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Applying the Mean Absolute Eigen-Deviation of Labour Commanded Prices from Labour Values to Actual Economies

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Abstract

This paper applies a recently proposed measure of production price-labour value deviation, i.e. the 'Mean Absolute Eigen-Deviation', to actual economies. The results suggest that that measure tends to the profit-wage ratio in the Sraffian Standard system divided by the number of produced commodities. This finding is reduced to the skew distribution of the eigenvalues of the vertically integrated technical coefficients matrices.

Mathematics Subject Classification: 91B24, 91B38, 91B66, 91B74, 91B82

Keywords: Actual economies, eigenvalue distribution, mean absolute deviation, price-labour value deviation, Standard commodities

1 Introduction

In a Sraffian world, long-period relative prices can change in a complicated way as income distribution changes, a fact that has critical implications for the traditional theories of capital, value, distribution and international trade ([14, §§19 and 48] and, for example, [5, chs 4-5 and 14]). Thus, it has often been found necessary to measure in some way the deviation between with-profit prices and zero-profit prices (or 'labour values').

In diagonalizable and 'regular' (in the sense of Schefold [12, pp. 11-23]) $n \times n$ single-product systems, the 'labour commanded' price-labour value ratios, $\pi_j \omega_j^{-1}$, $j = 1, 2, ..., n$, of the Standard (Sraffian and non-Sraffian) commodities (or the labour commanded eigenprices-eigenlabour value ratios)[1] depend in a *simple* way on the uniform profit rate, r, and the eigenvalues of the 'vertically integrated technical coefficients matrix', $\mathbf{H} \equiv \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1}$ (where $\mathbf{A}$ denotes the matrix of input-output coefficients and $\mathbf{I}$ the identity matrix; [11]).[2] Hence, it has recently been proposed ([6]) that the 'Mean Absolute Eigen-Deviation' (MAED) of labour commanded prices from labour values, i.e. the 'Mean Absolute Deviation' of $\boldsymbol{\pi}$ from $\boldsymbol{\omega}$, which is defined as

$$\text{MAED} \equiv n^{-1} \sum_{j=1}^{n} \left| (\pi_j - \omega_j) \omega_j^{-1} \right|$$

constitutes a workable measure of price-labour value deviation. In effect, this measure (i) leads to an algebraically simple, monotonic and economically interpretable expression; (ii) does not require the prior computation of the values and prices; and (iii) provides a transparent separation between the effects of income distribution and of the technical conditions of production.

On the basis of the available *empirical* evidence, this paper shows that the MAED of labour commanded prices from labour values tends to the profit-wage ratio in the Sraffian Standard system divided by the number of produced commodities: Section 2 presents the adopted measure of deviation and its properties. Section 3 brings in the empirical evidence by examining actual input-output data. Finally, Section 4 concludes.

2 The Mean Absolute Eigen-Deviation

Let $\rho \equiv rR^{-1}$ denote the relative profit rate ($0 \leq \rho \leq 1$ and $R \equiv \lambda_{\mathbf{A}1}^{-1} - 1 = \lambda_{\mathbf{H}1}^{-1}$ the maximum profit rate of the system) and let $\mathbf{J} \equiv R\mathbf{H}$ denote a normalized matrix of vertically integrated technical coefficients ($\lambda_{\mathbf{J}1} = R\lambda_{\mathbf{H}1} = 1$ and $|\lambda_{\mathbf{J}k}| < 1$). From the Sraffian price-wage-profit system and the definition of the MAED it follows that ([6, pp. 606-608]): $\pi_j \omega_j^{-1} = (1 - \rho\lambda_{\mathbf{J}j})^{-1}$ and

[1] For the non-Sraffian, real and/or complex, Standard commodities-systems, see [15, §42, footnote 2, and §§56, 64]; [2], [3] and [18, Part III].

[2] Matrices (and vectors) are delineated in boldface letters. The transpose of a $1 \times n$ vector $\mathbf{y} \equiv [y_j]$ is denoted by $\mathbf{y}^{\mathrm{T}}$. $\lambda_{\mathbf{A}1}$ denotes the Perron-Frobenius eigenvalue of a semi-positive $n \times n$ matrix $\mathbf{A}$ and $(\mathbf{x}_{\mathbf{A}1}^{\mathrm{T}}, \mathbf{y}_{\mathbf{A}1})$ the corresponding eigenvectors, whilst $\lambda_{\mathbf{A}k}$, $k = 2, ..., n$ and $|\lambda_{\mathbf{A}2}| \geq |\lambda_{\mathbf{A}3}| \geq ... \geq |\lambda_{\mathbf{A}n}|$, denotes the non-dominant eigenvalues. Finally, $\mathbf{e}_j$ denotes the $j-$th unit vector.

