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KMF Algorithm for Solving the Cauchy Problem for Helmholtz Equation

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Abstract

In this paper, we consider a Cauchy problem for Helmholtz equation which is to find the missing conditions on an inaccessible part of the boundary from additional conditions on the other part of the boundary. To solve this inverse problem known to be ill-posed , we use the iterative algorithm (KMF standard algorithm) proposed by Kozlov, Mazya and Fomin. We describe a new formulation named (KMF developed algorithm) to reduce the number of iterations required to achieve convergence, and that with better accuracy. The numerical implementation of the new algorithm is made by the finite element method, using the software FreeFem. Numerical tests are presented for judged of its effectiveness.

Keywords: Cauchy problem, inverse problem, Helmholtz equation, iterative method, FreeFem

1 Introduction

In this work, we consider a Cauchy problem for the Helmholtz equation which is to find conditions on a missing part of the boundary from additional conditions on the other part. This type of problem arises and can be encountered in several areas of science and engineering when conditions on the whole boundary cannot be obtained, we only know the noisy data on a part of the boundary or at some interior points of concerned domain. This type of problem is often used to describe the vibration of a structure [4], the acoustic cavity problem [9], the radiation wave [16], the scattering of a wave [19] and the problem of heat conduction in fins, see [1],[15]. The direct problem where Dirichlet, Neumann

or mixed boundary value problems for the Helmholtz equation are known in the entire boundary, is well-posed via the removal of the eigenvalues of the Laplacian operator [4]. Many authors have studied this direct problem, and have proposed different methods for its resolution, you can see [20],[8]. The inverse problem is known to be severely ill-posed, i.e. the existence, uniqueness and stability of their solutions, that are the three properties required to define well-posed problem according to Hadamard, are not always guaranteed [7]. Unlike in direct problems, the uniqueness of the Cauchy problem is guaranteed without the necessity of removing the eigenvalues for the Laplacian. However, existence and stability are the most delicate problem since a small change in the Cauchy data may result in a dramatic change in the solution. In order to solve the Cauchy problem for the Helmholtz equation we have proposed several performing methods to overcome of the ill-posed nature of this kind of problem, such as the Landweber method with boundary element method (BEM) [10], the conjugate gradient method [11], the method of fundamental solutions (MFS) [12]. Recently, a lot of regularization methods have been provided. For computational aspects, the readers can consult for example [5],[6],[17].
There are also a group of iterative method that has the advantage to allow any physical constraint to be easily taken into account directly in the scheme of the iterative algorithm, simplicity of the implementation schemes and the similarity of schemes for problems with linear and non linear operators. One possible disadvantage of this kind of method is the large number of iterations that may be required in order to achieve convergence. Based on these reasons, we have decided in this work to consider the KMF algorithm addressed by Kozlov, Mazya and Fomin since 1991 [18]. For Helmholtz equation, in the work of Lesnic and al [13], we present the implementation of this method using the BEM; and in [14] they present this method in a comparative study whith other methods where we conclude that the alternating iterative algorithm is less accurate than the other methods considered.
Based on the iterative procedure proposed by [2],[3] for the Cauchy problem for Laplace equation, we propose a new algorithm called KMF developed algorithm to reduce the number of iterations needed to achieve convergence with more precision. Furthermore, in order to cease the iterative procedure before the effects of the accumulation of noise become dominant, and the errors in the numerical solution start increasing, a discrepancy stopping criterion is also proposed. The numerical implementation of the algorithm is performed by the finite element method using the software FreeFem.
The second section is devoted to the presentation of the Cauchy problem for Helmholtz equation. Section 3 presents a classical KMF algorithm which enables one to find an approximate solution to that problem. Section 4 considers an alternating KMF algorithm and exhibits the relationship it has with the classical KMF algorithm. Finally, section 5 presents a numerical example

showing the feasibility of the alternating formulation, its ability to find an approximate solution more accurately in less iteration.

