A New Dual Based Approach for the Unbalanced Fuzzy Transportation Problem

A. Edward Samuel and M. Venkatachalapathy

1 Ramanujan Research Centre, P.G. & Research Department of Mathematics
Government Arts College (Autonomous)
Kumbakonam-612 001, Tamil Nadu, India

2 Department of Mathematics, Oxford Engineering College
Tiruchirappalli-620 009, Tamil Nadu, India

Abstract: This paper proposes a new approach to solve the trapezoidal fuzzy unbalanced transportation problem. The proposed method is very efficient in terms of computation. The algorithm of this approach is presented, and explained briefly. Finally, a numerical example is described in the paper to show its efficiency.

Keywords: Generalized trapezoidal fuzzy Number, Fuzzy unbalanced transportation problem, Ranking function

1. Introduction

The basic transportation problem was originally developed by Hitchcock. The transportation problems can be modelled as a standard linear programming problem, which can then be solved by the simplex method. However, because of its very special mathematical structure, it was recognized early that the simplex method applied to the transportation problem can be made quite efficient in terms of how to evaluate the necessary simplex method information? (variable to enter the basis, variable to leave the basis and optimality conditions). Charnes and Cooper [3] developed the stepping stone method which provides an
alternative way of determining the simplex method information. Dantzig and Thapa [8] used simplex method to the transportation problem as the primal simplex transportation method. An initial basic feasible solution (IBFS) for the transportation problem can be obtained by using the north-west corner rule, row minima, column minima, Matrix minima, or Vogel’s approximation method. The modified distribution method is useful for finding the optimal solution for the transportation problem. In general, the transportation problems are solved with the assumptions that the coefficients or cost parameters are specified in a precise way i.e., in crisp environment.

In real life, there are many diverse situations due to uncertainty in judgments, lack of evidence etc. Sometimes it is not possible to get relevant precise data for the cost parameter. This type of imprecise data is not always well represented by random variable selected from a probability distribution. Fuzzy number may represent this data. So Fuzzy decision making method is in need here.

Zimmermann [15] showed that solutions obtained by fuzzy linear programming method are always efficient. Subsequently, Zimmermann’s fuzzy linear programming has developed into several fuzzy optimization methods for solving the transportation problems. Chanas, Kolodziejczyk, Machaj [4] presented a fuzzy linear programming model for solving transportation problems with crisp cost coefficient and fuzzy supply and demand values. Chanas and Kuchta [5] proposed the concept of the optimal solution for the transportation problem with fuzzy coefficients expressed as fuzzy numbers, and developed an algorithm for obtaining the optimal solution. Saad and Abbas discussed the solution algorithm for solving the transportation problem in fuzzy environment.

Liu and Kao described a method for solving fuzzy transportation problem based on extension principle. Gani and Razak presented a two stage cost minimizing fuzzy transportation problem (FTP) in which supplies and demands are trapezoidal fuzzy numbers. A parametric approach is used to obtain a fuzzy solution and the aim is to minimize the sum of the transportation costs in two stages.

Lin introduced a genetic algorithm to solve transportation problem with fuzzy objective functions. Dinagar and Palanivel [12] investigated FTP, with the aid of trapezoidal fuzzy numbers. Fuzzy modified distribution method is proposed to find the optimal solution in terms of fuzzy numbers. Pandian and Natarajan [13] proposed a new algorithm namely, fuzzy zero point method for finding a fuzzy optimal solution for a FTP, where the transportation cost, supply and demand are represented by trapezoidal fuzzy numbers.

In this paper, a new algorithm is proposed for solving a special type of fuzzy transportation problem by assuming that a decision maker is uncertain about the precise values of transportation fuzzy cost only but there is no uncertainty about the supply and demand of the product. In the proposed algorithm transportation fuzzy costs are represented by generalized trapezoidal fuzzy numbers. To illustrate the proposed algorithm a numerical example is solved and
the proposed approach is very easy to understand and to apply on real life transportation problems for the decision makers.