$$\text{MAED} = n^{-1}(d_1 + \sum_{k=2}^{n} d_k) \tag{1}$$

where $d_1 \equiv \rho(1-\rho)^{-1}$ equals the profit-wage ratio in the Sraffian Standard system (and, at the same time, the elasticity of π_1 with respect to ρ), a strictly increasing and convex function of ρ, tending to infinity as ρ approaches 1 from below, and

$$d_k \equiv \left|\rho\lambda_{\mathbf{J}k}(1-\rho\lambda_{\mathbf{J}k})^{-1}\right| = \rho\left|\lambda_{\mathbf{J}k}\right|\left|1-\rho\lambda_{\mathbf{J}k}\right|^{-1} \tag{2}$$

equals the moduli of the profit-wage ratios in the non-Sraffian Standard systems.

Now consider the following four cases:

(i). If $\rho\left|\lambda_{\mathbf{J}k}\right| > 0$, then

$$d_k \le \rho\left|\lambda_{\mathbf{J}k}\right|(1-\rho\left|\lambda_{\mathbf{J}k}\right|)^{-1} = \rho(\left|\lambda_{\mathbf{J}k}\right|^{-1} - \rho)^{-1} < d_1 \tag{3}$$

which implies that

$$\text{MAED} < n^{-1}[d_1 + (n-1)d_1] = d_1 \tag{4}$$

(ii). If the moduli of the last $n-v$, $1 \le v \le n-1$, eigenvalues are sufficiently small that can be considered as negligible, then equation (1) reduces to

$$\text{MAED} \approx \text{MAED}_1 \equiv n^{-1}d_1 \text{, if } v = 1 \tag{5}$$

and

$$\text{MAED} \approx \text{MAED}_v \equiv n^{-1}(d_1 + \sum_{k=2}^{v} d_k) \text{, if } v \ge 2 \tag{5a}$$

(iii). If $\rho\left|\lambda_{\mathbf{J}k}\right| << 1$, which implies that

$$\pi_j\omega_j^{-1} = (1-\rho\lambda_{\mathbf{J}j})^{-1} = 1 + \rho\lambda_{\mathbf{J}j} + (\rho\lambda_{\mathbf{J}j})^2 + ... \approx 1 + \rho\lambda_{\mathbf{J}j}$$

then

$$\text{MAED} \approx \text{MAED}_{\text{L}} \equiv n^{-1}\rho(1 + \sum_{k=2}^{n}\left|\lambda_{\mathbf{J}k}\right|) \tag{6}$$

which is a linear approximation of the MAED.

(iv). If $\mathbf{J}$ has rank 1, then $\lambda_{\mathbf{J}k} = 0$, for all k, and

$$\text{MAED} = \text{MAED}_1 \tag{7}$$

i.e. the MAED equals the profit-wage ratio in the Sraffian Standard system divided by the number of produced commodities. By the Schur triangularization theorem (e.g. [10, pp. 508-509]) it follows that $\mathbf{J} = (\mathbf{y}_{\mathbf{J}1}\mathbf{x}_{\mathbf{J}1}^{\text{T}})^{-1}\mathbf{x}_{\mathbf{J}1}^{\text{T}}\mathbf{y}_{\mathbf{J}1}$ ($rank[\mathbf{J}] = 1$) can be transformed, via a semi-positive similarity matrix $\mathbf{S}$, into

$$\tilde{\mathbf{J}} \equiv \mathbf{S}^{-1}\mathbf{J}\mathbf{S} = \begin{bmatrix} 1 & \tilde{\mathbf{J}}_{12} \\ \mathbf{0}_{(n-1)\times 1} & \mathbf{0}_{(n-1)\times(n-1)} \end{bmatrix}$$

where the first column of $\mathbf{S}$ is $\mathbf{x}_{\mathbf{J}1}^{\text{T}}$ (the remaining columns are arbitrary), and $\tilde{\mathbf{J}}_{12}$ is a $1\times(n-1)$ *positive* vector (for example, $\mathbf{S} = [\mathbf{x}_{\mathbf{J}1}^{\text{T}}, \mathbf{e}_2^{\text{T}}, ..., \mathbf{e}_n^{\text{T}}]$). Thus, the original system is similar to a system with only one 'basic' commodity and without 'self-reproducing non-basics' (in the sense of Sraffa [14, Appendix B]) or, in other words, behaves as a 'corn-tractor' system (e.g. [1, ch. 10]), even if **J** is *irreducible*.

