Two-Dimensional Model of Lexicographic

Preferences

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Abstract

This paper characterizes lexicographic utility based on the consumer preferences on the available sets of commodities by focusing in the Islamic approach. It begins with the introduction of several general preference axioms in the conventional sense that only restricted to the worldly commodity which is one-dimensional utility model. Next, the paper explores the preference ordering grounded by the notion of Islamic teachings that includes worldly and ukhrawi commodity in which a rational consumer should apply in making any economic decision. The preference herein illustrates the connectivity between both of the commodities which generate two-dimensional utility model that is crucial to be hold in the lexicographic manner.
1 Introduction

The concept of a lexicographic involves a set \( I = \{1, 2, \ldots, n\} \) and a preference order relation \( \prec_i \) on a nonempty set \( X_i \) for each \( i \in I \). We let \( \sim_i \) denote the symmetric complement of \( \prec_i \), so that \( x_i \sim_i y_i \) iff (if and only if) not \( (x_i \prec_i y_i \) or \( y_i \prec_i x_i) \). With \( x = (x_1, \ldots, x_n) \) and \( y = (y_1, \ldots, y_n) \), \( x \) precedes \( y \) lexicographically under the natural order on \( I \) and with respect to the \( \prec_i \), or \( x <_L y \) for short, iff \( \{i : i \in I \& (x_i <_i y_i \text{ or } y_i <_i x_i)\} \) is nonempty and \( x_i <_i y_i \) for the first (smallest) \( i \) in this set. For this reason, a lexicographic order is also referred to as an order by first difference [1], lexical order [2] and lexicographical ordering [3].

A prime example of a lexicographic order as mentioned in [4] arises from the alphabetical order of words in a dictionary, or lexicon. To show this, let \( I = \{1, 2, \ldots, n\} \), \( X_i = A = \{\emptyset, a, b, \ldots, z\} \) with \( \emptyset \prec_i a \prec_i b \prec_i \ldots \prec_i z \) for each \( i \), take \( n \) as large as the longest listed word, and let the English word \( \alpha_1 \alpha_2 \ldots \alpha_m \) with \( m \leq n \) correspond to \( (\alpha_1, \alpha_2, \ldots, \alpha_m, \emptyset, \ldots, \emptyset) \) in \( A^n \). Then \( <_L \) on the subset of \( A^n \) that corresponds to the "legitimate" words orders these words in their natural alphabetical orders. For example, "so" precedes "son" since \( (s, o, \emptyset, \ldots, \emptyset) <_L (s, o, n, \emptyset, \ldots, \emptyset) \), which is to say that \( s \sim_1 s, o \sim_2 o, \) and \( \emptyset <_3 n \).

Accordingly, the idea of this paper is to derive the two-dimensional utility model in accordance with the endeavor to develop the preferences that correspond with Islamic approach in the lexicographic manner. For that purpose, it is crucial to bear in mind that Islam associates with the belief in the Day of Judgement, which considered as \( \text{ukhrawi} \) commodity. This prolongs the time horizon beyond worldly commodity. In this way, it is convenient to reflect that the conventional economic system is based only on one-dimensional utility model which is worldly commodity while Islamic economic system apply two-dimensional utility model that consists of the present life (worldly commodity) and the life in the hereafter (\( \text{ukhrawi} \) commodity).

Thus, we present the paper over six sections including the introduction. Section 2 and 3 explains the general preferences as well as the axioms in the conventional sense. This includes notation and the main contradiction of the axiom with Islamic norms. Next, section 4 and 5 enlightens the different sets of commodities in the Islamic economic system and hence develops the framework of lexicographic utility model that conform to the Islamic principles. Finally, section 6 provides a remark for the paper.
2 Notations and Axioms

It is important to explain how consumers form preferences in the process of making decision in the economic system. Thus, the ultimate decision of a consumer to choose a bundle from the consumption set is closely relates with his or her tastes and desires. These are represented by the concept of preference relation ($\succeq$) which is a binary relation on $X$. The most famous attempt to illustrate a preference relation is probably the theory of revealed preference that is the Weak Axiom of Revealed Preference (WARP) \[5\] \[6\]. This axiom infers that if an individual choose A out of alternative set of options including B, they should never choose B when faced with a choice of a different alternative set of options that also includes A and B. On the other hand, Houthakker \[7\] introduced the Strong Axiom of Revealed Preference that says, if $x^n \succ x^{n+1}$, $n = 0, 1, 2, \ldots, s-1$, then $x^0 \succ x^s$.

