Modeling Boomerang Motion in Euclidean Plane

Senay Baydas

Yuzuncu Yil University, Faculty of Science
Department of Mathematics, Van, Turkey
senay.baydas@gmail.com

Bulent Karakas

Yuzuncu Yil University, Faculty of Economics and Administrative Sciences, Numerical Methods, Van, Turkey
bulentkarakas@gmail.com

M. Nuri Almali

Yuzuncu Yil University, Faculty of Engineering and Architect Department of Electrical and Electronics Engineering, Van, Turkey
mna1@yyu.edu.tr

Abstract

We define a special motion in Euclidean plane inspired by the movement of the boomerang. We are called this motion as "modeling boomerang motion in plane" (MBM). Different forms of MBM are defined, analyzed and simulated at Matlab.

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1 Introduction

A boomerang is a flying device known for returning to its initial position of the throw. When you throw check, it returns to beginning point. Aerodynamic performance of the boomerangs is studied [5, 7,8].

The boomerang must be thrown a perpendicular route for orbiting a close elliptical orbit. According to these, the motion as the boomerang motion can be represented as at Fig. 1.[8]. The motion showed with MBM moves similar to the boomerang motion along a planar curve. The motion which is defined along a curve relates with properties of this curve.
2 Modeling Boomerang Motion Along A Planar Curve

A displacement in n-dimensional space is defined by matrix-vector pair $D = (A, d)$, where $A$ is an orthogonal matrix and $d$ is a colon vector. Consider a particle moving in space so that its position at time $t$ is given by $x(t)$. We think of $x(t)$ as moving along a curve $C$ parameterized by a function $f$, where

$$f : R \rightarrow R^n$$

Hence we have $x(t) = f(t)$, or, more simply, $x = f(t)$. In this study $n$ will always be 2 but there are physical situations in which it is reasonable to have larger values of $n$ [6].

At a given time $t_0$, the vector $x(t_0 + h) - x(t_0)$ represents the magnitude and direction of the change of position of the particle along $C$ from time $t_0$ to time $t_0 + h$ [1,2]. For a general planar displacement there is a point that does not move, which means that its coordinates are the same in both reference frames fixed and moving. This point is called the pole point of displacement then its pole $P$ satisfies the equation $D_p = D = (A, d)$ be a displacement. If $p$ is a pole point then $p$ satisfies the equation

$$p = [A]p + d.$$  \hspace{1cm} (2)

The general form of rotation matrix on $R^2$ is as follows.

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, 0 < \theta < 2\pi$$

If $p \in R^2$ is a pole point of the displacement $D = (A, d)$, then
\[
\begin{bmatrix}
  p_1 \\
  p_2
\end{bmatrix} = \begin{bmatrix}
  \cos \theta & -\sin \theta \\
  \sin \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
  p_1 \\
  p_2
\end{bmatrix} + \begin{bmatrix}
  d_1 \\
  d_2
\end{bmatrix}
\]
and we have pole point as

\[p_1 = \frac{1}{2(1 - \cos \theta)} (d_1(1 - \cos \theta) - d_2 \sin \theta)\]
\[p_2 = \frac{1}{2(1 - \cos \theta)} (d_1 \sin \theta + d_2(1 - \cos \theta)).\]

If we know pole point then we can write \((A, d)\), where \(d = (d_1, d_2)\),

\[d_1 = (1 - \cos \theta)p_1 + \sin \theta p_2\]
\[d_2 = \sin \theta p_1 + (1 - \cos \theta)p_2\]

[3,4].

**Definition 2.1.** Let the edge points C and D of CD bar are located on closed and elliptical curve. Let one of them is pole point on the curve and the other makes a rotation. This motion is called "modeling boomerang motion in plane".