Hence, for the general case, we can write

$$\text{MAED}_v \le \text{MAED} \le d_1 \tag{8}$$

Furthermore, since $\lambda_{\mathbf{J}k} = \alpha_k \pm i\beta_k$, where $i \equiv \sqrt{-1}$, and $|\lambda_{\mathbf{J}k}| \equiv \sqrt{\alpha_k^2 + \beta_k^2}$, equation (2) can be rewritten as

$$d_k = \rho|\lambda_{\mathbf{J}k}|D_k$$

where $D_k \equiv (1 - 2\rho\alpha_k + \rho^2|\lambda_{\mathbf{J}k}|^2)^{-0.5}$. Taking the first partial derivatives of d_k^2 with respect to ρ, α_k and β_k, i.e.

$$2|\lambda_{\mathbf{J}k}|^2 \rho(1-\rho\alpha_k)D_k^4,\ \ 2\rho^2[\alpha_k + \rho(\beta_k^2 - \alpha_k^2)]D_k^4 \text{ and } 2\beta_k\rho^2(1-2\rho\alpha_k)D_k^4$$

respectively, it follows that d_k (i) increases with increasing ρ, since $\rho\alpha_k < 1$; (ii) increases with $\alpha_k > 0$; (iii) decreases with $\alpha_k < 0$, when $\beta_k \le \alpha_k$; (iv) decreases with $\alpha_k < 0$ for $\rho < -\alpha_k(\beta_k^2 - \alpha_k^2)^{-1}$ (<1), when $\alpha_k(\alpha_k - 1) < \beta_k^2$; (v) increases with $|\beta_k|$, when $\alpha_k \le 0$; and (vi) increases with $|\beta_k|$ for $\rho < (2\alpha_k)^{-1}(>1/2)$, when $\alpha_k > 0$ (see, for example, Figure 1a, where $\alpha_k = -0.4$ or -0.1 and $\beta_k = 0.8$, and Figure 1b, where $\alpha_k = 0.8$ and $\beta_k = 0.3$ or $\beta_k = 0.5$; the dotted line represents d_1 and the solid lines represent d_k). Finally, let '$'$' denote the first derivative with respect to ρ. The negativity of

$$[(d_k d_1^{-1})^2]' = 2(d_k d_1^{-3})(d_k' d_1 - d_k d_1') = -2|\lambda_{\mathbf{J}k}|^2(1-\rho)[(1-\alpha_k)(1-\rho\alpha_k) + \rho\beta_k^2]D_k^4$$

implies $d_k d_1' - d_k' d_1 > 0$, which in its turn implies $(d_1 - d_k)' = d_1' - d_k' > 0$, since $0 < d_k < d_1$ for $\rho > 0$ (see relation (2)). It then follows that (i) the relative error between the MAED and the MAED$_1$, i.e.

$$\text{RE}_1 \equiv 1 - \text{MAED}_1(\text{MAED})^{-1} = 1 - (1 + \sum_{k=2}^{n} d_k d_1^{-1})^{-1} \tag{9}$$

or, alternatively, the accuracy of the approximation (5), is a strictly decreasing function of ρ, tending to $(\sum_{k=2}^{n}|\lambda_{\mathbf{J}k}|)(1+\sum_{k=2}^{n}|\lambda_{\mathbf{J}k}|)^{-1}$ (tending to 0) as ρ tends to 0 (tends to 1); and (ii) $d_1 - d_k$ increases with ρ. Nevertheless, the relative error between the MAED and the MAED$_v$, $v \ge 2$, or, alternatively, the accuracy of the approximation (5a), does not necessarily decrease monotonically with ρ, since $d_{k-1}d_k^{-1}$, $k \ge 3$, is not necessarily an increasing function of ρ (See, for example, Figure 2, where $n = 3$ and (i) $\lambda_{\mathbf{J}2} = 0.9, \lambda_{\mathbf{J}3} = 0.6$ (upper solid line); (ii) $\lambda_{\mathbf{J}2} = -0.9, \lambda_{\mathbf{J}3} = 0.6$ (dotted line); (iii) $\lambda_{\mathbf{J}2} = 0.9, \lambda_{\mathbf{J}3} = -0.6$ (dashed line); and

(iv) $\lambda_{\mathbf{J}2} = -0.9, \lambda_{\mathbf{J}3} = -0.6$ (lower solid line). It is easily checked that a necessary condition for a non-monotonic RE$_2$ is $\{\lambda_{\mathbf{J}2} < 0, \lambda_{\mathbf{J}3} > 0\}$).