This paper is organized as follows: In section 2, formulation of fuzzy linear programming’s. In section 3 and 4, preliminaries and basic definitions are discussed. In section 5, arithmetic operations are presented. In section 6, ranking function is presented. In section 7, proposed method for finding the IBFS by using ranking function. In section 8, a numerical example is solved. The conclusion is discussed in section 9.

2. Formulation of Fuzzy linear programming

This problem can be expressed as fuzzy linear programming model as follows:

\[ \text{Minimize } \phi = \sum_{i=0}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} x_{ij} \]

Subject to \( \sum_{j=1}^{n} x_{ij} \leq a_{i}, i = 1,2, \ldots m \) ...........................(1)

\( \sum_{i=1}^{m} x_{ij} \geq b_{j}, j = 1,2, \ldots n \) ...........................(2)

\( x_{ij} \geq 0, i = 1,2, \ldots m; \ j = 1,2, \ldots n \)

LP1

Here, all \( a_{i} \) and \( b_{j} \) are assumed to be positive, and \( a_{i} \) are normally called supplies and \( b_{j} \) are called demands, as shown in figure 1. The fuzzy cost \( \tilde{c}_{ij} \) are all non-negative. If \( \sum_{i=1}^{m} a_{i} = \sum_{j=1}^{n} b_{j} \), it is a balanced transportation problem. If this condition is not met, a dummy origin or destination is generally introduced to make the problem balanced. The proposed method are presented later the does not require a transportation problem to be balanced. The approach can be applied to both balanced and unbalanced problems. So the constrains set (1) represented as “\( \leq \)” and constraint set (2) as “\( \geq \)” instead of “\( = \)” for both cases in a balanced problem. This is one of the advantages of this new proposed method of stepping stone method. The unbalanced problem is not required to be converted into a balanced problem, no dummy origin or destination is introduced, so some time space are saved.

If model LP1 is considered as the primal, then its dual can be formulated as follows:
Maximize $\psi = \sum_{j=1}^{n} b_j \tilde{v}_j - \sum_{i=1}^{m} a_i \tilde{u}_i$

subject to $\tilde{v}_j - \tilde{u}_i \leq \tilde{c}_{ij}, i = 1,2,..,m; j = 1,2,..,n \quad \ldots \ldots \ldots \quad (3)$

$\tilde{u}_i, \tilde{v}_j \geq 0, i = 1,2,..,m; j = 1,2,..,n; \quad \text{(LP2)}$

Here, all $\tilde{u}_i$ and $\tilde{v}_j$ are dual variables.

3. Preliminaries

In this section, some basic definition, arithmetic operations and an existing method for comparing generalized fuzzy numbers are presented.

4. Basic Definitions

In this section, some basic definitions are presented.

4.1 Definition [1]

The characteristic function $\mu_A$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each number in $X$. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set $X$ fall within a specified range i.e. $\mu_{\tilde{A}}: X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set $A$.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

4.2 Definition [1]

A fuzzy set $\tilde{A}$, defined on the universal set of real numbers $\mathbb{R}$, is said to be generalized fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: \mathbb{R} \rightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (\infty, a] \cup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ Strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b,c]$, where $a < b < c < d$. 

}$
4.3 Definition [1]

A fuzzy number \( \widetilde{A} = (a, b, c, d) \) is said to be trapezoidal fuzzy number if its membership function is given by

\[
\mu_{\widetilde{A}}(x) = \begin{cases} 
(x - a) / (b - a), & a \leq x < b \\
1, & b \leq x \leq c \\
(x - d) / (c - d), & c < x \leq d \\
0, & \text{otherwise}
\end{cases}
\]

4.4 Definition [1]

A fuzzy set \( \widetilde{A} \), defined on the universal set of real numbers \( \mathbb{R} \), is said to be generalized fuzzy number if its membership function has the following characteristics:

1. \( \mu_{\widetilde{A}} : \mathbb{R} \rightarrow [0, \omega] \) is continuous.
2. \( \mu_{\widetilde{A}}(x) = 0 \) for all \( x \in (-\infty, a] \cup [d, \infty) \).
3. \( \mu_{\widetilde{A}}(x) \) Strictly increasing on \( [a, b] \) and strictly decreasing on \( [c, d] \).
4. \( \mu_{\widetilde{A}}(x) = \omega \), for all \( x \in [b, c] \), where \( 0 < \omega \leq 1 \).