*Modeling boomerang motion in plane is showed with MBM in this study.*

The points C and D have two different motion. They are these:

**First Form:** While one of the points C and D move as joint, the other rotates about this point. When the orbit of second point and the underlying curve intersect, instantaneously first point stops and the other to begin rotates about the first point(Fig.2). So first step is finished. And than the points change role

**Second Form:** While one of the points C and D moves along the curve as joint point, the other point rotates continuously about at this point along a curve (planar or spatial).

This article study the first form.

### 2.1 The MBM Motion Along A Curve At The Plane for First Form

In the beginning, underlying curve can be chosen is a circle. So MBM is more understandable.

**Circle form:** Let the orbit curve be a circle which its center is \((0, 0)\) and its radius is \(r\).
Let $CD$ bar be located on circle and be $d(C, D) = L$. Central angle for $CD$ is

$$A = 2 \arcsin \left( \frac{L}{2r} \right).$$

(3)

The bar $CD$ rotates on circle with $B$ angle. After rotating, the new position of $CD$ is $C''D'$ and then the point $D'$ rotates about $C'$ with $\alpha$ degree and located on circle. This rotating has $R_\alpha(C')$, rotating matrix, and $\alpha = \pi + A$.

If $\frac{2\pi}{A+B}$ is a integer, the moving is finished at the beginning point. The number $\frac{2\pi}{A+B}$ is called step number.

The relations between steps:

**First step**: Let $D = (1, 0)$ and $C = (\cos A, \sin A)$. After the CD bar rotates about O, the new position of CD is

$$C' = (\cos(A + B), \sin(A + B))$$

(4)

and the point $D' = (\cos B, \sin B)$ rotates about $C'$ with angle $\alpha$. And then $D'$ takes the position $D'' = R_{(A+B)}(C')(D')$.

**Second step**: At second step, if we take $D''$ as $C$ at first step and $C'$ as $D$ at first step, we can repate first step as second step.

If we write $P_i$ and $E_i$ for $C$ and $D$, respectively, we have following rotating points and rotated points.

![Figure 2: Changing of the joint points](#)
Rotating point | Rotated points
--- | ---
\(P_0 = (\cos(A), \sin(A))\) | \(E_0 = (1, 0)\)
\(P_2 = (\cos(A+B), \sin(A+B))\) | \(E_2 = (\cos(B), \sin(B))\)
\(P_4 = (\cos(2A+2B), \sin(2A+2B))\) | \(E_4 = (\cos(A+2B), \sin(A+2B))\)
\(P_6 = (\cos(3A+3B), \sin(3A+3B))\) | \(E_6 = (\cos(2A+3B), \sin(2A+3B))\)
\(P_8 = (\cos(4A+4B), \sin(4A+4B))\) | \(E_8 = (\cos(3A+4B), \sin(3A+4B))\)
\(P_{10} = (\cos(5A+5B), \sin(5A+5B))\) | \(E_{10} = (\cos(4A+5B), \sin(4A+5B))\)
\(P_{12} = (\cos(6A+6B), \sin(6A+6B))\) | \(E_{12} = (\cos(5A+6B), \sin(5A+6B))\)
\(\ldots\) | \(\ldots\)
\(P_{2n} = (\cos(2nA+2nB), \sin(2nA+2nB))\) | \(E_{2n} = (\cos((2n - 1)A + 2nB), \sin((2n - 1)A + 2nB))\)

2.1.1 The Displacement Matrix For MBM

The matrix for MBM is

\[
R_C = \begin{bmatrix}
\cos \alpha & -\sin \alpha & r(\cos t - \cos \alpha \cos t + \sin \alpha \sin t) \\
\sin \alpha & \cos \alpha & r(\sin t - \sin \alpha \cos t - \cos \alpha \sin t) \\
0 & 0 & 1
\end{bmatrix}
\]

where \(C = (r \cos t, r \sin t)\) are pole points.

We must know what \(L\) and \(r\) are. If we want initial point at the finish, we can use relations \(L\) and \(r\).

