Mathematical Model of the Development of Districts

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Abstract. We make a review about the treatments of territorial, social-economic problems in the industrially high developed countries. In this work is motivated the structure of the concept about social-economic development, the content of each interpretation of the concept. In this concept an important role plays the problem of homogenizing the labor and life conditions of the population. Concerning this fact the author suggests a formalized method of solving the problem. The author proposes a modality of simulation the process of social-economic development of the territorial-administrative subdivisions on the computer because the function-goal from the mathematical model is not differentiable. In order to solve the problem the author introduces a virtual indicator-standard. When the economic state, the number of population is changed, the district-standard also is changed. The author models the real situations through such treatment.

Further, in the work a modality of evaluation the dynamics in the changes of territorial indicators is proposed. This permits to the author to bring a range of useful conclusions as for scientific researches as for probationers. In the work is made a systemic treatment using the mathematical device and the calculation technique for solving the regional social-economic problems. The program products realized in the thesis permit the solving a set of problems in complex, in comparison with conventional program products which solve only one problem.

Keywords: Mathematical model, districts, social-economic

Any territory is characterized by a series of social-economic indicators, and the establishment of an order of districts, according to the level of development, would seem a question without answer.

Before suggesting the idea of solving the respective problem is necessary to establish the territories, cities that will be considered and the social-economic indicators, which can be put at the basis of taking some decisions regarding the territorial development.
Each district (city) is characterized by a range of social-economic indicators that refer to:

1. **Economic problems:**
   - $A_1$ — level of unemployment (rate of unemployment - RU; $A_1 = 1 - RU$).
   - $A_2$ — quantum of the final product of the territory per capita.
   - $A_3$ — volume of all investments from the territory.
   - $A_4$ — quota of the industry of raw material processing in the global produc.
   - $A_5$ — quota of new articles in the global volume of the deliveries of industrial goods.
   - $A_6$ — export of production, including of the goods with added value and of services per capita in relation to GNP.
   - $A_7$ — quota of import in the consumption of goods ($A_7 = 1 - QI$, where $QI$ - the quota of the respective import).
   - $A_8$ — number of authorizations for building new houses reported to the total volume of the floor space.
   - $A_9$ — quota of enterprisers, firms to 10000 inhabitants of the district (city).
   - $A_{10}$ — number of telephones reported to 1000 inhabitants.

2. **Public order:**
   - $A_{11}$ — area occupied by offices and bureaus to 1000 inhabitants.
   - $A_{12}$ — rate of delinquency ($A_{12} = 1 - RD$, where $RD$ - the respective rate).
   - $A_{13}$ — rate of crime discovery.
   - $A_{14}$ — number of citizens involved in acute social conflicts (strikes, unauthorized meetings) ($A_{14} = 1 - CI$, where $CI$ - the respective number).
   - $A_{15}$ — participation in local elections, within the country, reported to the number of voters.

3. **Development of the human capital:**
   - $A_{16}$ — annual average income per capita.
   - $A_{17}$ — buying power of the population of district, city.
   - $A_{18}$ — quota of population with incomes smaller towards the minimum of living ($A_{18} = 1 - QP$, where $QP$ - the respective quota).
   - $A_{19}$ — average life expectancy.
   - $A_{20}$ — insurance of population with floor space.
   - $A_{21}$ — population that posses limos and other expensive technique.
   - $A_{22}$ — number of population with professional studies.
   - $A_{23}$ — share of pupils.
   - $A_{24}$ — share of people with superior studies.
   - $A_{25}$ — number of students.
   - $A_{26}$ — rate of morbidity.
   - $A_{27}$ — death rate ($A_{27} = 1 - DR$, $DR$ - the respective rate).
   - $A_{28}$ — number of computers reported to the number of population.
   - $A_{29}$ — balance of migration ($A_{29} = 1 - RE$, $RE$ - the rate of emigration).
   - $A_{30}$ — volume of services provided to the population.
4. Financial stability:
- A\textsubscript{31} – budget deficit (A\textsubscript{31}=1-BD, where BD-the respective deficit).
- A\textsubscript{32} – quota of external loans in covering the budgetary deficit (A\textsubscript{32}=1-quota).
- A\textsubscript{33} – own capital of banks from the district.
- A\textsubscript{34} – financial balance, the quota of profitable firms.
- A\textsubscript{35} – insurance of industrial branches with reserves of imported raw materials.
- A\textsubscript{36} – agricultural areas per capita.

