A Probabilistic Analysis of Three Levels of Manpower Affecting Business - Continuous Time Markov Chain Model

C. Mohan

Department of Mathematics
Vel Tech Multi Tech Dr. Rangarajan Dr. Sakunthala Engineering College
Avadi, Chennai, Tamilnadu, India
vrvpun@gmail.com

R. Ramanarayanan

Controller Of Examinations
VelTech Dr. RR And Dr. SR Technical University
Avadi, Chennai, Tamilnadu, India
r.ramanarayanan@gmail.com

Abstract

In this paper we consider a business organization under varying conditions which are restricted to depend on manpower, money and business under fluctuating conditions of availability of man power and business with a special emphasis given to a new and prevailing idea of frequent changes taking place in manpower. We consider three phases of manpower viz manpower is fully available, moderately or insufficiently available and manpower not at all available or scarcely available and the transitions take place from full availability to nil availability and from nil availability to full availability in case of money and business but in the case of manpower it is assumed that the transitions can be from nil availability to moderate availability and vice versa and from moderate availability to full availability and vice versa. No transitions take place between the lowest and the highest. The different states have been discussed under the assumption that changes from one state to another in any three characteristic occur in exponential times with different parameters. An expression for “Rate of Crisis under steady state ($C_\infty$)” is arrived and steady state cost have also been worked by assuming different costs for the parameters under different conditions.

Keywords: Manpower planning, crisis state and steady state probabilities
1 Introduction

Nowadays we find that labor has become a buyers market as well as seller’s market. Any company normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet company’s requirement at any time during the course of the business and manpower is not an exception. This is spelt in the sense that a company does not want to keep manpower more than what is required. Hence, retrenchment and recruitment ate common and frequent in most of the companies now. Recruitment is done when the business is busy and shed manpower when the business is lean. Equally true with the labor. The workers have the option to switch over to other jobs because of better working condition, better emolument, proximity to their living place or other reasons. Under such situations the company may face crisis because business may be there but manpower may not be available. If skilled laborers and technically qualified persons leave the business the seriousness is worst felt and the company has to hire paying heavy price or pay overtime to employees.

Approach to manpower problems have been dealt in very many different ways as early as 1947 by Vajda [10] and others. Models in manpower planning has been dealt in depth in Bartholomew [1], Grinold and Marshal [3] and Vajda [10]. The methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals has been dealt by Lesson [4]. Markov models are designed for wastages and promotion in manpower system by Vasilou [11]. V. Subramaniam [9] in his thesis has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in man power planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [8]. A two unit stand by system has been investigated by Chandrasekar and Natrajan [2] with confidence limits under steady state. For n unit standby system one may refer to Ramanarayanan and Usha [7]. Yadhavalli and Botha [12] have examined the same for two unit system with introduction of preparation time for the service facility and the confidence limits for stationary rate of disappointment of an intermittently used system. For three characteristics system involving manpower, money and machine one may refer to C. Mohan and R. Ramanarayanan [6]. For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5].

In this model we consider three characteristics namely manpower, money and business and derive a formula for steady state rate of crisis and and the steady state probabilities. The situations may be that the manpower may be fully available, moderately available or hardly available, but business and
money may fluctuate between full availability to nil availability. The steady state probabilities of the continuous two unit time Markov chain describing the transitions in various states are derived and critical states are identified for presenting the cost analysis. Numerical illustrations are provided.

2 Assumptions

1. The random length of time the staff strength is assumed to remain inadequate is exponentially distributed with parameter $\lambda_s$, the length of time required for filling up of vacancies from nil to moderate level (under staff) because of scarcity of skilled labor or the manpower becoming costly is exponentially distributed with parameter $\mu_d$. The length of time the Staff strength is assumed to remain insufficient (minimal optimal) is assumed to be exponentially distributed with parameter $\lambda_c$. The length of time required for filling up of vacancies from moderate to full is also exponentially distributed with parameter $\mu_s$. The above periods of transitions are random variables are independent.

2. The random length of time for which the fund is fully available is exponentially distributed with parameter and the length of time required to meet shortages is also exponentially distributed with parameter $\beta$ and the period of transitions are independent random variables.

3. The random length of time for which the business is busy is exponentially distributed with parameter $a$ and the random length of time for which the business remains lean is also exponentially distributed with parameter $b$. The periods of transitions are independent random variables.