5. Arithmetic operations

In this section, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on the universal set of real numbers \( \mathbb{R} \), are presented.

Let \( \widetilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1) \) and \( \widetilde{A}_2 = (a_2, b_2, c_2, d_2; \omega_2) \) be two generalized trapezoidal fuzzy numbers then

(i) \( \widetilde{A}_1 + \widetilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \text{minimum}(\omega_1, \omega_2)) \)

(ii) \( \widetilde{A}_1 - \widetilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2; \text{minimum}(\omega_1, \omega_2)) \)

(iii) \( \widetilde{A}_1 \times \widetilde{A}_2 \cong (a, b, c, d; \text{minimum}(\omega_1, \omega_2)) \), where

\[
a = \text{minimum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2) \\
b = \text{minimum}(b_1b_2, b_1c_2, c_1b_2, c_1c_2) \\
c = \text{maximum}(b_1b_2, b_1c_2, c_1b_2, c_1c_2) \\
d = \text{maximum}(a_1a_2, a_1d_2, a_2d_1, d_1d_2)
\]

(iv) \( \lambda \widetilde{A}_1 = \begin{cases} \lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; \omega_1, & \lambda > 0 \\
\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; \omega_1, & \lambda < 0
\end{cases} \)
6. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of ranking function \([1], \mathcal{R} : F(\mathbb{R}) \rightarrow \mathbb{R}\), where \(F(\mathbb{R})\) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) \(\tilde{A} >_{\mathcal{R}} \tilde{B}\) if and only if \(\mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{B})\)

(ii) \(\tilde{A} <_{\mathcal{R}} \tilde{B}\) if and only if \(\mathcal{R}(\tilde{A}) < \mathcal{R}(\tilde{B})\)

(iii) \(\tilde{A} =_{\mathcal{R}} \tilde{B}\) if and only if \(\mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{B})\)

Let \(\tilde{A}_1 = (a_1, b_1, c_1, d_1; \omega_1)\) and \(\tilde{B} = (a_2, b_2, c_2, d_2; \omega_2)\) be two generalized trapezoidal fuzzy numbers and \(\omega = \text{minimum}(\omega_1, \omega_2)\) then

\[\mathcal{R}(\tilde{A}) = \frac{\omega(a_1 + b_1 + c_1 + d_1)}{4}\]

and

\[\mathcal{R}(\tilde{B}) = \frac{\omega(a_2 + b_2 + c_2 + d_2)}{4}\]

7. The Proposed Method

The main idea of the dual approach is to obtain a feasible solution to the dual problem and its corresponding matrix. Then the duality theory is used to check the optimal condition and to get the leaving cell. All non-basic cells are evaluated in order to get the entering cell. Finally, the entering cell replaces the leaving cell and the matrix is updated. The dual-matrix approach is presented as follows:

Step 0: Initialization

Step 0.1: Set \(A = (b_1, b_2, b_3, \ldots, b_n, a_1, a_2, \ldots, a_m)\).

Step 0.2: Set \(\bar{v}_i = (0, 0, 0, 0; i_1)\); \((i = 1, 2, \ldots, m)\) and

let \(\bar{v}_j = \tilde{c}_j = \min(\tilde{c}_j, i = 1, 2, \ldots, m)\); \((j = 1, 2, \ldots, n)\). Ties can be broken arbitrarily. The corresponding cells to \(\tilde{c}_j\) are \((i, j)\), \((j = 1, 2, \ldots, n)\), respectively.