5. Ecological security:
- A\textsubscript{37} – resources of water per capita.
- A\textsubscript{38} – quota of population that lives in ecologically dangerous zones (A\textsubscript{38}=1-QP, where QP-the respective quota).
- A\textsubscript{39} – insurance of the district with power resources.
- A\textsubscript{40} – quality of drinking water, according to the international standards.
- A\textsubscript{41} – frequency of accidents, fires (A\textsubscript{41}=1-FA, where, FA is the frequency of accidents).

This vector of indicators represents a reference system, which serves for the presentation (then also for evaluation) of the socio-economic performances of districts. The final user, the expert in the domain, can elaborate without the assistance of the elaborator, proper reference systems: to materialize and to modify the dimension of the system, to exclude some of its components and to include others.

Theoretically, the enumerated indicators contribute to the solving of a complex of economic problems (A\textsubscript{1}-A\textsubscript{11}), about the public order and trust towards authorities (A\textsubscript{12}-A\textsubscript{15}), the welfare and life quality, the human capital’s development (A\textsubscript{16}-A\textsubscript{30}), the financial stability of the district (city) (A\textsubscript{31}-A\textsubscript{34}), the ecological security (A\textsubscript{37}-A\textsubscript{41}).

The problems of social-economic development of each district (city) from Jordan [2,1] cannot be resolved without using the device of the processes’ modeling, algorithmization, of performing the respective methods on the computer. For this purpose is necessary: to make a complex diagnosis of the social-economic state and to elaborate some strategic plans of development for each district (city) with a view to reduce the inequality between these, to use the advantages of their diversification and competitive priorities, to combine, rationally, the national and local interests; to elaborate some distraction special state programmes, to establish, in a legislative way, the norms of social insurance, as well as to elaborate the mechanism of homogenization of the levels of districts’ development, of resolving the conflicts of interests, of limitation the local and national authority’s powers; to improve the fiscal basis of districts, of the financial and staff’s professional training system.

Each indicator $A_j, j=1,..., 41$, reflects the positive tendencies in the territorial development. If, for instance, it is about the death rate (negative tendency) or other negative processes, then the respective indicator is replaced with the supplement of it, namely, instead of counting sick people we count those healthy and the respective rate of 2% of patients is replaced with the rate of 98% of healthy. These transformations are necessary for further calculations.
A model is an isomorphic representation of the reality, offering an intuitive image, but accurate, in the sense of the logical structure of the studied phenomenon, facilitating the discovery of some connections and rules impossible or very difficult to find in other ways [5,4].

Advantages of using the mathematical models are the following:

- Compress or extend the time (the processes from macrounivers and microunivers can be modeled in real time on the computer).
- Permit the analysis of a very big number of possible solutions and of determination of the best one from those studied.
- Contribute to the training of the manager, ensure the raising of the level of managerial culture.
- Reduce the cost of the experiment.
- Permit to avoid some managing mistakes.

The models examined in management are frequently based on a single objective. In fact, including in thesis, are discussed the decisional situations with more objectives. We will present a mathematical model for evaluation of the socio-economic performances of the districts within the reference system proposed, which can be used at the distribution of resources on districts at the simulation on the computer of the socio-economic consequences of these distributions.

For each considered district, on the basis of statistical data, we will create the following table (table 1), the last line of which contains the information for an imaginary district (virtual), which has the best results of the socio-economic indicators. Following, we will name it standard-district. It will be considered as reference-district for evaluation of performances of the country’s districts.