The set of all consumption bundles that are possible is called the consumption set. This is a non-empty subset of a commodity space, denoted by $X$. Among the alternative commodity bundles in the consumption set, for any two bundles of $x$ and $y$, $x \in X$, $y \in X$, the statement $x \succeq y$ means that $x$ is at least as good as $y$. The axioms imposed on the preference relation that is often regards as a definition of a rational consumer in the conventional sense are as follows:

**Axiom 1 (Reflexivity)**
For all $x \in X$, $x \succeq x$, i.e. any bundle is as good as itself.

**Axiom 2 (Transitivity)**
For any three bundles $x$, $y$, $z$ in $X$ such that $x \succeq y$ and $y \succeq z$ it is true that $x \succeq z$.

**Axiom 3 (Completeness)**
For any two bundles $x$ and $y$ in $X$, $x \succeq y$ or $y \succeq x$.

A preference relation $\succeq$ that satisfies these three axioms is a complete preordering on $X$ and is called a preference order. Like any preordering, a preference relation can be decomposed into a symmetric and asymmetric components by defining the following two sub relations that is the strict preference relation ($x \succ y$ iff $x \succeq y$ and not $y \succeq x$) and the indifference relation ($x \sim y$ iff $x \succeq y$ and $y \succeq x$). With $\succeq$ being reflexive and transitive, the strict preference relation is clearly irreflexive and transitive. It is assumed throughout that there exist at least two bundles $x'$ and $x''$ such that $x' \succ x''$. The indifference relation $\sim$ defines an equivalence relation on $X$, i.e. $\sim$ is reflexive, symmetric and transitive. Axioms 1-3 describes order properties of a preference relation that have intuitive meaning in the context of the theory of choice. This is much less so with the topological conditions, which are usually assumed as well. The most common one is given in Axiom 4 below.
Axiom 4 (Continuity)
For every $x \in X$, the sets $\{y \in X | y \succsim x\}$ and $\{y \in X | x \succsim y\}$ are closed relative to $X$.

The set $\{y \in X | y \succsim x\}$ is called the upper contour set and $\{y \in X | x \succsim y\}$ is called the lower contour set. Intuitively, Axiom 4 requires that the consumer behaves consistently in the “small”, i.e. given any sequence of bundles $y^n$ converging to a bundle $y$ such that for all $n$, each $y^n$ is at least as good as some bundle $x$, then $y$ is also at least as good as $x$. For a preference order, i.e. for a relation satisfying Axioms 1-3, the intersection of the upper and lower contour sets for a given point $x$ defines the indifference class $I(x) = \{y \in X | y \sim x\}$ which is a closed set under Axiom 4. Axioms 1-4 together also imply that the upper and the lower contour sets of the derived strict preference relation $\succ$ are open, i.e. $\{y \in X | y \succ x\}$ and $\{y \in X | x \succ y\}$ are open. However, many known relations do not display the continuity property. The most commonly known of them is the lexicographic order, which is in fact a strict preference relation that is transitive and complete as well as its indifference classes consists of single elements.