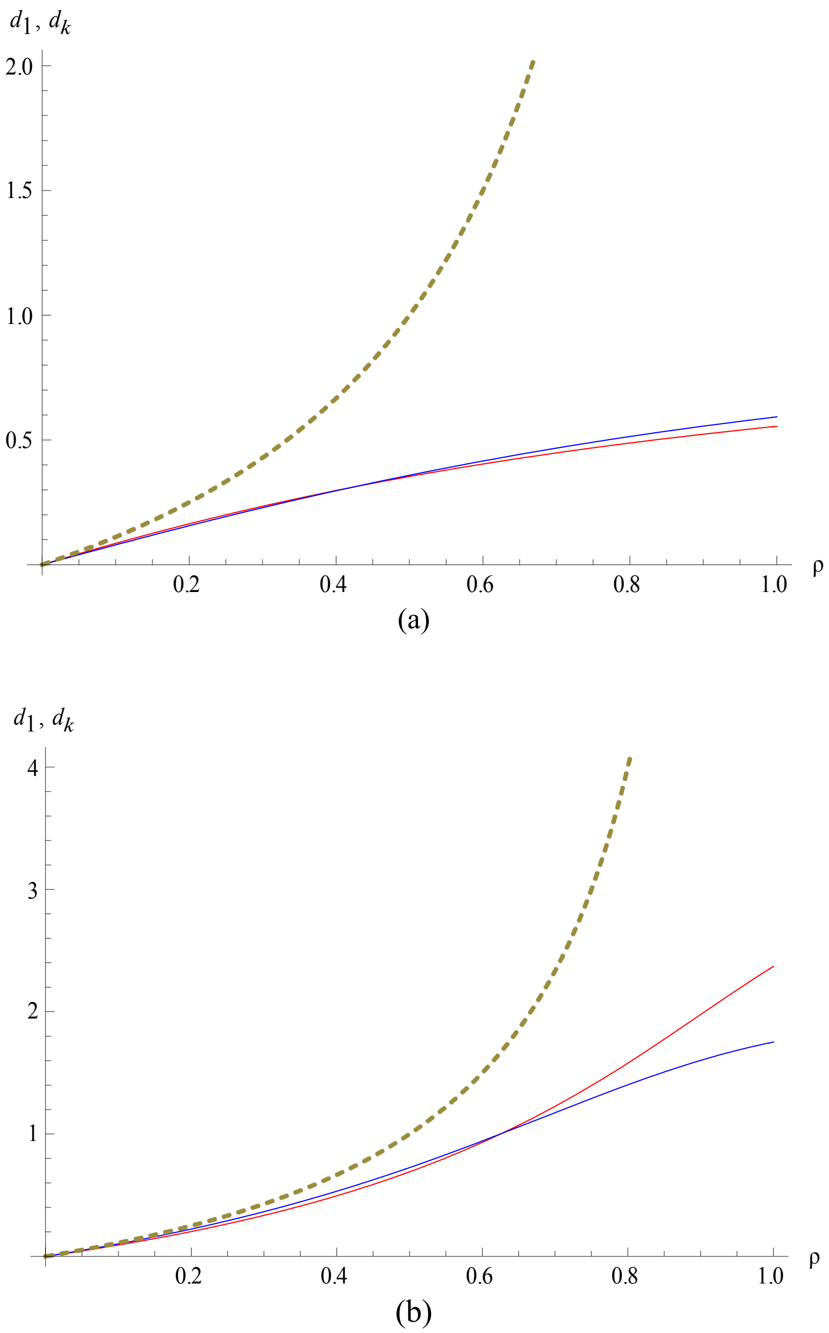

Figure 1. Components of the MAED as functions of the relative profit rate

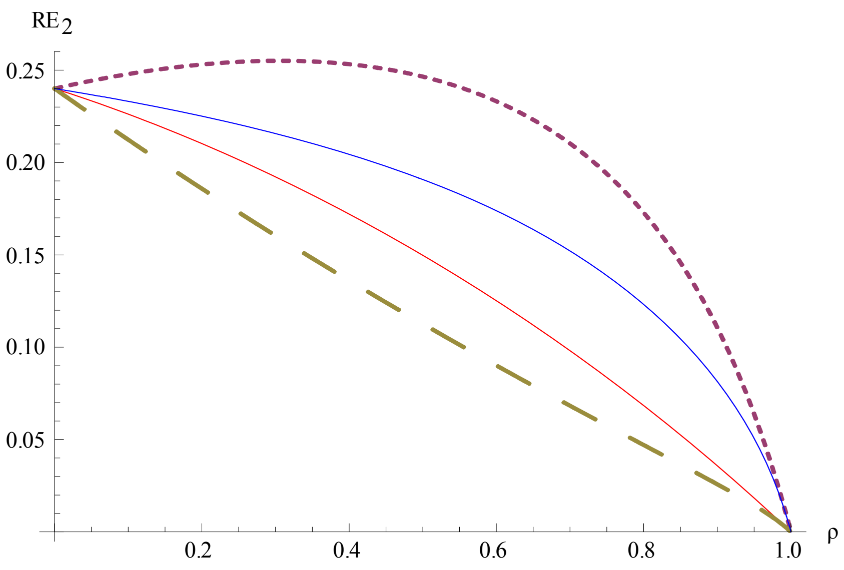

Figure 2. Relative errors between the MAED and the $MAED_2$ as functions of the relative profit rate

3 Empirical Evidence

The application of the previous analysis to the symmetric input-output tables (SIOT) of the Greek economy gives the results summarized in Tables 1 to 3.