2 Mathematical Formulation

Let Ω be an open bounded domain in $\mathbb{R}^2$, with a smooth boundary Γ. We consider a partition of this boundary: $\Gamma = \Gamma_0 \cup \Gamma_1$ where $\Gamma_0 \cap \Gamma_1 = \emptyset$ and $mes(\Gamma_1) \neq 0$.
The problem is to find the temperature T solution of the following problem:

$$\begin{cases} LT = 0 & on \quad \Omega \\ T = f & in \quad \Gamma_0 \\ \partial_n T = g & in \quad \Gamma_0 \end{cases} \tag{1}$$

$L = -\Delta + k^2$ where $k = \alpha + i\beta$ with $\alpha, \beta \in \mathbb{R}$ and $i = \sqrt{-1}$.
Γ_0 is overspecified by prescribing both the temperature $T_{/\Gamma_0}$ and the flux $\partial_n T_{/\Gamma_0}$
$\Gamma_1 = \Gamma/\Gamma_0$ is underspecified since both the temperature $T_{/\Gamma_1}$ and the $\partial_n T_{/\Gamma_1}$ are unknown and have to be determined.
It should be noted that the problem studied in this paper is of practical importance. For example, the Cauchy problem (1), where $k \in \mathbb{C}/\mathbb{R}$, represents the mathematical model for the heat conduction in plate finned heat exchangers, see [15] for example, for which the temperature and the flux can be measured at some points on the fin, whilst both the temperature and the flux are unknown at the fin base.

3 Description of the standard algorithm

The Cauchy problem (1) is ill-posed and cannot be solved numerically by using a direct approach. Instead we use a convergent iterative algorithm which was proposed by Kozlov et al. [18] for Cauchy problems associated to linear, elliptic, self-adjoint and positive-definite operators.
This iterative algorithm (KMF algorithm standard) investigated is based on reducing this ill-posed problem to a sequence of mixed well-posed boundary value problems and consists of the following steps:
Step1. specify an initial approximation v_0 for the flux on Γ_1.
Step2. Solve the following mixed well-posed boundary value problem:

$$\begin{cases} LT^{(0)} = 0 & on \quad \Omega \\ \partial_n T^{(0)} = v_0 & in \quad \Gamma_1 \\ T^{(0)} = f & in \quad \Gamma_0 \end{cases} \tag{2}$$

$$\text{to obtain} \quad u_0 = T^{(0)}_{/\Gamma_1} \tag{3}$$

Step3. for $n \geq 1$, solving alternatively the following two mixed well-posed boundary value problems:

$$(a) \quad \begin{cases} LT^{(2n-1)} = 0 & on \quad \Omega \\ T^{(2n-1)} = u_{n-1} & in \quad \Gamma_1 \\ \partial_n T^{(2n-1)} = g & in \quad \Gamma_0 \end{cases} \quad \text{and}(b) \quad \begin{cases} LT^{(2n)} = 0 & on \quad \Omega \\ \partial_n T^{(2n)} = v_n & in \quad \Gamma_1 \\ T^{(2n)} = f & in \quad \Gamma_0 \end{cases} \tag{4}$$

$$\text{to obtain } v_n = \partial_n T^{(2n-1)}_{/\Gamma_1} \qquad \text{to obtain } u_n = T^{(2n)}_{/\Gamma_1} \tag{5}$$

Step4. repeat the step3 until a prescribed stopping criterion is satisfied.

3.1 Observations

- Although not illustrated here, an important conclusion is reported, namely, that the alternating iterative algorithm described is not convergent for the differential operator $L = \Delta + k^2$ for $k = \alpha + i\beta$, $\alpha \in \mathbb{R}$,and $\beta = 0$. The reason is that the proof of convergence of the iterative algorithm of Kozlov et al. [18] requires, as a necessary condition, $L = \Delta + k^2$ to be positive-definite differential operators and this is not the case when k is real.

- Let $H^1(\Omega)$ be the sobolev space and $H^{\frac{1}{2}}(\Gamma)$ be the space of traces on Γ corresponding to $H^1(\Omega)$. We denote by $H^{\frac{1}{2}}(\Gamma_i)$ the space of functions from $H^{\frac{1}{2}}(\Gamma)$ that are bounded on Γ_i, and by $H^{\frac{1}{2}}(\Gamma_i)^*$ the dual space of $H^{\frac{1}{2}}(\Gamma_i)$ for $i = 0, 1$.
  Kozlov et al showed that if Γ is smooth, $f \in H^{\frac{1}{2}}(\Gamma_0)$ and $g \in H^{\frac{1}{2}}(\Gamma_0)^*$ and k is purely imaginary, i.e. $\alpha = 0$ then the alternatng algorithm on step1-4 produces two sequences of approximate solution $(T^{2n-1})_{n\geq 1}$ and $(T^{2n})_{n\geq 1}$ which both converge to the solution $T \in H^1(\Omega)$ to the Cauchy problem (1) for any initial guess $v_0 \in H^{\frac{1}{2}}(\Gamma_1)^*$. Also the same conclusion is obtained if at the step1, we specify an initial guess $u_0 \in H^{\frac{1}{2}}(\Gamma_1)$, instead of an intial guess for $v_0 \in H^{\frac{1}{2}}(\Gamma_1)^*$ and we modify accordingly the steps2-3 of the algorithm.