4. Busy and lean periods are assumed to occur in cycles.

3 System Analysis

The stochastic process $x(t)$ describing the state of the system is continuous Markov chain with 12 points state space and is given by,

$$ S = \{(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1/2, 0, 0), (1/2, 0, 1), (1/2, 1, 0), (1/2, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1) \} $$

0 refers to nil availability or shortages of resources, 1/2 refers to semi availability and manpower and 1 refers to full availability of resources.
Where
\[
\epsilon_1 = -(b + \beta + \mu_d), \quad \epsilon_2 = -(a + \mu_d + \beta), \quad \epsilon_3 = -(\mu_d + b + \alpha), \\
\epsilon_4 = -(\mu_d + \alpha + a), \quad \epsilon_5 = -(\lambda_s + b + \beta + \mu_s), \quad \epsilon_6 = -(\lambda_s + a + \beta + \mu_s), \\
\epsilon_7 = -(\lambda_c + \alpha + b + \mu_s), \quad \epsilon_8 = -(\lambda_s + \alpha + a + \mu_s), \quad \epsilon_9 = -(\lambda_c + b + \beta), \\
\epsilon_{10} = -(\lambda_c + a + b), \quad \epsilon_{11} = -(\lambda_c + \alpha + b), \quad \epsilon_{12} = -(\lambda_c + \alpha + a).
\]

The crisis states are \{(0, 0, 1)(0, 1, 1)(1, 0, 1)(1/2, 0, 1)\}. Here, we consider a state to be in crisis if full business may be there but there is shortage of manpower or money. Using the above infinitesimal matrix the steady state probability vector can be derived by using \(\Pi Q = 0\) and \(\pi e = 1\) where \(\pi = [1, 1, 1, \ldots, 1]^T\) is a vector of the type \(12 \times 1\) and \(\Pi\) is given by,

\[
\Pi = [\Pi_{0,0,0}, \Pi_{0,0,1}, \Pi_{0,1,0}, \Pi_{0,1,1}, \Pi_{1/2,0,0}, \Pi_{1/2,0,1}, \Pi_{1/2,1,0}, \Pi_{1/2,1,1}, \\
\Pi_{1,0,0}, \Pi_{1,0,1}, \Pi_{1,1,0}, \Pi_{1,1,1}]
\]

Which is a \(12 \times 1\) vector.

The steady state probabilities can be derived easily taking into consideration the independent nature of the states and are given by,

\[
\Pi_{0,0,0} = \frac{\lambda_c \lambda_s a \alpha}{XYZ}, \quad \Pi_{0,0,1} = \frac{\lambda_c \lambda_s a b}{XYZ}, \quad \Pi_{0,1,0} = \frac{\lambda_c \lambda_s \beta a}{XYZ}, \\
\Pi_{0,1,1} = \frac{\lambda_c \lambda_s \beta b}{XYZ}, \quad \Pi_{1/2,0,0} = \frac{\lambda_c \mu_d a \alpha}{XYZ}, \quad \Pi_{1/2,0,1} = \frac{\lambda_c \mu_d a b}{XYZ}, \\
\Pi_{1/2,1,0} = \frac{\lambda_c \mu_d \beta a}{XYZ}, \quad \Pi_{1/2,1,1} = \frac{\lambda_c \mu_d \beta b}{XYZ}, \quad \Pi_{1,0,0} = \frac{\mu_s \mu_d a \alpha}{XYZ}, \\
\Pi_{1,0,1} = \frac{\mu_s \mu_d a b}{XYZ}, \quad \Pi_{1,1,0} = \frac{\mu_s \mu_d \beta a}{XYZ}, \quad \Pi_{1,1,1} = \frac{\mu_s \mu_d \beta b}{XYZ}
\]

Where \(X = (\lambda_c \lambda_s + \lambda_c \mu_d + \mu_s \mu_d), \quad Y = (\alpha + \beta)\) and \(Z = (a + b)\).
Now the rate of crisis in steady state ($C_\infty$) is obtained as follows.

\[
P(\text{crisis in } [t(t + \Delta t)])
\]
\[
= p[x(t + \Delta t) = (0, 0, 1)/x(t) = (0, 0, 0)] \times p[x(t) = (0, 0, 0)]
\]
\[
+ p[x(t + \Delta t) = (0, 1, 1)/x(t) = (1/2, 1, 1)] \times p[x(t) = (1/2, 1, 1)]
\]
\[
+ p[x(t + \Delta t) = (1, 0, 1)/x(t) = (1, 1, 1)] \times p[x(t) = (1, 1, 1)]
\]
\[
+ p[x(t + \Delta t) = (1/2, 0, 1)/x(t) = (1/2, 0, 0)] \times p[x(t) = (1/2, 0, 0)]
\]
\[
+ p[x(t + \Delta t) = (1/2, 0, 1)/x(t) = (1/2, 1, 1)] \times p[x(t) = (1/2, 1, 1)]
\]
\[
+ p[x(t + \Delta t) = (1, 0, 1)/x(t) = (1, 0, 0)] \times p[x(t) = (1, 0, 0)]
\]
\[
+ p[x(t + \Delta t) = (0, 1, 1)/x(t) = (0, 1, 0)] \times p[x(t) = (0, 1, 0)]
\]
\[
+ O\Delta(t).
\]