Tables 1 and 2 are associated with the 19×19 flow SIOT ([17]), spanning the period 1988-1997. Table 1 reports (i) $\lambda_{\mathbf{J}k}$ (listed in descending order of moduli); and (ii) the '*actual*' relative profit rate, ρ^{a}.[3]

[3] For the estimation of ρ^{a} we assume that wages are paid at the end of the common production period, and then we follow the usual procedure (e.g. [7, pp. 616-617]). *Mathematica 7.0* is used in the calculations, whilst the precision in internal calculations is set to 16 digits. The analytical results are available on request from the author.

Table 1. Non-dominant eigenvalues and ‘actual’ relative profit rate; Greek economy, 1988-1997

1988	1989	1990	1991	1992
$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$
0.643	0.683	0.675	0.657	0.624
0.416	0.436	0.418	0.397	$0.442 \pm i\ 0.023$
0.409	$0.376 \pm i\ 0.025$	$0.376 \pm i\ 0.011$	$0.382 \pm i\ 0.016$	0.406
0.361	0.308	0.311	0.326	0.308
0.259	$0.199 \pm i\ 0.060$	$0.207 \pm i\ 0.069$	$0.217 \pm i\ 0.065$	$0.230 \pm i\ 0.075$
$0.178 \pm i\ 0.0588$	0.104	0.110	0.101	0.108
$0.065 \pm i\ 0.052$	$0.067 \pm i\ 0.047$	$-0.058 \pm i\ 0.068$	$-0.066 \pm i\ 0.068$	$-0.075 \pm i\ 0.073$
$-0.054 \pm i\ 0.057$	$-0.052 \pm i\ 0.062$	$0.066 \pm i\ 0.046$	$0.063 \pm i\ 0.046$	$0.068 \pm i\ 0.044$
$0.070 \pm i\ 0.013$	$0.028 \pm i\ 0.013$	0.039	$0.030 \pm i\ 0.015$	0.053
$-0.017 \pm i\ 0.022$	$-0.015 \pm i\ 0.018$	0.028	$-0.013 \pm i\ 0.018$	0.029
0.020	-0.007	$-0.013 \pm i\ 0.018$	0.008	$-0.017 \pm i\ 0.021$
-0.009	0.006	0.009	-0.005	-0.005
0.006	---	-0.006	---	0.003
ρ^{a}	ρ^{a}	ρ^{a}	ρ^{a}	ρ^{a}
0.411	0.414	0.399	0.409	0.420

contd.

1993	1994	1995	1996	1997
$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$	$\lambda_{\mathbf{J}k}$
0.667	0.678	0.655	0.664	0.641
0.433	0.420	$0.382 \pm i\ 0.008$	$0.382 \pm i\ 0.008$	0.350
0.353	0.357	0.281	0.313	0.307
0.320	0.327	0.246	0.233	0.279
0.268	0.261	$0.196 \pm i\ 0.050$	$0.204 \pm i\ 0.062$	$0.238 \pm i\ 0.072$
$0.224 \pm i\ 0.069$	$0.199 \pm i\ 0.057$	0.098	0.098	0.210
0.110	0.109	$-0.066 \pm i\ 0.064$	$0.083 \pm i\ 0.029$	0.103
$-0.075 \pm i\ 0.073$	$-0.071 \pm i\ 0.066$	$0.077 \pm i\ 0.036$	$-0.058 \pm i\ 0.064$	$-0.066 \pm i\ 0.072$
0.083	$0.071 \pm i\ 0.041$	$0.019 \pm i\ 0.013$	0.072	0.087
$0.058 \pm i\ 0.037$	0.059	$-0.010 \pm i\ 0.011$	$-0.016 \pm i\ 0.024$	0.042
$-0.015 \pm i\ 0.021$	$-0.013 \pm i\ 0.023$	0.005	0.019	$0.027 \pm i\ 0.022$
0.017	0.023	-0.004	0.002	$-0.009 \pm i\ 0.015$
-0.006	-0.007	---	-0.001	0.013
0.002	0.006	---	---	0.001
ρ^{a}	ρ^{a}	ρ^{a}	ρ^{a}	ρ^{a}
0.388	0.421	0.419	0.423	0.438