Table 1. Distribution of relative data that correspond to the indicators on districts

Theoretically, the enumerated indicators contribute to the solving of a complex of economic problems (A1-A11), about the public order and trust towards authorities (A12-A15), the welfare and life quality, the human capital’s development (A16-A30), the financial stability of the district (city) (A31-A34).
The territorial development of each district can be characterized with the help of \( n \) socio-economic indicators\[3\]. In the examined case (\( n=41 \)) the district \( j (j=1, 2, \ldots, m) \) is characterized by a vector \((O_{1j}, O_{2j}, \ldots, O_{ij}, \ldots, O_{nj})\). For each \( i \) we determine:

\[
\hat{a}_i = \max_{j=1,m} O_{ij},
\]

which will be considered further the components of the vector that corresponds to the standard-district (see the table 1). Each district can be characterized by the share of the respective indicator in the formula:

\[
\bar{O}_j = \frac{O_{ij}}{\hat{a}_j}, i = 1,41, j = 1,m,
\]

respectively, the share of the standard district for each indicator is 1.

We note through \( S_j \) – standard of the vector \((\bar{O}_{1j}, \bar{O}_{2j}, \ldots, \bar{O}_{ij}, \ldots, \bar{O}_{41j})\), \( j=1, 2, \ldots, m \), then

\[
S_j = \sqrt{\left(\frac{O_{ij}}{\hat{a}_i}\right)^2 + \left(\frac{O_{ij}}{\hat{a}_i}\right)^2 + \ldots + \left(\frac{O_{ij}}{\hat{a}_i}\right)^2}, j = 1, m,
\]

and for the standard district (SD) we will have:

\[
S_{DE} = \sqrt{1^2 + 1^2 + \ldots + 1^2 + \ldots + 1^2} = \sqrt{41} = 6,4031
\]

Districts can be arranged in the decreased order of the results.

\[
\max \{ 6,4031', 6,4031', \ldots, 6,4031', \ldots, 6,4031 \}.
\]

On the last place will be placed the district with the results

\[
\min \{ 6,4031', 6,4031', \ldots, 6,4031', \ldots, 6,4031 \}.
\]

The government of Jordan carries out its functions of territorial planning through 12 districts. The necessary for different districts is different. This fact has some explanations:

- In different districts the categories of age, of work capacity of population are different.
- The branches from the production field (sphere) are unhomogenously located in the country.
- The geo – climatic conditions are not the same, etc.

So, the government of Jordan has to ensure equal conditions for the inhabitants of the country, no matter where these live. In order to formulate better the problem we will use the matrix

\[
O = \{O_{ij}\}; \ i=1, 2, \ldots, 16; \ j = 1, 2, \ldots, 12,
\]

where \( j \) represents one of the districts: Ajlun, Al Aqaba, Al Balqa, Al Mafrak, Al Tafilah, Amman, Irbid, Jerash, Karak, Maan, Madaba, Zarga.

The columns of the matrix \( O \) express the distribution of data about the indicators \( i, i=1, 2, \ldots, 16 \) on districts:
• floor space – thousands of m².
• institutions of professional training – thousands of graduates/year.
• aquatic transport lines – millions of passengers/year.
• air transport lines – millions of passengers/year.
• railway transport lines - millions of passengers/year.
• motor transport lines – millions of passengers/year.
• tourist lines – millions of tourists/year.
• jobs – thousands.
• the number of inhabitants – thousands.
• cultural institutions – thousands of graduates.
• bookshops – thousands of m² of commercial area.
• libraries – millions of copies of books.
• training institutions – thousands of graduates.
• health assistance institutions – thousands of places (beds).
• the useful resources of water – thousands of m³.
• arable ground – thousands of hectares.

