A Way of Giving Priority to the Incoming Vehicles 
and the Method of Servicing in M/M/1 Queueing System

P. Saravanamoorthi

Department of Mathematics
Bannari Amman Institute of Technology
Sathyamangalam, Tamilnadu, India
kp_sm@rediffmail.com

V. Navaneethakumar

Adhiyamaan College of Engineering
Hosur, Tamilnadu, India

Abstract

Queueing theory is a well known method for evaluating the performance of vehicles which transports raw materials from the source to destinations. If there is only one service provider, one can use M/M/1 queueing model to find the waiting time and waiting cost of the vehicle. But the vehicles with different capacity and travelling for a long distance are not considered for giving service. If these things are taken into account, the loss due to putting the vehicle in queue, is very much reduced. In this paper, we described a model for priority selection of vehicles and the way in which the vehicles are being served. And the waiting time of a vehicle in the queue is also discussed.

Mathematics Subject Classification: 60K25, 90B06

Keywords: Transportation, Transportation cost and Queueing theory

1. Introduction

In recent years, factories are more concentrating on transportation of raw materials from the sources to destinations. This destination is considered as the factory. This transportation may be done through by the own vehicles of the factory concerned or hiring the vehicles from the operator. If the vehicles used are their own, then there is no extra cost incurred to the factory because of waiting for getting service in the queue. But by hiring the vehicles, they
have to pay extra attention on the waiting time and the waiting cost. And the main reason is that the vehicles which are travelling from a long distance may suffer a lot in terms of loss in the weight.

2. Literature Review

In [9], he described a vehicle routing problem. In which, it is assumed that there is only one depot where the routes start and end for each vehicle. A homogeneous fleet consisting of several vehicles with fixed capacity, while each customer’s demand is pre-determined. In this, they found minimum cost vehicle route. In [1], [2] used for the case of uniformly distributed demand locations and Poisson arrivals to minimize a total average cost over an infinite horizon. In such a setting, the economical delivery objective corresponds to minimizing the average distance travelled per demand served. In [4], queueing theory was first postulated for studying queueing phenomena in Commerce, telephone traffic, transportation, industrial servicing etc. There are many applications and derivations of queueing models like priority queues with interruptions have been discussed. In [5], proposed a systematic way to classify different kind of interruptions in a single machine system. In [3], considered a discrete time finite buffer queue with correlated arrivals and service interruptions and the corresponding infinite buffer queue. In [8], discussed the concept of merging of more than one queueing channels. In this, study a N-station discrete material flow merge system model feeding a buffer connected to a down stream station is presented. It helps us to know about merging of more than one channels. In [6,7], we knew the applications queueing theory and their models.

3. Defining the problem

Transportation of raw material from the source to destination is an important task to all factories. After transportation, the initial task is to make a queueing process to the vehicles which are carrying the materials to the factory. In our process and calculations, we used the queueing channels with one server serving for the unloading the raw materials. But the incoming vehicles are treated in different classifications. The classifications are given below.

(a) \( R_1 \): Long route vehicles which are carrying maximum load. Let it be denoted by \( C_1 \)

(b) \( R_2 \): Route which is less than \( R_1 \) and let the capacity of the vehicle be denoted by \( C_2 \) and also \( C_1 \geq C_2 \). Like wise the vehicles are classified based on the distance travelled and the load capacity.

We tried to separate the vehicles as per the classifications given above. We identified the arriving vehicles at a particular time period. At a particular time period, the vehicles are to be grouped as open balls using some conditions and
the vehicles are to be put in a sequence. The calculation is done using the following procedure and algorithm.

4. classification of the incoming vehicles

4.1. Definition. Let \( n > 0 \) be an integer. An ordered set of \( n \) real numbers \((X_1, X_2, X_3, \ldots)\) is called an \( n \)-dimensional vector with \( n \) components. i.e., each vector \( X \) has the following \( n \) components like

\[
X_1 = \{x_1, x_2, x_3, \ldots, x_n\}
\]

Similarly the vectors \( X_2, X_3, \ldots \) having different components.

Let there be \( n \) number of vehicles arriving at the entrance of the factory concerned. The vehicles are carrying different capacity of raw materials and travelling different distances. Then there is a constant, called as a \( k \)-factor and it is defined by

\[
k\text{-factor} = \frac{L}{D} \text{tons/kms}
\]

where \( L \)- load carried by the vehicle and \( D \)- distance travelled by the vehicle.

Then form a vector with the above \( k \)-factors of all the vehicles. Let \( K = \{k_1, k_2, k_3, \ldots, k_n\} \) since there are \( n \) number of vehicles are arriving and obviously, there will be \( n \) number of \( k \)-factors.

4.2. Properties of all \( K \) vectors.

Let \( K = \{k_1, k_2, k_3, \ldots, k_n\} \) and \( L = \{l_1, l_2, l_3, \ldots, l_n\} \) be any two \( k \)-factors in a vector space. Then they should satisfy the following.