Table 2 reports, in percentage terms, the MAED, MAED_1 and MAED_5 of $\boldsymbol{\pi}$ from $\boldsymbol{\omega}$, and the relevant relative errors (see relations (1), (5), (5a) and (9)) at

$\rho = \rho^{a}$ and $\rho = 0.9$ (i.e. a high, somewhat unrealistic value),[4] whilst Figure 3 displays these errors as functions of ρ, for the representative years 1988 (dotted lines) and 1997 (solid lines; compare with Figure 2). Thus, it is observed that approximations of the MAED on the basis of only a few of the largest modulus eigenvalues are not so bad and the RE_5 decrease monotonically with ρ. Clearly, these findings are due, respectively, to the following facts identified in input-output tables of a number of diverse economies *and* over the years: (i) the moduli of the first non-dominant eigenvalues fall quite rapidly and the rest constellate in much lower values forming a 'long tail'; and (ii) the negative eigenvalues, as well as the complex eigenvalues with negative real part, tend to appear in the lower ranks, whilst the (positive) real part of complex eigenvalues that appear in the higher ranks is much larger than the imaginary part. More specifically, in examining the input-output tables of Canada (1997), China (1997), Denmark (2000 and 2004), Finland (1995 and 2004), France (1995 and 2005), Germany (2000 and 2002), Japan (1970, 1975, 1980, 1985, 1990, 1995, 2000 and 2005), Korea (1995 and 2000), Sweden (1995 and 2005), UK (1998) and USA (1947, 1958, 1963, 1967, 1972, 1977 and 1997) we detected a strong tendency towards uniformity in the distribution of the moduli of the eigenvalues and, in fact, the moduli follow an exponential pattern of the form $y = c + b\exp(x^{a})$, $c < 0$, $b > 0$ and $a < 0$ ([9, pp. 101-109], [4, Section 3]; see also [13, pp. 14-15]).

Table 2. The MAED (%) and its approximations; Greek economy, 1988-1997

		1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
$\rho = \rho^{a}$	MAED	11.3	11.8	11.3	11.6	12.5	10.7	11.8	11.3	11.7	11.7
	$MAED_1$	3.7	3.7	3.5	3.6	3.8	3.3	3.8	3.8	3.9	4.1
	$MAED_5$	8.6	8.9	8.3	8.5	9.2	7.8	8.8	8.5	8.7	8.7
	RE_1	67.3	68.6	69.1	69.0	69.6	69.2	67.8	66.4	66.7	65.0
	RE_5	23.9	24.6	26.5	26.7	26.4	27.1	25.4	24.8	25.6	25.6
$\rho = 0.9$	MAED	69.8	71.6	71.4	71.0	72.0	70.6	70.4	68.7	69.5	67.6
	$MAED_1$	47.4	47.4	47.4	47.4	47.4	47.4	47.4	47.4	47.4	47.4
	$MAED_5$	63.3	64.6	64.1	63.4	64.1	63.2	63.5	62.2	62.8	60.7
	RE_1	32.1	33.8	33.6	33.2	34.2	32.9	32.7	31.0	31.8	29.9
	RE_5	9.3	9.8	10.2	10.7	11.0	10.5	9.8	9.5	9.6	10.2

[4] To the best of my knowledge, there is no relevant empirical study where the 'actual' ρ is considerably greater than 0.4 (than 0.5), provided that wages are paid at the beginning (end) of the production period ([8], [7, pp. 616-617] and the references therein).

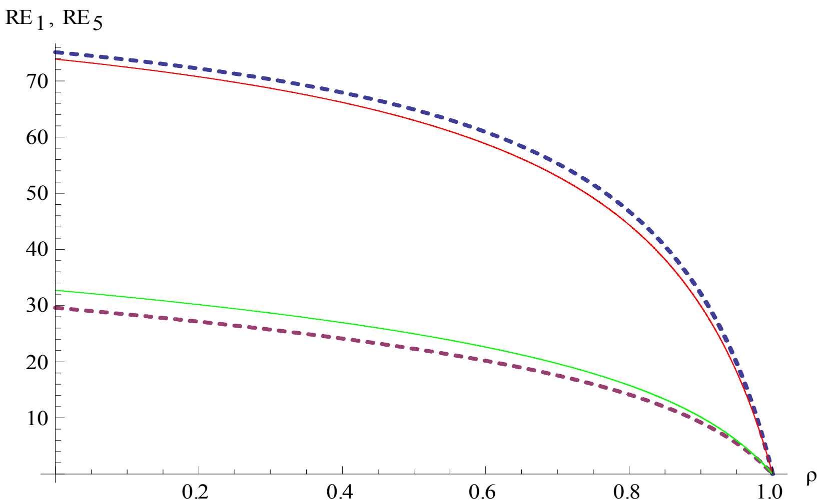

Figure 3. Relative errors (%) between the MAED and its approximations as functions of the relative profit rate; Greek economy, 1988 and 1997