Step 0.3: Let the basic cell set

\[\Gamma = \{(i_1, 1), (i_2, 2), \ldots, (i_n, n), (1, 0), (2, 0), \ldots, (m, 0)\}\]. The cells \((1, 0), (2, 0), \ldots, (m, 0)\) are called virtual cells because they do not exist in the original transportation problem matrix.
Step 0.4: Let the matrix $D=\begin{bmatrix} d_{ij} \end{bmatrix}$, $i, j = 1, 2, \ldots, m+n$;

where $d_{ij} = \begin{cases} 1, & i, j = 1, 2, 3, \ldots, n \\ -1, & i = 1, 2, 3, \ldots, n; j = n + i_1, n + i_2, \ldots, n + i_n \\ -1, & i, j = 1, 2, \ldots, n+1, n+2, \ldots, m+n \\ 0, & otherwise \end{cases}$

and compute the objective $\psi = \sum_{j=1}^{n} b_j \tilde{v}_j - \sum_{i=1}^{m} a_i \tilde{u}_i$

Step 1: Determination of the leaving cell

Step 1.1: Compute $Y = AD$.

Step 1.2: Find the smallest value $y_k$ in the elements of $Y$, that is, the value of the $k$-th element in $Y$ is the smallest. Ties can be broken arbitrarily.

Step 1.3: If $y_k \geq 0$, the solution is optimal (both the dual and primal), stop. Otherwise, the leaving cell is the $k^{th}$ cell in $\Gamma$, that is, $(i_k, j_k)$.

Step 2: Determination of the entering cell

Step 2.1: let

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{bmatrix} = \begin{bmatrix} d_{1,k} \\ d_{2,k} \\ \vdots \\ d_{m,k} \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_m \end{bmatrix} = \begin{bmatrix} d_{n+1,k} \\ d_{n+2,k} \\ \vdots \\ d_{n+m,k} \end{bmatrix}$$

Step 2.2: For all non-basic cells, if $p_i - q_j \leq 0$, then the dual problem is not bounded, and the original primal problem has no feasible solution, and stop. Otherwise, compute $\tilde{\theta}_i = \tilde{c}_{ij} + \tilde{u}_i - \tilde{v}_j$ if $p_i - q_j > 0$.

Step 2.3: Find the smallest value $\tilde{\theta}_{st}$ in all $\tilde{\theta}_i$, and the cell $(s, t)$ is the entering cell. Ties can be broken arbitrarily.

Step 3: Updating

Step 3.1: Update the matrix $D$.

Step 3.1.1: For the elements of the columns $k$ in $D$: 

...
\[ \hat{d}_{lr} = -d_{rl}, l = 1, 2, \ldots, m + n \]

Step 3.1.2: For the elements of other columns in D:

\[ \hat{d}_{lr} = d_{lr} + (d_{s+n,r} - d_{s+n,r})d_{rl} \]

\[ \left\{ \begin{array}{ll}
  r = 1, 2, \ldots, k - 1, k + 1, \ldots, m + n \\
  l = 1, 2, \ldots, m + n
\end{array} \right. \]

Step 3.2: Update the basic cell set \( \Gamma \): replace the \( k^{th} \) cell \((i_k, j_k)\) in \( \Gamma \) With the entering cell\((s,t)\).

Step 3.3: Update the objective value:

Compute:

\[ \tilde{u}_i = \tilde{u}_i - \tilde{\theta}_i p_i, i = 1, 2, \ldots, m \]

\[ \tilde{v}_j = \tilde{v}_j - \tilde{\theta}_j q_j, j = 1, 2, \ldots, n \]

and the objective:

\[ \psi = \sum_{j=1}^{n} b_j \tilde{v}_j - \sum_{i=1}^{m} a_i \tilde{u}_i \]

Go to Step 1.

8. Numerical Example

This is an unbalanced fuzzy transportation problem. A dummy destination can be added to make the problem balanced. However, this is not required in the dual-matrix approach.

<table>
<thead>
<tr>
<th></th>
<th>Destinations 1</th>
<th>Destinations 2</th>
<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orgin 1</td>
<td>(8,11,17,20;5)</td>
<td>(2,6,11,13;5)</td>
<td>5</td>
</tr>
<tr>
<td>Orgin 2</td>
<td>(0,5,10,15;8)</td>
<td>(6,9,12,13;8)</td>
<td>15</td>
</tr>
<tr>
<td>Orgin 3</td>
<td>(1,3,8,12;5)</td>
<td>(9,12,19,20;6)</td>
<td>9</td>
</tr>
<tr>
<td>Demand</td>
<td>15</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Step 0: Initialization

\[ A = (15, 6, -5, -15, -9) \quad \tilde{u}_i = (0,0,0,0;1); \quad (i = 1, 2, \ldots, m) \]

\[ \Gamma = \{(3,1), (1,2), (1,0), (2,0), (3,0)\} \]
New dual based approach

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & -1 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]

and \( \psi = \sum_{j=1}^{5} b_{j}v_{j} - \sum_{i=1}^{5} a_{i}u_{i} = (27, 81, 186, 258; 5) \)

