nodes in each concrete situation are random. Therefore the mathematical expectation of a random variable $p_0(t)$ is taken as a true value of consumer-node stability coefficient. It corresponds to a sample mean

$$p_0^{\text{out}}(t) = \frac{1}{N_c} \sum_{i=1}^{N_c} p_0(i)$$

Where $N_c$ - the number of carried out series of fully tests (series of all possible elementary events exhaustive search).

The series of 30-40 tests, i.e. $N=40$, is enough to achieve 3-5% precision in defining the coefficient $p_0^{\text{out}}$. These considerations have formed a basis for
numerical experiments. The results of these experiments are presented below.

The following example is more complicated. There is one consumer-node and its total demand for production is 300 000 units. It has five peer supplier-nodes. The statistical stability each of them is equal, for instance $p_{1j} = 0.9$. The total demand for production of 300 000 units is distributed over the contracts among all suppliers equally – 60 000 of units per one supplier and possible increase of supplies specified in each contract is zero. (Table 3)

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Shipments</th>
<th>Reserve</th>
<th>Priority</th>
<th>Stability</th>
<th>Actual Shipments</th>
<th>Use Reserve</th>
<th>Total Shipments</th>
<th>Feasibility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vendor1</td>
<td>60000</td>
<td>0</td>
<td>6</td>
<td>0.9</td>
<td>51957.86</td>
<td>0</td>
<td>51957.86</td>
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<td>6</td>
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<td>0</td>
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<td>9</td>
<td>0.9</td>
<td>50812.42</td>
<td>0</td>
<td>50812.42</td>
<td>0.8469</td>
</tr>
<tr>
<td>Vendor4</td>
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<td>0.9</td>
<td>59768.42</td>
<td>0</td>
<td>59768.42</td>
<td>0.9961</td>
</tr>
<tr>
<td>Vendor5</td>
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<td>2</td>
<td>0.9</td>
<td>55020.24</td>
<td>0</td>
<td>55020.24</td>
<td>0.9170</td>
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**Parameters**

<table>
<thead>
<tr>
<th>Total supply</th>
<th>Shipments without reserve</th>
<th>272141.34</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Involved reserve</td>
<td>0.00</td>
</tr>
<tr>
<td>300000</td>
<td>Total shipments</td>
<td>272141.34</td>
</tr>
</tbody>
</table>

**Summary Stability** 0.59

Table 3. The network diagram “consumer-node – a set of supplier-nodes”

In this example the final stability of a consumer-node is 0.59 that is significantly lower than the stability of supplier-nodes. It can be explained quite easily: the more peer suppliers who are not connected with extra-commitments in the form of possible increase of suppliers for $\Delta_{ij}$ exist, the higher probability of failure at least one of them is. The total demand for 300 000 units is distributed over suppliers without any reserve hence there is a high probability of shortage in delivery.

This is an example of extremely unsuccessful contract policy of a consumer-node. One should discuss the possible increase of product delivery on $\Delta_{ij}$ to maintain the consumer-node stability at the level of supplier-nodes stability or more.

It is obvious that the increase of specified in contracts reserves of suppliers leads to increase of consumer-node stability. However, it is not hard to understand that it is connected with significant material consumer costs because a consumer-node is forced to bear costs for the product reservation that, in fact, is not always necessary.