(a) Equality \( K = L \) if and only if \( k_1 = l_1 \), \( k_2 = l_2 \) etc.,

(b) Addition of two \( K \)-factor spaces. Let \( K \) and \( L \) be any two \( K \)-factor spaces, then \( K + L = \{k_1 + l_1, k_2 + l_2, \ldots, k_n + l_n\} \)

(c) Multiplication by a real number. Let \( a \) be any constant and \( K \) be a \( K \)-factor space, then \( aK = \{ak_1, ak_2, \ldots, ak_n\} \)

(d) Inner product of two \( K \)-factor spaces. Let \( K \) and \( L \) be any two \( K \)-factor spaces, then \( K.L = \sum k_m l_m \) where \( m = 1, 2, 3, \ldots, n \)

(e) Norm of a \( K \)-factor spaces. \( \|k\| = (k.k)^{\frac{1}{2}} \)

4.3. Theorem Let \( K \) and \( L \) be any two \( k \)-factor spaces. Then the following properties are true.

(a) \( a\|k\| = a\|k\| \), where \( a \) is a constant.

(b) \( \|k\| \geq 0 \) and \( \|k\| = 0 \) if and only if \( k = 0 \) for every \( k \).
(c) $\|k - l\| = \|l - k\|$.

(d) $\|k.l\| \leq \|k\|\|l\|

(e) $\|k + l\| \leq \|k\| + \|l\|

4.4. Definition of open balls. Let $a_1$, $a_2$, $a_3$, .... $a_p$ be the $p$ number of points in $R^n$ and let $r_1$, $r_2$, $r_3$, .... $r_p$ be the longest and also they are positive numbers. Let $k$ point in $R^n$ such that $\|k - a_1\| \leq r_1$, $\|k - a_2\| \leq r_2$ and etc., $\|k - a_p\| \leq r_p$. The relation between the constants $r_i's$ is given by $r_1 \geq r_2 \geq r_3 \geq .... \geq r_p$. Then the constants $a_i's$ and $r_i's$ are called as the centre and radii of the open balls defined in $R^n$.

4.5. Groups of open balls. Let $k$ be any point in $R^n$. Then the vectors are categorised as below

$G_1 = \|k - a_1\|$

$G_2 = \|k - a_2\|$

etc., $G_p = \|k - a_p\|$

Here, $G_1$, $G_2$, ......., $G_p$ are the group of $p$ open balls satisfying the above conditions.

4.6. Theorem If $G_1$, $G_2$, etc., and $G_p$ are the group of $p$ open balls in $R^n$. Then the following conditions are true.

(i) $G_1 \cap G_2 \cap ....... \cap G_p = 0$

(ii) $G_1 \cup G_2 \cup G_3 \ldots \cup G_p = K$

5. Algorithm for the selection of open balls to put the vehicles in a sequence

Based on the number of vehicles carrying the raw materials, capacity of the vehicles and the distance travelled by the vehicles, they are classified as the open balls. Here, the open balls reflects the properties of the vehicles exactly. The following algorithm gives the place in the sequence to get service to the vehicles which are selected from the above calculations.

Step:1 Identify the balls $G_1$, $G_2$, ......., $G_p$ and take a ball with maximum radii.

Step:2 After the selection of the ball, the corresponding vehicle should be put in a sequence.

Step:3 Go for selecting next ball which is having a less radius than the above. Proceed the above calculations till all vehicles are being put in the sequence.

6. Calculation of Total Waiting Time of a vehicle
It is assumed that there is only one server providing service to the vehicles. In [9], a queueing approach was discussed. Here, the roads on which the vehicles are sequenced are subdivided into segments. These segments are equal to the space required by the vehicle. But, in this paper, we classified the waiting time of a vehicle based on the Sequencing and the Servicing of the vehicles. So, the total waiting time of a vehicle is given by

\[
\text{Total waiting time} = \text{waiting in the sequence} + \text{servicing time of the vehicle}
\]

### 6.1. Calculation of Waiting Time of a vehicle in the Sequence and in the service

Let there be \( n_1, n_2, n_3, \) etc., number of vehicles waiting in the sequence in the first, second and third category of vehicles for getting service. Let the service time of the vehicle may be denoted by \( \mu_1, \mu_2, \mu_3 \) etc., for the first, second and the third category of vehicles. But, out of the above number of vehicles, not all the vehicles are given priority to get the service at a time. Only a few may be given first priority and the others will be given after. Let \( p_1 = \frac{m_1}{n_1}, p_2 = \frac{m_2}{n_2} \) etc., be the probabilities of the vehicles to go for service from the sequence, where \( m_1, m_2, m_3 \) etc., are the number of vehicles getting priorities for service in the sequence.

For the first priority vehicle, the waiting time is only the time which is waiting during the service. Because, it gets service first of all. For the second kind of vehicle, the waiting time is based on the service time of first kind of vehicle and the service time of second kind and it is equal to \( p_1\mu_1 + \mu_2 \)

Similarly for the third kind of vehicle, Waiting time is equal to \( p_1\mu_1 + p_2\mu_2 + \mu_3 \)

In general, the waiting time of \( n^{th} \) kind of vehicle is equal to \( \sum p_i\mu_i + \mu_n \), where \( i = 1, 2, 3, ... n - 1 \).

Let \( C_i \) be the cost of the vehicle, which is put in waiting and is given by

\[
\text{Cost of waiting} = C_i[\sum p_i\mu_i + \mu_n]
\]

### References


Received: March, 2012