On the basis of the expressed data we absolutely determine the matrix of the relative results (per capita), that is the matrix [23]

\[ B = \{a_{ij}\}; \quad i = 1, 2, \ldots, 16; \quad j = 1, 2, \ldots, 12. \]

The standard district is characterized by the following vector of relative indexes (per capita):

\[ d = \left( \max_j a_{ij}, \ldots, \max_j a_{ij}, \ldots, \max_j a_{16j} \right) \]

The level of endowment (convenience, insurance) of each district in comparison with the standard, to which corresponds the vector \( d \), is characterized by the sizes:

\[ \frac{a_{ij}}{\max_j a_{ij}}, \quad i = 1, 16, \quad j = 1, 12. \]

Natural is that, the numbers from above represent positive sub unitary factions. The classification of the districts according to the general level of convenience is made on the basis of norms

\[ \sqrt{\sum_{j=1}^{16} \left( \frac{a_{ij}}{\max_j a_{ij}} \right)^2}, \quad j = 1, 12 \]

and the norm of the standard vector \( d \), \( \sqrt{16} = 4 \).

So, the level of endowment (insurance, convenience) of each district will be

\[ \sqrt{\sum_{j=1}^{16} \left( \frac{a_{ij}}{\max_j a_{ij}} \right)^2}, \quad \text{for} \; j = 1, 2, \ldots, 12. \]

The performing of this algorithm on the computer permits the government of Jordan to justify and to motivate the inhomogeneous distribution of the investments,
subventions in the division of districts. The real information persuades us, that the Jordan’s policy of centralization of financial resources is welcome. Otherwise (decentralization) would create for a big part of the country’s population unbearable living conditions and, so, would generate the economic unbalance, the encroachment of human rights. The finding of the level of socio-economic development of the districts (cities) is an approach necessary only for taking decisions, but not sufficient one. Further is proposed a mathematical model of this process. Solving of the examined problems is made through the application of the program me products of the unconventional informational technologies.

The problem formulated in such a way has a categorically new aspect: the standard district (city) changes in each time moment, becomes other. This variation is brought about by a range of factors, including by the dynamics of the enumerated indicators $A_1 - A_{41}$. And, so, in the process of finding, the authorities put at the basis not some normative-indicators that can be unreal, but are orientated to the successes already obtained in Jordan by a district (city) or other. The number and content of the indicators, like the number of the territorial subjects (of districts, cities) refer to other studies from the economic, sociologic, demographic etc. branches.

In the paper we focus on the formulation of the respective problem, the elaboration of the adequate algorithm and the performing of it on the computer. The made calculations do not pretend to unquestionable results, but the treatment of the problem needs to be put at the basis of the elaboration of some methods for the territorial authorities, at the formulation of database in the territorial division of Jordan.

We note with:

- $j$ – districts of Jordan, $j = 1, 2, \ldots, 12$;
- $i$ – socio-economic indicators, $i = 1, 2, \ldots, 16$;
- $c_{ij}$ – specific efficiency of the indicator $i$ in the district $j$;
- $a_{ij}$, $x_{ij}$ – insurance per capita, respectively growth of this insurance according to the indicator $i$ in the district $j$.

By analogy with [12], the socio-mathematical model is represented in the following way:

$$L(X) = \sum_{i=1}^{16} \sum_{j=1}^{12} \frac{c_{ij}(a_{ij} + x_{ij})}{\max(a_{ij} + x_{ij})} \Rightarrow \max$$

In the conditions:

$$a_{ij}^{(1)} \leq x_{ij} \leq a_{ij}^{(2)}, \quad i = 1, 2, \ldots, 16; \quad j = 1, 2, \ldots, 12;$$

$$\sum_{i=1}^{16} \sum_{j=1}^{12} k_{ij}^{(s)}(a_{ij} + x_{ij}) \leq M^{(s)}, \quad s = 1, 2, 3; \quad x_{ij} \geq 0, \quad i = 1, 2, \ldots, 16; \quad j = 1, 2, \ldots, 12,$$

where $k_{ij}^{(s)}$ is the matrix of specific expenditures, and $X = (x_{11}, x_{12}, \ldots, x_{1,12}, \ldots, x_{16,1}, x_{16,2}, \ldots, x_{16,12})$, 

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\(a_{ij}^{(1)}, a_{ij}^{(2)}\) represents respectively the possible minimal and maximal value of insurance per capita according to the indicator \(i\) in the district \(j\).