3 Islamic Perspectives of Rational Consumer Preferences

Islam teaches rationality with the same emphasis on how consumer should spend and where they are supposed to spend. As stated by Khan [8] in his paper, consumer’s total spending can be classified into two major categories that is to achieve satisfaction in this world which includes present and future consumption and spending for others with a view to earn rewards in the hereafter. This matter arises in order to explain the unsuitability of conventional perspectives with the Islamic perspective. Thus, moderation and balanced in Islam implies the rationality in consumption that Islam does not sermonize complete rejection of all worldly pleasure in view of the hereafter, but can be partaken in the present life as long as it is on the right way that is in total obedience to the Almighty in all affairs. However, consumers are taught to keep in mind that the hereafter life is their ultimate goal and not become too engrossed to the worldly things. For this purpose, consumer should act accordance to the Islamic teachings and avoidance of miser or extravagance activities. This certainly emphasizes the importance of consumer to always know their priority over the preferences. Sometimes, we did not even realize that we have spent more than what we require while still many people out there is struggling to fulfill their basic needs.

Besides, the concept of rationality holds from the conventional sense does not necessary true as human behavior is sometimes hardly to be predicted in certain circumstances depending on their religious values and morality. In the conventional economic phenomenon for example, the concept of rationality carries the obliga-
tions to fulfill individual interest where it leads to the hedonic phenomenon. This phenomenon visualizes the situation in which the rich people keep on accumulating wealth and pursue prestige without realizing their responsibility towards the poor and the needy. This once again differs from Islamic principles that significantly concern about maslahah (society interest). In addition, an individual is judged by his or her economic strength and material affluence which diverse from the Islamic perspective that is not only subjected to material but also spiritual attainment. As according to Khursid [9] the Islamic concept of rationality does not deny either the internal consistency of axioms and postulates self-interest as one of the main determinants of human behavior, but it put restrain on self-interest by linking it with individual and social responsibility as well as morality.

On the other hand, Islamic economics uses a framework derived from the divine text as a basic frame of reference that is the sacred of Quran and Sunnah. Human criticism does not apply on these divine texts. This is distinct from the conventional framework where the fundamental theory is subject to criticism and can undergo change as well as modification.

4 Islamic Consumer Set of Commodities

Islam has its own norms and ethical values with regard to the consumer behavior that leads to seek the priority for the fulfillment of basic needs, fairness distribution of consumption among the family members, spending on religious and social ceremonies, savings for the rainy days and others expenditures. Thus, preference in the Islamic economic system is mainly a matter of the element one would choose from the consumption set which can be divided into two parts that is worldly and ukhrawi commodity. For the worldly commodity, it is further classified into the subsets of dharuriyat (essentials), hajiyat (comforts) and kamaliyat or tahsiniyat (perfection) [10] [11]. First, the subset of dharuriyat consists of any basic needs commodities compulsory to be fulfilled by each individual such as foods, clothes, and a place to live etc. Next, is the hajiyat commodity that is needed to improve the quality of life and reduce hardness. The commodity includes the usage of vehicles, home equipment’s, and others in purpose of making life easier. Finally, kamaliyat commodity comprises any commodities that lead to the perfection and contented life such as the usage of luxury vehicles, luxury houses, and any other luxury commodities. However, this commodity is subject to the capability of an individual in possessing properties.

As for ukhrawi commodity, it relates with the actions that is dependent on the rules of wajib (obligation), mandoob (recommended), mubaah (permissible), makrooh (discouraged or abominable) and haram (prohibition) as stated in the rulings of syariah that linked to the Islamic behavior [12]. In this paper, we
discuss these five categories of actions only in the framework of Islamic economic activities. For example, *wajib* action requires the obligation to pay *zakah* and prohibits any transactions where the element of *riba* (interest), *gharar* (uncertainty or speculation) and *maysir* (gambling) exists which considered as *haram*. For *mandoob* action, it includes the acts of *shadaqah* or any charitable acts with the intention to require blessedness from the Almighty. *Mubaah* is any act that is left to the personal decision and to individual liberty. Although there is neither reward for doing *mubaah* nor punishment for avoiding it, the intention might turn a *mubaah* act into a rewarding act. Investing in the Islamic bank by performing *mudharabah* method for example, with the sincere intention to help needy people is an act of reward. This division clearly shows that Islamic law has various mechanisms and dynamic devices that are flexible and elastic enough to suit its application to the relevancies of life in every time and place. Besides, the classification may also be looked at as generic priorities for resource use towards achieving human well-being both material and spiritual [13].