Let’s finally consider a common example of calculation the consumer-node stability if there are several suppliers with various statistical stability factors, orders distribution and all kinds of reserves and ranking. Two intermediate scenarios of probable events (1) and (2) are shown in the Table 4 and 5
Table 4. The results of playback a scenario 1.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Shipments</th>
<th>Reserve</th>
<th>Priority</th>
<th>Stability</th>
<th>Actual Shipments</th>
<th>Use Reserve</th>
<th>Total Shipments</th>
<th>Feasibility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vendor1</td>
<td>100000</td>
<td>7000</td>
<td>2</td>
<td>0.92</td>
<td>100000.00</td>
<td>1811.54</td>
<td>101811.54</td>
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</tr>
<tr>
<td>Vendor2</td>
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<td>3000</td>
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<td>0</td>
<td>53506.77</td>
<td>0.8918</td>
</tr>
<tr>
<td>Vendor3</td>
<td>40000</td>
<td>0</td>
<td>9</td>
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<td>0</td>
<td>39681.69</td>
<td>0.9920</td>
</tr>
<tr>
<td>Vendor4</td>
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<td>6</td>
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<td>70000.00</td>
<td>4000.00</td>
<td>74000.00</td>
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</tr>
<tr>
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<td>1000</td>
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<td>0.92</td>
<td>30000.00</td>
<td>1000.00</td>
<td>31000.00</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5. The results of playback a scenario 2.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>Shipments</th>
<th>Reserve</th>
<th>Priority</th>
<th>Stability</th>
<th>Actual Shipments</th>
<th>Use Reserve</th>
<th>Total Shipments</th>
<th>Feasibility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vendor1</td>
<td>100000</td>
<td>7000</td>
<td>2</td>
<td>0.92</td>
<td>100000.00</td>
<td>7000</td>
<td>107000.00</td>
<td>1</td>
</tr>
<tr>
<td>Vendor2</td>
<td>60000</td>
<td>3000</td>
<td>8</td>
<td>0.9</td>
<td>55764.96</td>
<td>0</td>
<td>55764.96</td>
<td>0.9294</td>
</tr>
<tr>
<td>Vendor3</td>
<td>40000</td>
<td>0</td>
<td>9</td>
<td>0.95</td>
<td>40000.00</td>
<td>0</td>
<td>40000.00</td>
<td>1</td>
</tr>
<tr>
<td>Vendor4</td>
<td>70000</td>
<td>4000</td>
<td>6</td>
<td>0.87</td>
<td>67939.53</td>
<td>0</td>
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<tr>
<td>Vendor5</td>
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<td>1000</td>
<td>6</td>
<td>0.92</td>
<td>28995.83</td>
<td>0</td>
<td>28995.83</td>
<td>0.9665</td>
</tr>
</tbody>
</table>

And, finally, there is the total consumer-node stability in the given initial conditions after series of N=40 tests is equal 0,9567.

It has to be noted that according to the given and quite realistic initial conditions from a practical point of view, the final stability is quite acceptable \( \cdot p_0 = 0,9567 \) – and it corresponds reality intuitively. This is an excellent confirmation of the constructed mathematical model adequacy, of the surrounding environment accordance and of the correct realization of the developed simulation method.

Let’s playback the scenario of the node stability depending on the number of suppliers, their stability and supplies reserves. The Fig. 2 shows the graph of node stability changes depending on the number of supplier-nodes provided that there are no reserves.
Local stability of enterprise to its vendors

Increasing the number of supplier-nodes causes increasing the probability of failure at least one of them and decreasing of the consumer-node stability. However the consumer-node stability rises sharply with entering a reserve (Fig. 3, 4, 5, 6).

Fig. 2 The dependence of the consumer-node stability from the number of supplier-nodes with their stability 0.95, 0.9, 0.8, 0.7

Fig. 3 The dependence of the consumer-node stability from supply reserve and two two suppliers with their stability 0.95 and 0.9

Fig. 4. The dependence of the consumer-node stability from supply reserve and free suppliers with their stability 0.95, 0.9 and 0.8

Fig. 5. The dependence of the consumer-node stability from supply reserve and five suppliers with their stability 0.95, 0.9 and 0.8

Fig. 6. The dependence of the consumer-node stability from supply reserve and ten suppliers with their stability 0.95, 0.9

Fig. 7. The dependence of a consumer-node from the stability of five supplier-nodes with 3, 5, 20% reserves
Thus the stability of a consumer-node in all scenarios will be higher than 0.8 when 5% reserve exists. Further reserve increasing leads to asymptotically approximation of stability to 1. In this case one can state that 10% reserve ensures consumer-node stability within 0.98.

Hence, there is one more important conclusion: the consumer-node stability can achieve the required level working with supplier-nodes with extremely low stability, for example 0.5 (Fig. 7)

In this case it is necessary to plan reserves at the volume 20% and more for all supplier-nodes. However, increase of their stability leads to reducing demand in reserves sharply and the stability of a consumer-node will asymptotically approximates to 1 when the stability is at 3% volume. The confirmation of this conclusion can be also observed in the Fig. 8.

Fig. 8. The dependence of a consumer-node from the number of supplier-nodes without reserves of supply and with 3% reserve.

References


Received: August 23, 2013