Table 3 is associated with the 33×33 SIOT of the Greek economy for the year 1970, and the case of fixed capital with a uniform profit rate ([16]).[5] It reports (i) the non-zero non-dominant eigenvalues of $\mathbf{J}$ (*all* the zero eigenvalues come from the fact that $\mathbf{K}$ is a reducible matrix without self-reproducing non-basics); and (ii) the MAED, MAED$_v$, where $v=1$ or 5, $\text{MAD}_{\text{L}} \cong 33^{-1}(\rho 1.141)$ (see relation (6)), RE$_v$ and RE$_\text{L}$, at $\rho=\rho^{\text{a}} \cong 0.328$ ('actual' value) and $\rho=0.9$. Thus, it is observed that the moduli of the non-dominant eigenvalues fall even *more* abruptly and, therefore, the approximation of the MAED through the MAED$_1$ works pretty well (See also Figure 4, which displays the MAED, MAED$_1$ and MAED$_\text{L}$ as functions of ρ; it is easily checked that MAED$_1$ > MAED$_\text{L}$ for $\rho>0.124$). Finally, it is important to note that we have also experimented with the relevant input-output tables of Korea (for the years 1995 and 2000) and USA (for the years 1947, 1958, 1963, 1967, 1972 and 1977), and the results were quite similar, since $|\lambda_{\mathbf{J}2}|^{-1}$ is in the range of 1.8 (USA, 1972)-15.9 (Korea, 2000) and $|\lambda_{\mathbf{J}3}|^{-1}$ is in the range of 8.5 (USA, 1947)-16.9 (Korea, 1995) ([9, pp. 109-111];[6] consider also the evidence provided by [15]). On this basis, it is reasonable to

[5] In this case, we postulate that $\mathbf{H}\equiv\mathbf{K}[\mathbf{I}-(\mathbf{A}+\mathbf{A}_{\text{D}})]^{-1}$, where $\mathbf{K}$ denotes the matrix of capital stock coefficients and $\mathbf{A}_{\text{D}}$ the matrix of depreciation coefficients ('Leontief-Bródy approach'; see, e.g. [5, pp. 270-271]). It is noted that, setting aside the year 1970, in the available SIOT of the Greek economy we do not have data on fixed capital stocks. Moreover, the flow SIOT for the year 1970 gives results similar to those reported in Tables 1 and 2.

[6] It should be noted that, in the relevant flow SIOT, $|\lambda_{\mathbf{J}2}|^{-1}$ is in the range of 1.46 (Korea, 2000)-1.90 (USA, 1977) and $|\lambda_{\mathbf{J}3}|^{-1}$ is in the range of 1.72 (USA, 1963)-2.60 (USA, 1977).

expect that the $MAED_1$ is a good (and easily computed) approximation for empirical work.

Table 3. The non-zero non-dominant eigenvalues, the MAED (%) and its approximations for the case of fixed capital; Greek economy, 1970

$\lambda_{\mathbf{J}k}$	ρ	MAED	$MAED_1$	$MAED_5$	$MAED_L$	RE_1	RE_5	RE_L
-0.035	0.328	1.62	1.48	1.59	1.13	8.64	1.85	30.25
$-0.002 \pm i\, 0.033$	0.900	27.65	27.27	27.59	3.11	1.37	0.22	88.75
-0.015								
0.011								
$0.002 \pm i\, 0.004$								
0.002								
-0.002								
0.0004								
$-0.0002 \pm i\, 0.0003$								
0.0001								
$(-2.62 \pm i\, 3.56)$E-19								
-2.03E-21								

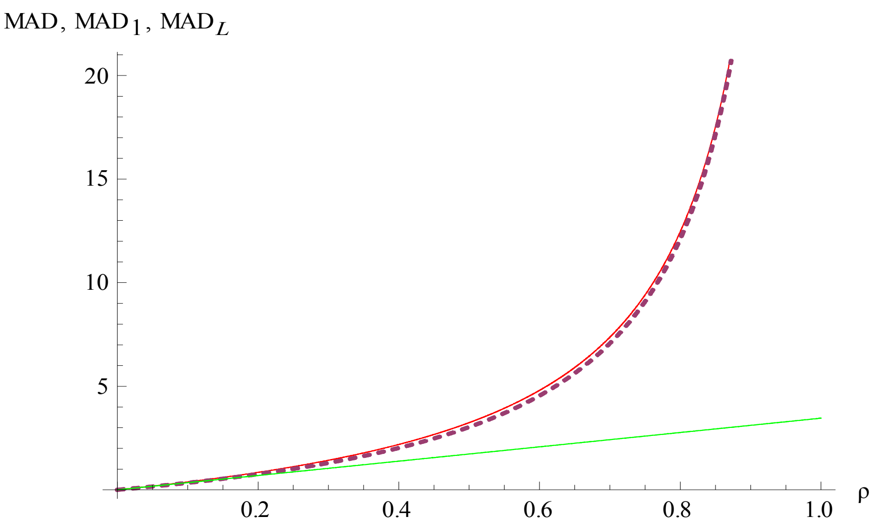

Figure 4. The MAED (%), $MAED_1$ (%) and $MAED_L$ (%) as functions of the relative profit rate for the case of fixed capital; Greek economy, 1970