5 Lexicographic Preferences in Islamic Framework

The theory of lexicographic preferences was first formalized by Georgescu-Roegen [14], then followed by Encarnacion [3] that explored a class of $L^*$ lexicographic orderings that model Simon’s [15], bounded-rationality concept of satisficing that is choosing an alternatives that is satisfactory or that suffices, rather than seeking to optimize. Day [16] [17] developed the theory further by establishing the continuity and convexity properties of $L^*$ choices and the existence of competitive equilibria in an economy of $L^*$ agents. According to the model, an individual which is the decision maker first consider the alternative that satisfies the most important or top-ranked attribute, then restricting attention to alternatives that attain this level, considers the alternatives that satisfies the second-ranked attribute, and so on until the decision process focuses on a single alternative or a set of alternatives among which the individual is indifferent. However, the problem arises as the individual preferences are not fixed but instead are predictably changing.

Preferences also may be fuzzy, imprecise or vague [18] [19]. That is why Islam came up with the guiding principles of an Islamic economic framework that are derived from the teachings of the *Quran* and *Sunnah* to make the right decisions in sorting out priorities in preferences. This can be directly relates with the rational behavior of consumer which implies the theory of choice. Thus, in order to choose, a preference or ordering is needed. This process may under certain assumptions yield the analytical tools such as revealed preference theorems, indifference curve etc. In the case of Islamic approach, the process of preference is quite similar with the standard conventional economic theory. However, the difference will be that the preferences are supposed to be compatible with Islamic teachings by assimila-
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The preference orders as well as the axioms shown in the conventional sense seems not to meet with Islamic principles as it only concern in the range of worldly commodity while it has to be connected with the belief to the ukhrawi commodity. This should be in line with an attempt to develop and improve the Islamic economic system that is not merely around this world that is the objective. The approach that Islam seeks extends to the life in the hereafter and there should be no conflict between the two. This dimension is missing in the contemporary concept of economic. In addition, the concept of rationality also is diverse from the Islamic concept. Thus, each consumer particularly Muslims who strongly have faith in the accountability on the Day of Judgement and the Divine reward, will acts within a milieu of Islamic rules.

Our starting point is identified by the following definition.

Definition 1
Let \( K_U \) be the set of ukhrawi commodity and \( K_D \) be the set of worldly commodity in the universal set space \( Z \), assumed to be finite. The individual’s preference relation \( \succ \) on the product set of \( K_U \times K_D \) is an asymmetric weak order.

We admit that everyday decisions are often made from sets of alternatives that vary on a number of attributes, aspects, or criteria. Hence, to formalize the lexicographic utility model in Islamic framework, we supposed that the consumer make a preferences from the set of ukhrawi and worldly commodity and that each alternative \( K_U \) and \( K_D \) is represented by a vector \( K = (K_U, K_D) \) with \( K_U = (K_1^U, K_2^U, K_3^U) \) and \( K_D = (K_1^D, K_2^D, K_3^D) \). Each of the commodity consists of three elements: \( K_1^U = wajib, K_2^U = mandoob \) and \( K_3^U = mubaah; K_1^D = dharuriyat, K_2^D = hajiyat, K_3^D = tahsiniyat \). In this manner, we form the product set where \( K_U \times K_D \equiv \{ (K_1^U, K_1^D), (K_1^U, K_2^D), (K_1^U, K_3^D), (K_2^U, K_1^D), (K_2^U, K_2^D), (K_2^U, K_3^D), (K_3^U, K_1^D), (K_3^U, K_2^D), (K_3^U, K_3^D) \} \) and we proposed that the following axiom needs to be hold for the preference to be well-suited with Islamic preference principles.

Axiom 1
For every pair of \( K_U \times K_D \) of a well-ordered set in the \( Z \) commodity, contains at least a pair of \( (K_1^U, K_1^D) \in K_U \times K_D \) such that \( (K_1^U, K_1^D) \succ (\emptyset, K_D) \) and \( (K_U, K_D) \succ (\emptyset, K_D) \).