We made a programme for MBM at Matlab with codes as follows.

```matlab
{clear all, close all, clc
u=0:1:360;
x=cosd(u);
y=sind(u);
plot(x,y, ’r’)
r=r_{0};
l=l_{0};
n=n_{0};
AA=2*asind(1/(2*r))
```
B=2*AA;
Q=180+AA;
r=1;
step=(360/(AA+B))
for aa=1:1:step
    O=(AA+B)*(aa-1)
    A=(aa*AA+(aa-1)*B)
        for i=0:i_{0}:B
            E=[(r*cosd(O+i));(r*sind(O+i));1];
            T=[(r*cosd(A+i));(r*sind(A+i));1];
            line([E(1),T(1)],[E(2),T(2)],'Marker','.','LineStyle','-')
            pause(0.2)
        end
for i=0:i_{0}:Q
    R=[cosd(i) -sind(i) r*(cosd(A+B)-cosd(i)*cosd(A+B)+sind(i)*sind(A+B));
        sind(i) cosd(i) r*(sind(A+B)-sind(i)*cosd(A+B)-cosd(i)*sind(A+B));
        0 0 1];
    M=R*E;
    line([T(1),M(1)],[T(2),M(2)],'Marker','.','LineStyle','-')
    pause(0.1)
end
end

**Example 2.2.** If we choose $A=30$, $B=30$ and $r=1$ we can check easily our defined motion. In this case we have

```matlab
{clear all, close all, clc
u=0:1:360;
x=cosd(u);
y=sind(u);
plot(x,y,'r')
axis([-12 12 -12 12])
0=0;
A=30;
B=30;
AA=30;
Q=180+A;
r=1;
step=(360/(A+B))
for aa=1:1:step
    O=(AA+B)*(aa-1) ;
```
A=(aa*AA+(aa-1)*B); 
for i=0:15:B 
  E=[(r*cosd(0+i));(r*sind(0+i));1]; 
  T=[(r*cosd(A+i));(r*sind(A+i));1]; 
  line([E(1),T(1)],[E(2),T(2)], 'Marker','.','LineStyle','-') 
  pause(0.1) 
end 
for i=0:15:Q 
  R=[cosd(i) -sind(i) r*(cosd(A+B)-cosd(i)*cosd(A+B)+sind(i)*sind(A+B)) 
      sind(i) cosd(i) r*(sind(A+B)-sind(i)*cosd(A+B)-cosd(i)*sind(A+B)) 
      0 0 1]; 
  M=R*E 
  line([T(1),M(1)],[T(2),M(2)], 'Marker','.','LineStyle','-') 
  pause(0.3) 
end 

The motion has six steps. The centers of every step are the following.

\[
\begin{array}{ll}
P_0 = (0.866, 0.5) & P_1 = (1, 0) \\
P_2 = (0.5, 0.866) & P_3 = (0, 1) \\
P_4 = (-0.5, 0.866) & P_5 = (-0.866, 0.5) \\
P_6 = (-1, 0) & P_7 = (-0.866, -0.5) \\
P_8 = (-0.5, -0.866) & P_9 = (0, -1) \\
P_{10} = (0.5, -0.866) & P_{11} = (0.866, -0.5) \\
P_{12} = (1, 0) & P_{13} = (0.866, 0.5) \\
\vdots & \vdots \\
P_{(2n+1)} = D_n & P_{2n} = C_n.
\end{array}
\] (6)

The graph and points of first step are shown at Figure 3.

3 Conclusion

MBM makes a repetitional motion. The motion MBM we defined in this article has a lot of expansion. Especially, since the motion of the second point is free and orbit of first point can be chosen different, we can define many versions of MBM. We discussed a special motions of MBM. The first and second points move as joint along the curve. On the application periodic point, while the first point stops, second point returns around first point and then they change their moving properties.
Figure 3: Pole curve and orbit of CD for A=30

References


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