Step 1: Determination of the leaving cell
\( Y = AD = (15, 6, -1, 15, -6) \)
So, \( k = 5 \) and the leaving cell is \((3, 0)\) in \( \Gamma \).

Step 2: Determination of the entering cell
\[
Q = \begin{bmatrix} q_{1} \\ q_{2} \end{bmatrix} = \begin{bmatrix} d_{15} \\ d_{25} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix} p_{1} \\ p_{2} \\ p_{3} \end{bmatrix} = \begin{bmatrix} d_{35} \\ d_{45} \\ d_{55} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}
\]

Among the all non basic cells, the cell \((3, 2)\) have non-positive \((p_{i} - q_{j})\)'s while the cells\((1, 1)\) and \((2, 1)\) have positive \((p_{i} - q_{j})\)'s, so \( \theta_{st} = \min \{ \theta_{11}, \theta_{21} \} \)
\( = (-12, -3, 7, 14; 5) \)
Now, the entering cell is \((s, t)\) is \((2, 1)\).

Step 3 Updating:
\[
\begin{bmatrix}
\hat{d}_{15} \\
\hat{d}_{25} \\
\hat{d}_{35} \\
\hat{d}_{45} \\
\hat{d}_{55}
\end{bmatrix} = - \begin{bmatrix}
d_{15} \\
d_{25} \\
d_{35} \\
d_{45} \\
d_{55}
\end{bmatrix} + \begin{bmatrix}
-1 \\
0 \\
0 \\
0 \\
-1
\end{bmatrix}
\]
\[
\begin{bmatrix}
\hat{d}_{11} \\
\hat{d}_{21} \\
\hat{d}_{31} \\
\hat{d}_{41} \\
\hat{d}_{51}
\end{bmatrix} = - \begin{bmatrix}
d_{11} \\
d_{21} \\
d_{31} \\
d_{41} \\
d_{51}
\end{bmatrix} + (d_{41} - d_{11}) \begin{bmatrix}
\hat{d}_{15} \\
\hat{d}_{25} \\
\hat{d}_{35} \\
\hat{d}_{45} \\
\hat{d}_{55}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-1
\end{bmatrix}
\]

Similarly, other elements in \( D \) can be updated and
\[
D = \begin{bmatrix}
0 & 0 & 0 & -1 & 1 \\
0 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & -1 & 1 \\
\end{bmatrix}
\]

\[
\Gamma = \{(3,1)(1,2)(1,0)(2,0)(2,1)\}
\]

Step 3.3: Update the objective value
\[
\begin{bmatrix}
\tilde{u}_1 \\
\tilde{u}_2 \\
\tilde{u}_3 \\
\tilde{v}_1 \\
\tilde{v}_2 \\
\end{bmatrix} = 
\begin{bmatrix}
\tilde{u}_1 \\
\tilde{u}_2 \\
\tilde{u}_3 \\
\tilde{v}_1 \\
\tilde{v}_2 \\
\end{bmatrix} - \tilde{\theta}_{21} 
\begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
q_1 \\
q_2 \\
\end{bmatrix} = 
\begin{bmatrix}
(0,0,0,0;1) \\
(0,0,0,0;1) \\
(0,0,0,0;1) \\
(1,3,8,12;5) \\
(2,6,11,13;5) \\
\end{bmatrix} - \begin{bmatrix}
(-12,-3,7,14;5) \\
0 \\
\end{bmatrix} 
\]

and \(\psi = \sum_{j=1}^{2} b_j \tilde{v}_j - \sum_{i=1}^{3} a_i \tilde{u}_i = (-279,-27,318,576;5)\)

Now \(Y = AD = (9, 6, -1, 9, 6)\)

So, \(k=3\) and the leaving cell is \((1,0)\) in \(\Gamma\).