The axiom above revealed the set of basic preference in Islamic economic system comprises both the worldly and ukhrawi commodity that become the main item which supposed to be fulfilled. We assumed that this set of commodities have been fulfilled before the consumer proceeds to the next level of preferences. Thus, the following definition sets the stage for our lexicographic utility model.
Definition 2
Preferences are lexicographic if and only if there is a binary relation $>_{i}$ on $Z_{i}$ for $i = 1, \ldots, n$ and a linear order $<_0$ on $\{1, \ldots, n\}$ such that, for all $(K_{U}, K_{D})$ and $(L_{U}, L_{D})$ in $Z$,

$$(K_{U}, K_{D}) > (L_{U}, L_{D}) \text{ if and only if } (K_{U}, K_{D}) >_{i} (L_{U}, L_{D}), \text{ for some } i \text{ and, for every } k \in \{1, \ldots, n\} \text{ for which } (K_{U}, K_{D}) >_{k} (L_{U}, L_{D}) \text{ there is a } j <_0 k \text{ such that } (K_{U}, K_{D}) >_{j} (L_{U}, L_{D}).$$

Hence, we propose the following lexicographic utility model by replacing the $al$-$falih$ function as $f: K_{U} \times K_{D} \rightarrow \mathcal{N}^{2}$. Thus, a linear order on $\mathcal{N}^{2}$ is defined as $(K_{U}, K_{D}) > (L_{U}, L_{D})$ if and only if $f(K_{U}, K_{D}) > f(L_{U}, L_{D})$. We also define the $al$-$falih$ function as follows:

$$(K_{U}, K_{D}) \mapsto (f_{1}(K_{U}, K_{D}), f_{2}(K_{U}, K_{D}), f_{3}(K_{U}, K_{D}))$$

such that $K_{U} = (K_{U}^{1}, K_{U}^{2}, K_{U}^{3}) \subseteq K_{U}(K_{U}^{1}) \times K_{U}(K_{U}^{2}) \times K_{U}(K_{U}^{3})$, $K_{D} = (K_{D}^{1}, K_{D}^{2}, K_{D}^{3}) \subseteq K_{D}(K_{D}^{1}) \times K_{D}(K_{D}^{2}) \times K_{D}(K_{D}^{3})$ with the following properties:

i) $f_{1}(K_{U}, K_{D}) > f_{1}(L_{U}, L_{D})$ if and only if $(K_{U}, K_{D}) > (L_{U}, L_{D})$ or $[(K_{U} = L_{U} + L_{D} \text{ and } K_{U} > L_{U}] \text{ with } f_{1}(K) \succeq f_{1}(K')$

ii) $f_{2}(K_{U}, K_{D}) \geq f_{2}(L_{U}, K_{D})$ such that $(K_{U}, K_{D}) \geq (L_{U}, L_{D})$ with $K_{U} >_{(p)} L_{U}$

iii) $f_{3}(K_{U}, K_{D}) \leq f_{3}(K_{U}, L_{D})$ such that $K_{D} >_{(p)} L_{D}$ and such that $K_{D} \leq L_{D}$

The properties above justify the preferences in the lexicographic manner with the basic commodity that should be exist as the minimum level of consumerism which is bundle of $K^{*}$ commodity. This bundle is matched with Axiom 1 in order to produce the bundle set of consumer preferences that is

Set $Z = [(K, L): K \geq K^{*}]$  \hspace{1cm} (5.2)

6 Conclusion

Islam has provides guidelines and rules for every sphere of life as well as the society in order to create the spirit of Islamic economic system. Hence, the major contribution of Islam lies in making human life and effort purposive and value-oriented. The standards that has been set by Islam also is connected to the purpose of human existence as to act as a vicegerent for the sake of the Almighty. Thus, the economic behavior is indeed directly associated with the religious ideological foundation that guides human in making daily decision. Consumer should behave according to the Islamic rules and leading their life with the strong faith that they are always be monitored by the Almighty and will be responsible in every single action they made.
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