- We note that if the initial guess $v_0 \in H^{\frac{1}{2}}(\Gamma_1)^*$ and the boundary data $f \in H^{\frac{1}{2}}(\Gamma_0)$, and $g \in H^{\frac{1}{2}}(\Gamma_0)^*$, the problems (2)-(4) are well-posed and solvable in $H^1(\Omega)$, provided that k^2 is not an eigenvalue of the Laplacian operator Δ, see [17].

4 Description of the KMF developed algorithm

In this study we develop a new algorithm called KMF developed Algorithm in order to improve the rate of convergence of the iterative algorithm described. The main idea of the algorithm alternative proposed is based in completing the missing data in alternative way to the two sub-parts of the inaccessible boundary. The inaccessible part is subdivided in two parts, and the KMF standard algorithm is used to complete the data in the first part, then to complete the data in the second part in an alternative way.
For this, we consider $\Gamma_1 = \Gamma_{1,1} \cup \Gamma_{1,2}$ such that $\Gamma_{1,1} \cap \Gamma_{1,2} = \emptyset$ and $mes(\Gamma_{1,1}) = mes(\Gamma_{1,2})$.
The algorithm consists of the following steps:
Step1. specify an initial approximation v_0 for the flux on Γ_1
Step2. Solve the well-posed problem :

$$\begin{cases} LT^{(0)} = 0 & on & \Omega \\ \partial_n T^{(0)} = v_0 & in & \Gamma_{1,1} \cup \Gamma_{1,2} \\ T^{(0)} = f & in & \Gamma_0 \end{cases} \tag{6}$$

$$\text{to obtain} \quad u_{1,0} = u^{(0)}_{/\Gamma_{1,1}} \quad \text{and} \quad u_{2,0} = u^{(0)}_{/\Gamma_{1,2}} \tag{7}$$

Step3. for $n \geq 1$ solve the two well-posed problems:

$$(a) \begin{cases} Lu^{(2n-1)} = 0 & on & \Omega \\ u^{(2n-1)} = u_{1,n-1} & in & \Gamma_{1,1} \\ u^{(2n-1)} = u_{2,n-1} & in & \Gamma_{1,2} \\ \partial_n u^{(2n-1)} = g & in & \Gamma_0 \end{cases} \quad (b) \begin{cases} Lv^{(2n-1)} = 0 & on & \Omega \\ \partial_n v^{(2n-1)} = v_{1,n} & in & \Gamma_{1,1} \\ v^{(2n-1)} = u_{2,n-1} & in & \Gamma_{1,2} \\ \partial_n v^{(2n-1)} = g & in & \Gamma_0 \end{cases} \tag{8}$$

$$\text{to obtain } v_{1,n} = \partial_n u^{(2n-1)}_{/\Gamma_{1,1}} \qquad \text{to obtain } v_{2,n} = \partial_n v^{(2n-1)}_{/\Gamma_{1,2}} \tag{9}$$

Step4. for $n \geq 1$ solve the two well-posed problems:

$$(a) \begin{cases} Lu^{(2n)} = 0 & on & \Omega \\ \partial_n u^{(2n)} = v_{1,n} & in & \Gamma_{1,1} \\ \partial_n u^{(2n)} = v_{2,n} & in & \Gamma_{1,2} \\ u^{(2n)} = f & in & \Gamma_0 \end{cases} \quad (b) \begin{cases} Lv^{(2n)} = 0 & on & \Omega \\ v^{(2n-1)} = u_{1,n} & in & \Gamma_{1,1} \\ \partial_n v^{(2n)} = v_{2,n} & in & \Gamma_{1,2} \\ v^{(2n)} = f & in & \Gamma_0 \end{cases} \tag{10}$$

$$\text{to obtain } u_{1,n} = u^{(2n)}_{/\Gamma_{1,1}} \qquad \text{to obtain } u_{2,n} = v^{(2n)}_{/\Gamma_{1,2}} \tag{11}$$

Step5. repeat the step3-4 until a prescribed stopping criterion is satisfied.

