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Solution for Inverse Space-Dependent Heat Source Problems by Homotopy Perturbation Method

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Abstract

In this paper, the homotopy perturbation method is applied to solve the inverse space-dependent heat source problems. Some examples are given.The results reveal that the homotopy perturbation method is very effective and simple and gives the exact solution.

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1 Introduction

Many physical problems can be described by mathematical models that involve partial differential equations.The following parabolic partial differential equation is induced in the process of transportation, diffusion and conduction of natural materials:

$$u_t - a^2 \triangle u = f(x,t;u) \qquad (x,t) \in \Omega \times (0, t_{max}] \tag{1}$$

where u represents the state variable, a is the diffusion coefficient, Ω is abounded domain in R^d, and f denotes some physical laws,which is the source term here.

2 Homotopy perturbation method

To illustrate the basic idea of the homotopy technique on the nonlinear parabolic equation (1), we can construct a homotopy $v(r,p) : \Omega \times (0,1) \rightarrow R$ which satisfies

$$H(v,p) = (1-p)(\frac{\partial v}{\partial t} - \frac{\partial v_0}{\partial t}) + p(\frac{\partial v}{\partial t} - a^2 \triangle v - f(x,t;v)) = 0 \qquad p \in [0,1] \tag{2}$$

where p is an embedding parameter, and v_0 is the initial approximation of Equation (1) which satisfies the boundary conditions.According to the (HPM), we can first use the embedding parameter p a small parameter, and assume that the solutions of Eq. (2) can be written as a power series in p:

$$v = \sum_{i=0}^{\infty} v_i p^i \tag{3}$$

3 Numerical examples

Substituting (3) into (2), and comparing coefficients of terms with identical powers of p, leads to:

$$p^0 : (v_0)_t = 0$$

$$p^1 : (v_1)_t + (v_0)_t - a^2 \triangle v_0 - f(x,t;v) = 0$$

$$p^2 : (v_2)_t = (v_1)_t - a^2 \triangle v_1$$

$$\vdots$$

with initial condition $v_i(x,0) = 0$ for $i = 1, 2, 3, ...$

Example 3.1

In this paper for showing the efficiency of homotopy perturbation method for solving the inverse space-dependent heat source problems, here we present our numerical experiments by some example. Find the functions $f : [0,1] \to R$ and $u : [0,1] \times [0,1] \to R$ such that

$$u_t = \frac{1}{\pi^2} u_{xx} + f(x) \qquad 0 < x < 1, 0 < t \leq 1 \tag{4}$$

$$u(x,0) = sin(\pi x) \qquad 0 \leq x \leq 1 \tag{5}$$

$$u(0,t) = 0, \qquad 0 \leq t \leq 1 \tag{6}$$

$$u(1,t) = t \qquad 0 \leq t \leq 1 \tag{7}$$

with the over-specified condition

$$u(0.5,t) = \frac{t}{2} + e^{-t} \quad 0 \leq t \leq 1 \tag{8}$$

We can easily obtion the components of series (3) as

$$(u_0)_t = 0, u(x,0) = sin(\pi x) \to u_0 = sin(\pi x)$$

$$(u_1)_t = -sin(\pi x) + f(x) \to u_1 = -sin(\pi x)t + f(x)t$$

by continuing this process

$$(u_2)_t = sin(\pi x)t + \frac{1}{\pi^2} f_{xx}(x)t \rightarrow u_2 = sin(\pi x)\frac{t^2}{2} + \frac{1}{\pi^2} f_{xx}(x)\frac{t^2}{2}$$

Thus, we have the solution given by

$$u(x,t) = u_0 + u_1 + u_2 + ...$$

$$u(x,t) = sin(\pi x)(1 - t + \frac{t^2}{2} + ...) + f(x)t + \frac{1}{\pi^2} f_{xx}\frac{t^2}{2} + ...$$

$$= sin(\pi x)e^{-t} + f(x)t + \frac{1}{\pi^2} f_{xx}\frac{t^2}{2} + ...$$

by using (6) , (7) , (8) ,we have $u(x,t) = sin(\pi x)e^{-t} + xt$ that is the exact solution and $f(x) = x$.

Example 3.2

Find the functions $f : [0,1] \rightarrow R$ and $u : [0,1] \times [0,1] \rightarrow R$ such that

$$u_t = u_{xx} + f(t) \qquad 0 \le t \le 1 \qquad (9)$$

$$u(x,0) = sin(x) + \frac{x^4}{4} \qquad 0 \le x \le 1 \qquad (10)$$

$$u(0,t) = 0 \qquad 0 \le t \le 1 \qquad (11)$$

$$u(1,t) = sin(1)e^{-t} + 3t + \frac{1}{4} \qquad 0 \le t \le 1 \qquad (12)$$

with the over-specified condition

$$u(0.5,t) = sin(0.5)e^{-t} + \frac{3t}{4} + \frac{1}{64} \qquad 0 \le t \le 1 \qquad (13)$$

We can easily obtion the components of series (3) as

$$p^0 : (u_0)_t = 0, u(x,0) = sin(x) + \frac{x^4}{4} \rightarrow u_0 = sin(x) + \frac{x^4}{4}$$

$$p^1 : (u_1)_t = -sin(x) + 3x^2 + f(t) \rightarrow u_1 = -sin(x)t + 3x^2t + \int f(t)dt$$

$$p^2 : (u_2)_t = sin(x)t + 6t \rightarrow u_2 = sin(x)\frac{t^2}{2} + 6\frac{t^2}{2}$$

Thus, we have the solution given by

$$u(x,t) = u_0 + u_1 + u_2 + ...$$

$$u(x,t) = sin(x)(1 - t + \frac{t^2}{2} + ...) + \frac{x^4}{4} + 3x^2t + \int f(t)dt + 3t^2$$

$$= sin(x)e^{-t} + \frac{x^4}{4} + 3x^2t + \int f(t)dt + 3t^2$$

by using (11),(12),(13) we have

$$\int f(t)dt + 3t^2 = 0 \rightarrow f(t) = -6t$$

4 Conclusion

The homotopy perturbation method deforms a complex problem under study to a simple problem routinely. The method is of remarkable simplicity, while the obtained results are exact on the whole solution domain. The method can be applied to various other nonlinear problems without any difficulty.

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