Taking limit as $\Delta t \to 0$, we get,

\[
C_t = bP_{0,0,0}(t) + \lambda_s P_{1/2,1,1}(t) + \alpha P_{1,1,1}(t) + bP_{1/2,0,0}(t) + \alpha P_{1/2,1,1}(t) + bP_{0,0,0}(t) + bP_{0,1,0}(t)
\]

\[
C_\infty = \lim_{t \to \infty} [bP_{0,0,0}(t) + \lambda_s P_{1/2,1,1}(t) + \alpha P_{1,1,1}(t) + bP_{1/2,0,0}(t) + \alpha P_{1/2,1,1}(t) + bP_{1,0,0}(t) + bP_{0,1,0}(t)]
\]

that is

\[
C_\infty = [b\Pi_{0,0,0} + \lambda_s \Pi_{1/2,1,1} + \alpha \Pi_{1,1,1} + b\Pi_{1/2,0,0} + \alpha \Pi_{1/2,1,1} + b\Pi_{1,0,0} + b\Pi_{0,1,0}]
\]

Using the steady state probabilities, we get

\[
C_\infty = \frac{b}{XYZ} [\lambda_c \lambda_s (\alpha \alpha + \beta \alpha + \beta \mu_d) + \mu_s \mu_d (\alpha \beta + \alpha \alpha) \lambda_c \mu_d (\alpha \beta + \alpha \alpha)]
\]

4 Numerical Illustrations

The various parametric values are assigned as given to verify the nature of rate of crises by substitution of values in the equation of $C_\infty$.

$\lambda_c = 2, \lambda_s = 3, \mu_s = 9, \mu_d = 5; \alpha = 4, \beta = 6; a = 8, \quad b = 9, 11, 13, 15, 17$.

The table below gives the value of $C_\infty$ for various values $b$ keeping rest of the other variables as constants and the graph is drawn with $b$ against and is observed to have increasing crises rate $C_\infty$ as $b$ increases.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$C_\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2.8084</td>
</tr>
<tr>
<td>11</td>
<td>3.0712</td>
</tr>
<tr>
<td>13</td>
<td>3.2840</td>
</tr>
<tr>
<td>15</td>
<td>3.4593</td>
</tr>
<tr>
<td>17</td>
<td>3.6073</td>
</tr>
</tbody>
</table>
C. Mohan and R. Ramanarayanan

The steady state cost in different situations are determined by assuming the following values:
$C^0_{MP} = 25, C^{1/2}_{MP} = 15, C^1_{MP} = 10, C^1_M = 8, C^1_M = 5, C^0_B = 15, C^1_B = 8.$

Now applying these values in the steady state cost formula at $(ijk)$, we get the whole set steady state costs as given in the table

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Steady State Probability</th>
<th>Cost of state</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\Pi_{0,0,0}$</td>
<td>0.8400</td>
</tr>
<tr>
<td>2</td>
<td>$\Pi_{0,0,1}$</td>
<td>0.8938</td>
</tr>
<tr>
<td>3</td>
<td>$\Pi_{0,1,0}$</td>
<td>1.1790</td>
</tr>
<tr>
<td>4</td>
<td>$\Pi_{0,1,1}$</td>
<td>1.2464</td>
</tr>
<tr>
<td>5</td>
<td>$\Pi_{1/2,0,0}$</td>
<td>1.6878</td>
</tr>
<tr>
<td>6</td>
<td>$\Pi_{1/2,0,1}$</td>
<td>1.8615</td>
</tr>
<tr>
<td>7</td>
<td>$\Pi_{1/2,1,0}$</td>
<td>2.4035</td>
</tr>
<tr>
<td>8</td>
<td>$\Pi_{1/2,1,1}$</td>
<td>2.6208</td>
</tr>
<tr>
<td>9</td>
<td>$\Pi_{1,0,0}$</td>
<td>4.3263</td>
</tr>
<tr>
<td>10</td>
<td>$\Pi_{1,0,1}$</td>
<td>4.2614</td>
</tr>
<tr>
<td>11</td>
<td>$\Pi_{1,1,0}$</td>
<td>5.9010</td>
</tr>
<tr>
<td>12</td>
<td>$\Pi_{1,1,1}$</td>
<td>5.6557</td>
</tr>
<tr>
<td></td>
<td>Total expected steady state cost</td>
<td>32.8715</td>
</tr>
</tbody>
</table>

We observe that (i) rate of crisis increases as the value of b increases. (ii) the steady state cost when both money and manpower are full and the business is zero is highest because the business has to be got by paying premium. (iii) the steady cost when all are full is also highest which means that the business is doing well. The cost is the least when all the characteristics are zero is the least.
Acknowledgments

The authors thank the management of VelTechMultiTech Dr. Rangarajan, Dr. Sakunthala Engineering College, Avadi, Chennai for providing necessary facilities to bring out this research paper in a very short time and Mr. K.V.D. Kishore Kumar, Director, VelTech group of Educational Institutions who is always a source of encouragement and takes keen interest in our research activities. The authors thank Prof. Udayabaskaran and Prof. Jambulingam for their valuable suggestions and guidance.

References


Received: September, 2012