4.1 Remarks

- If we consider every iteration to consist of solving the four mixed well-posed problems from the Step3 and 4 of the algorithm, then for every $n \geq 1$ the following approximations are obtained at the iteration n:
  $u_{1,n}$ for the Dirichlet condition on the $\Gamma_{1,1}$.
  $u_{2,n}$ for the Dirichlet condition on the $\Gamma_{1,2}$.
  $v_{1,n}$ for the Neumann condition on the boundary$\Gamma_{1,1}$.
  $v_{2,n}$ for the Neumann condition on the boundary $\Gamma_{1,2}$.

- The KMF developed algorithm can be seen as two parallel problems of KMF standard algorithm. These two problems are initialized with the same initial data. Each problem allows to obtain approximation in each subpart $\Gamma_{1,i}$ where $i = 1, 2$ (for the approximation in $\Gamma_{1,1}$ the two well-posed problems (8-a) and (10-a), for the approximation in $\Gamma_{1,2}$ the two well-posed problems (8-b) and (10-b).

- The missing Dirichlet condition in the part Γ_1 can be obtained from the problem (10-b) since the condition $u_{1,n} = u^{(2n)}_{/\Gamma_{1,1}}$obtained from (10-a) is introduced in (10-b) which also provides $u_{2,n} = v^{(2n)}_{/\Gamma_{1,2}}$.

- The missing Neumann condition in the part Γ_1 can be obtained from the problem (8-b), since the condition $v_{1,n} = \partial_n u^{(2n-1)}_{/\Gamma_{1,1}}$ obtained from (8-a) is introduced in (8-b) which also provides $v_{2,n} = \partial_n v^{(2n-1)}_{/\Gamma_{1,2}}$.

5 Numerical results and discussion

In many practical applications Γ_0 and Γ_1 are two simple arcs having in common only the end-points. In this section, we illustrate the numerical results obtained using the KMF developed algorithm described in section 4 in comparison with the Standard KMF algorithm described in section 3. In addition, we investigate the convergence and the accuracy of the solution with respect the number of iterations.
In order to present the performance of the numerical method proposed, we solve the Cauchy problem for an example in a two-dimensional smooth geometry, namely the unit disc $\Omega = \{(x, y) \in \mathbb{R}^2/x^2 + y^2 \leq 1\}$, since the condition of a smooth domain is required by the theoretical analysis of Kozlov et al.
We assume that the boundary of the solution domain is divided into two disjointed parts Γ_0 and Γ_1 , namely;
$\Gamma_0 = \{(x, y) \in \mathbb{R}^2/x = cos(t), y = sin(t), \theta_0 \leq t \leq 2\pi\}$
$\Gamma_1 = \{(x, y) \in \mathbb{R}^2/x = cos(t), y = sin(t), 0 \leq t \leq \theta_0\}$

where θ_0 is a specified angle in the interval $(0, 2\pi)$
The analytical function to be retrieved is given by:
$T_{ex} = exp(ax + by)$, with $a = 0.5, b = \sqrt{\beta^2 - a^2}$
That is solution of the Cauchy problem of the Helmholtz equation where $k = \alpha + i\beta$ where $\alpha = 0$ and $\beta = 1.0$
The known data are given by:
$f = exp(ax + b\sqrt{1 - x^2})$ and $g = (an_1 + bn_2)(ax + b\sqrt{1 - x^2})$
For the implementation of the iterative algorithm we use the software FreeFem, and solve the well-posed problems in the algorithm by the finite element method. In this example, we use a finite element method with continuous piecewise linear polynomials to provide simultaneously the unspecified boundaries Dirichlet and Neumann.
An arbitrary function $v_0 \in (H^{1/2}(\Gamma_1))^*$ may be specified as an initial guess for the flux on Γ_1, but in order to improve the rate of convergence of the iterative procedure we have chosen a function which ensures the continuity of the flux at the endpoints of Γ_1 and which is also linear with respect to the polar angle θ. This initial guess is given by:

$$v_0(x) = (\tfrac{2\pi - \theta(x)}{2\pi - \theta_0})g(\overline{x}) + (\tfrac{\theta(x) - \theta_0}{2\pi - \theta_0})g(\overline{y}) \quad \text{where} \quad x \in \Gamma_1 \tag{12}$$

where $\overline{x}$ and $\overline{y}$ are the endpoints of Γ_1
The convergence of the algorithm may be investigated by evaluating at every iteration the error:

$$\mathrm{e}_T = \|u_n - T_{ex}\|_{0,\Gamma_1} \quad \text{and} \quad \mathrm{e}_v = \|v_n - \partial_n T_{ex}\|_{0,\Gamma_1} \tag{13}$$

where u_n is the approximation obtained for the function on the boundary Γ_1 after n iterations and T_{ex} is the exact solution of the problem (1). However, in practical applications the error e_T cannot be evaluated since the analytical solution is not known and therefore the following stopping criterion:

$$E = \|u_n - u_{n+1}\|_{0,\Gamma_1} \leq \epsilon. \tag{14}$$

where ϵ is a small real, has to be used.
In order to illustrate the typical numerical results, we have taken different choices of the polar angle θ_0 .
In figure 1, the comparison between the results for e_T obtained by the KMF standard algorithm and the KMF developed algorithm shows that the proposed algorithm reduces the number of iterations needed to achieve the convergence and allows obtaining more accurate results.
Table 1 illustrates the different results obtained for different choices of measure of the part of boundary during the iterative process.
For example; in the KMF standard algorithm, the error e_T obtained for $\theta_0 = \frac{\pi}{3}$ after 200 iterations is 9.510^{-3}. However, with the algorithm proposed, better

result is obtained in the iteration 100 with error equal to $2,9.10^{-3}$.
The same conclusion can be drawn for all the results.

θ_0	$\pi/6$		$\pi/4$		$\pi/3$		$\pi/2$	
iteration	30	100	30	100	100	200	130	200
$e_T stand \times (10^3)$	2.56	2.55	25.5	3.77	20.2	9.55	45.7	25.7
$e_T New \times (10^3)$	1.09	0.686	4	1.72	2.93	2.91	46	6.21

table1: The error e_T for standard and New algorithm given for different iterations

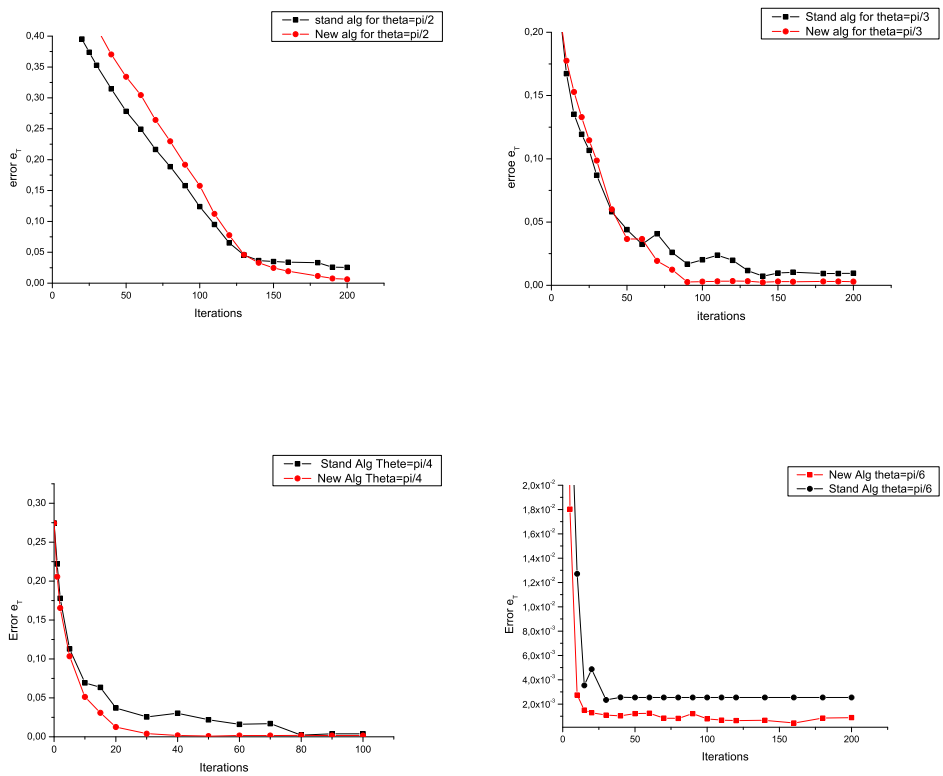

Figure 1: The error e_u as a function of the number of iterations for different choice of $\theta_0(\pi/2, \pi/3, \pi/4, \pi/6)$ obtained for KMF developed algorithm (New) in comparison with classical algorithm (Stand)

The figure 2 showed that the algorithm proposed decreases considerably the number of iteration necessary to achieve the convergence that can be reduced in approximationg the Neumann missing data.
Figure 3 shows the numerical results obtained in approximating the function u in the part of the boundary Γ_1, indicating that from a choice of an initial data, we obtain satisfying results for both algorithms. However, the KMF developed algorithm requires less iteration to achieve more accurate convergence.