Determination of the entering cell
\[
Q = \begin{bmatrix}
\hat{q}_1 \\
\hat{q}_2 \\
\end{bmatrix} = \begin{bmatrix}
\hat{d}_{13} \\
\hat{d}_{23} \\
\end{bmatrix} = \begin{bmatrix}
-1 \\
0 \\
\end{bmatrix} \quad \text{and} \quad P = \begin{bmatrix}
\hat{p}_1 \\
\hat{p}_2 \\
\hat{p}_3 \\
\end{bmatrix} = \begin{bmatrix}
\hat{d}_{35} \\
\hat{d}_{45} \\
\hat{d}_{45} \\
\end{bmatrix} = \begin{bmatrix}
-1 \\
0 \\
0 \\
\end{bmatrix}
\]

Among the all non basic cells, the cells \((1,1)\) have non positive \((p_i - q_j)\)'s while the cells\((2,2)\) and \((3,2)\) have positive \((p_i - q_j)\)'s.

So, \(\hat{\theta}_{tt} = \{\hat{\theta}_{22}, \hat{\theta}_{32}\} = (-7,-2,6,11;5)\)

Now, the entering cell is \((s,t)\) is \((2,2)\).
New dual based approach

Updating:

\[
\begin{bmatrix}
\hat{d}_{13} \\
\hat{d}_{23} \\
\hat{d}_{33} \\
\hat{d}_{43} \\
\hat{d}_{53}
\end{bmatrix} = \begin{bmatrix}
d_{13} \\
d_{23} \\
d_{33} \\
d_{43} \\
d_{53}
\end{bmatrix} - \begin{bmatrix}
0 \\
-1 \\
-1 \\
0 \\
0
\end{bmatrix},
\]

\[
\hat{d}_{ik} = d_{ik} + (d_{s+i,r} - d_{ik})\hat{d}_{ik}, \ l = 1,2,3,4,5.
\]

Similarly, other elements in D can be updated and

\[
D = \begin{bmatrix}
0 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & -1 & 0 \\
0 & -1 & 1 & -1 & 0 \\
0 & 0 & 0 & -1 & 0 \\
-1 & 0 & 0 & -1 & 1
\end{bmatrix}
\]

\[
\Gamma = \{(3,1)(1,2)(2,2)(2,0)(2,1)\}
\]

\[
\begin{bmatrix}
\hat{\nu}_1 \\
\hat{\nu}_2 \\
\hat{\nu}_3
\end{bmatrix} = \begin{bmatrix}
(-7, -2, 6, 11; 5) \\
(0, 0, 0, 0; 5) \\
(-12, -3, 7, 14; 5)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tilde{\nu}_1 \\
\tilde{\nu}_2 \\
\tilde{\nu}_3
\end{bmatrix} = \begin{bmatrix}
(-11, 0, 15, 26; 5) \\
(-5, 4, 17, 24; 5)
\end{bmatrix},
\]

and \( \psi = \sum_{j=1}^{2} b_j \tilde{v}_j - \sum_{i=1}^{3} a_i \hat{\nu}_i = (-266, -69, 364, 677; .5) \). Go to Step 1.

Now, \( Y = AD = (9, 5, 1, 8, 6) \)

The optimal solution is obtained with the objective function

\( \Psi = (-266, -69, 364, 677; .5) \) with \( x_{31} = 9, x_{12} = 5, x_{22} = 1, x_{20} = 8, x_{21} = 6 \).

9. Conclusion

A new approach, the dual-matrix approach, to the fuzzy transportation problem is presented in this paper. The approach considers the dual of the fuzzy transportation model, starts from a good feasible solution, and uses a matrix to get next better solution until an optimal solution is obtained. The dual-matrix approach can be applied to both balanced and unbalanced fuzzy transportation problems.
problems. An unbalanced fuzzy transportation problem is not required to be converted into a balanced fuzzy transportation problem. This approach has no path tracing.

References


10. L.R. Ford and D.R.Fulkerson, solving the transportation problem, Management science 3 (1956), 24-32.


Received: April, 2012