4 Concluding Remarks

It has been found that, in actual economies, the Mean Absolute Eigen-Deviation of labour commanded prices from labour values tends to the profit-wage ratio in the Sraffian Standard system divided by the number of produced commodities.

This is reduced to the skew distribution of the eigenvalues of the vertically integrated technical coefficients matrices, and implies that (i) there are considerable quasi-linear dependencies amongst the technical conditions of production in many vertically integrated industries; and, therefore, (ii) actual economies *tend* to behave as 'corn-tractor' systems. Thus, it can be concluded that, especially in the (more realistic) fixed capital case, "the quantity of labour that can be purchased by the [*Sraffian*] Standard net product" ([14, p. 32]) provides a tangible and useful measure of price-labour value deviation.

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References

[1] S. Ahmad, Capital in Economic Theory. Neo-classical, Cambridge and Chaos, Edward Elgar, Aldershot, 1991.

[2] R.M. Goodwin, Use of normalized general co-ordinates in linear value and distribution theory, in: K.R. Polenske and J.V. Skolka (eds), Advances in Input-Output Analysis, Ballinger, Cambridge, MA, 1976.

[3] R.M. Goodwin, Capital theory in orthogonalised general co-ordinates, in: R.M. Goodwin, Essays in Linear Economic Structures, Macmillan, London, 1983.

[4] F. Iliadi, T. Mariolis, G. Soklis and L. Tsoulfidis, Bienenfeld's approximation of production prices and eigenvalue distribution: further evidence from five European economies, Contributions to Political Economy (2014; forthcoming).

[5] H.D. Kurz and N. Salvadori, Theory of Production. A Long-Period Analysis, Cambridge University Press, Cambridge, 1995.

[6] T. Mariolis, A simple measure of price-labour value deviation, Metroeconomica, **62** (2011), 605-611.

[7] T. Mariolis and G. Soklis, On constructing numeraire-free measures of price-value deviation: a note on the Steedman-Tomkins distance, Cambridge Journal of Economics, **35** (2011), 613-618.

[8] T. Mariolis and L. Tsoulfidis, Measures of production price-labour value deviation and income distribution in actual economies: a note, Metroeconomica, **61** (2010), 701-710 (Enlarged version: Measures of production price-labour value deviation and income distribution in actual economies: theory and empirical evidence, MPRA Paper No. 43718, http://mpra.ub.uni-muenchen.de/43718/1/MPRA_paper_43718.pdf).

[9] T. Mariolis and L. Tsoulfidis, Eigenvalue distribution and the production price-profit rate relationship: theory and empirical evidence, Evolutionary and Institutional Economics Review, **8** (2011), 87-122.

[10] C.D. Meyer, Matrix Analysis and Applied Linear Algebra, New York, Society for Industrial and Applied Mathematics, 2001.

[11] L. Pasinetti, The notion of vertical integration in economic analysis, Metroeconomica, **25** (1973), 1-29.

[12] B. Schefold, Mr. Sraffa on joint production, Ph.D. thesis, University of Basel, Mimeo, 1971.

[13] B. Schefold, Approximate surrogate production functions, Cambridge Journal of Economics (2013, doi:10.1093/cje/bes056; forthcoming).

[14] P. Sraffa, Production of Commodities by Means of Commodities. Prelude to a Critique of Economic Theory, Cambridge University Press, Cambridge, 1960.

[15] A.E. Steenge and M.J.P.M. Thissen, A new matrix theorem: interpretation in terms of internal trade structure and implications for dynamic systems, Journal of Economics, **84** (2005), 71-94.

[16] L. Tsoulfidis and Th. Maniatis, Values, prices of production and market prices: some more evidence from the Greek economy, Cambridge Journal of Economics, **26** (2002), 359-369.

[17] L. Tsoulfidis and T. Mariolis, Labour values, prices of production and the effects of income distribution: evidence from the Greek economy, Economic Systems Research, **19** (2007), 425-437.

[18] K. Velupillai (ed.), Nonlinear and Multisectoral Macrodynamics, New York University Press, New York, 1990.

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