Such implementation of the algorithm allows us to notice that after the number of iterations is sufficiently increased, the error become small; this

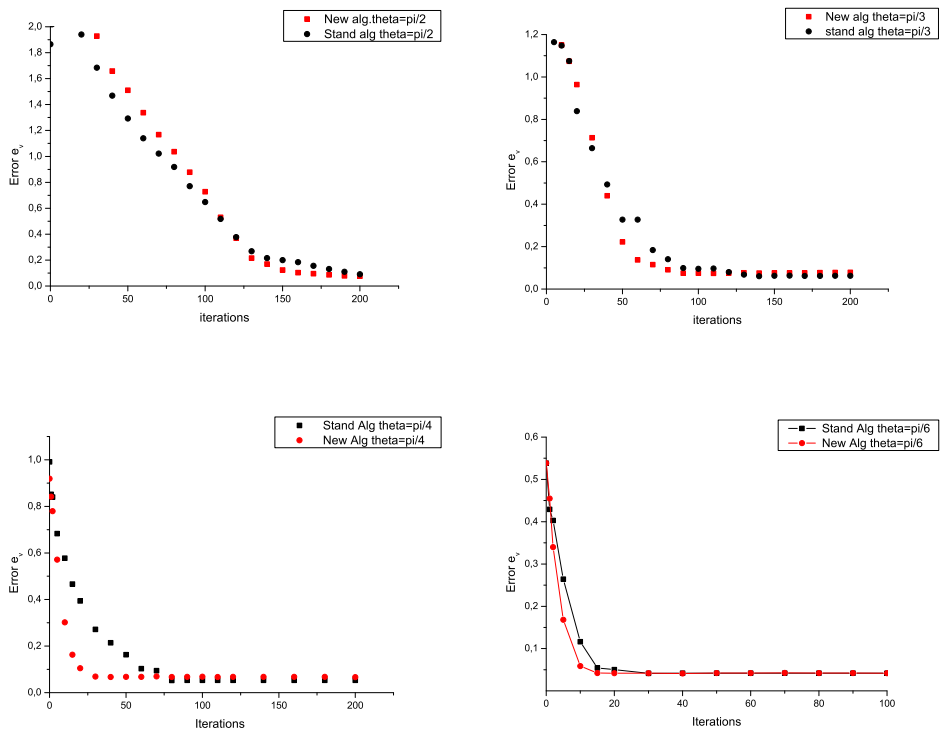

Figure 2: The error e_v as a function of the number of iterations for different choice of $\theta_0(\pi/2, \pi/3, \pi/4, \pi/6)$ obtained for KMF developed algorithm (New) in comparison with classical algorithm (Stand)

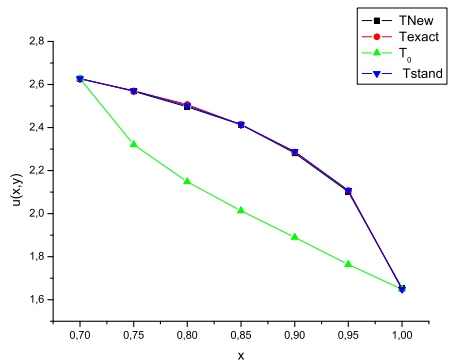

Figure 3: The numerical results for the temperature T on the boundary Γ_1 obtained with KMF developed algorithm (Tnew) in comparison with the analytical solution (Texact), the initial guess (T_0) and the solution with KMF standard algorithm (Tstand

shows that the numerical solution is accurate and consistent with the number of iterations.

Furthermore, when perturbations are introduced into the given data problem the numerical results obtained are stable.

6 Conclusion

In this paper we have investigated the KMF iterative algorithm for a Cauchy problem for Helmholtz equation. Comparison of numerical results with those obtained by the KMF standard algorithm show that the proposed algorithm significantly reduces the number of iterations needed to achieve the convergence and produces more accurate results. In addition, it can be concluded that the proposed algorithm is very efficient to reduce the rate of convergence. When perturbations are introduced into the given data problem the results are stable. Overall, it can be concluded that the alternating iterative algorithm proposed produces a convergent, stable and accurate numerical solution.

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