The signification of the matrix \(k_{ij}^{(s)}\) according to values (results) of \(s\) is the following:
- \(s=1\), expenditures of investments.
- \(s=2\), expenditures of the floating funds in circulation.
- \(s=3\), expenditures of work.

We will consider \(L(X)\) as objective-function of the country. Starting from the fact, that the number of the population in districts, \(n_1, n_2, \ldots, n_{12}\) are sizes which vary in time on certain segments of values and theoretically we will consider them as random variables on the respective segments.

The model will take the form:

\[
\bar{L}(X) = e \left( \sum_{i=1}^{16} \sum_{j=1}^{12} \frac{e \left( a_{ij} + x_{ij} \right)}{n_j} \right) \Rightarrow \max,
\]

In the conditions:

\[
e \left( \sum_{i=1}^{16} \sum_{j=1}^{12} k_{ij}^{(s)} \frac{a_{ij} + x_{ij}}{n_j} \right) \leq M^{(s)}, s = 1,2,3,
\]

\[
e \frac{x_{ij}}{n_j} \leq \max_j \left( \frac{a_{ij}^{(2)}}{n_j} - e \frac{a_{ij}^{(1)}}{n_j} \right).
\]

Through \(e(.)\) is understood the average value of the random variable (.).

The objective - functions from these models are not differentiable, so the traditional methods of determination of the extreme results (values) neither for the first model nor for the stochastic model can be applied. Solving of this problem is possible through performing of some unconventional programme products.

The districts, being the components of a complex system (Jordan), are reciprocally influenced by the fluctuation of prices \([6.7]\). The expenditures, income, consumption etc, necessary for calculation of the indicators \(A_1-A_{41}\) need to establish some forecast – prices, balanced in a certain way, or of their mathematical hope (average). This problem is solved beginning with the elaboration of the mathematical model of the process and of the solving algorithm. Solutions can be determined, starting from different premises, namely these can be simulated.

Let us elaborate the mathematical model of the respective problem.

CONCLUSIONS

In the result of investigations, based on the mathematical models and methods of the economic products (on the elaboration of a set of satellite-models, on the
systematic treatments of the problems of territorial development of the districts of Jordan) and on the elaboration of the respective soft's ("Forecast of population", "Comparison of the social-economic state of districts and the distribution of resources" and "Management of decisions") can be made the following conclusions:

1. Decisions of the public officials regarding the development of the socio-economic dimension can be adequate to the created real situations, only if these are implemented on the basis of a complex of mathematical models and of the respective soft.

2. The problem of territorial development of the districts of Jordan can be solved with the help of the mathematical device and of the soft's proposed in thesis if the statistical system of economic, social, demographic data, etc is going to correspond.

3. Implementation of the results of investigation in Jordan requires the supplementary training of the public officials, the publishing and popularization of the methods of calculation the satellite-indicators.

4. For implementing the system of evaluation of the territorial development is necessary to motivate the inclusion in the calculation of certain economic, ecological, social indicators that are in the competence of the government of Jordan.

5. The problem of territorial development can be solved successfully, only in situations of permanent up-dating of the initial information.

6. The system of evaluation of districts’ development permits to solve successfully many complicated problems, without big theoretical initiatives of the respective officials.

7. On the basis of the elaborated models was managed to reach the solutions of the formulated problems in an optimal way, on the basis of the classic algorithms this procedure having a more sophisticated character.

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Received: March, 2012