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Algorithmic Approach for Solving Intuitionistic Fuzzy Transportation Problem

R. Jahir Hussain and P. Senthil Kumar

PG and Research Department of Mathematics
Jamal Mohamed College, Tiruchirappalli – 620 020. India
hssn_jhr@yahoo.com, senthilsoft_5760@yahoo.com

Abstract

In this paper, we investigate transportation problem in which supplies and demands are intuitionistic fuzzy numbers. Intuitionistic fuzzy zero point method is proposed to find the optimal solution in terms of triangular intuitionistic fuzzy numbers. A new relevant numerical example is also included.

Keywords: Triangular intuitionistic fuzzy numbers, intuitionistic fuzzy transportation problem, intuitionistic fuzzy zero point method, optimal solution.

1. Introduction

The theory of fuzzy set introduced by Zadeh[8] in 1965 has achieved successful applications in various fields. The concept of Intuitionistic Fuzzy Sets (IFSs) proposed by Atanassov[1] in 1986 is found to be highly useful to deal with vagueness. The major advantage of IFS over fuzzy set is that IFSs separate the degree of membership (belongingness) and the degree of non membership (non belongingness) of an element in the set .The concept of fuzzy mathematical programming was introduced by Tanaka et al in 1947 the frame work of fuzzy decision of Bellman and Zadeh[2].

In [4], Nagoor Gani et al presented a two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy number. In [7], Stephen Dinager et al investigated fuzzy transportation problem with the aid of trapezoidal fuzzy numbers. In[6], Pandian.P and Natarajan.G presented a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problem. In [3], Ismail Mohideen .S and Senthil Kumar .P investigated a comparative study on transportation problem in fuzzy environment.

In this paper, a new ranking procedure which can be found in [5] and is used to obtain an optimal solution in an intuitionistic fuzzy transportation problem[IFTP]. The paper is organized as follows: section 2 deals with some terminology, section 3 provides the definition of intuitionistic fuzzy transportation problem and its mathematical formulation, section 4 deals with solution procedure, section 5 consists of numerical example, finally conclusion is given.

2. Terminology

Definition 2.1 Let A be a classical set, $\mu_A(x)$ be a function from A to [0,1]. A fuzzy set A^* with the membership function $\mu_A(x)$ is defined by

$$A^* = \{(x, \mu_A(x)); x \in A \text{ and } \mu_A(x) \in [0,1]\}.$$

Definition 2.2 Let X be denote a universe of discourse, then an intuitionistic fuzzy set A in X is given by a set of ordered triples,

$\tilde{A}^I = \{< x, \mu_A(x), \vartheta_A(x) >; x \in X\}$

Where $\mu_A, \vartheta_A: X \to [0,1]$, are functions such that $0 \le \mu_A(x) + \vartheta_A(x) \le 1, \forall x \in X$. For each x the membership $\mu_A(x)$ *and* $\vartheta_A(x)$ represent the degree of membership and the degree of non – membership of the element $x \in X$ to $A \subset X$ respectively.

Definition 2.3 An Intuitionistic fuzzy subset A = $\{<x, \mu_A(x), \upsilon_A(x)> : x \in X\}$ of the real line R is called an intuitionistic fuzzy number (IFN) if the following holds:

i. There exist $m \in R$, $\mu_A(m) = 1$ and $\upsilon_A(m) = 0$, (m is called the mean value of A).
ii. μ_A is a continuous mapping from R to the closed interval [0,1] and $\forall x \in R$, the relation $0 \le \mu_A(x) + \vartheta_A(x) \le 1$ holds.

The membership and non – membership function of A is of the following form:

$$\mu_A(x) = \begin{cases} 0 & for -\infty < x \le m - \alpha \\ f_1(x) & for\ x \in [m-\alpha, m] \\ 1 & for\ x = m \\ h_1(x) & for\ x \in [m, m+\beta] \\ 0 & for\ m+\beta \le x < \infty \end{cases}$$

Where $f_1(x)$ and $h_1(x)$ are strictly increasing and decreasing function in $[m-\alpha, m]$ and $[m, m+\beta]$ respectively.

$$\vartheta_A(x) = \begin{cases} 1 & for -\infty < x \le m - \alpha' \\ f_2(x) & for\ x \in [m-\alpha', m]; 0 \le f_1(x) + f_2(x) \le 1 \\ 0 & for\ x = m \\ h_2(x) & for\ x \in [m, m+\beta']; 0 \le h_1(x) + h_2(x) \le 1 \\ 1 & for\ m+\beta' \le x < \infty \end{cases}$$

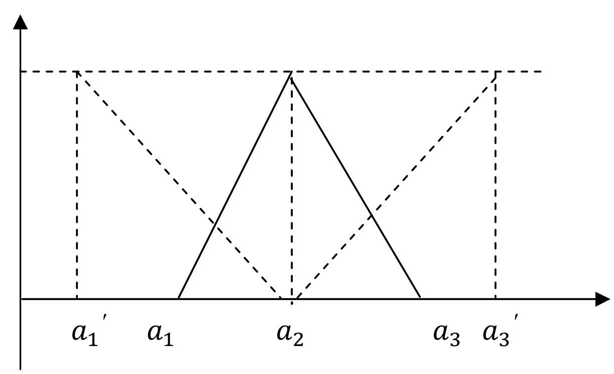
Here m is the mean value of A. α and β are called left and right spreads of membership function $\mu_A(x)$, respectively. α' *and* β' represents left and right spreads of non membership function ϑ_A(x), respectively. Symbolically, the intuitionistic fuzzy number $\tilde{A}^I$ is represented as A_{IFN} =(m; α, β; α', β').

Definition 2.4 A Triangular Intuitionistic Fuzzy Number ($\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function μ_A(x) and non membership function ϑ_A(x):)

$$\mu_A(\mathrm{x}) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & for\ a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & for\ a_2 \leq x \leq a_3 \\ 0 & Otherwise \end{cases}$$

$$\vartheta_A(\mathrm{x}) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'} & for\ a_1' \leq x \leq a_2 \\ \dfrac{x - a_2}{a_3' - a_2} & for\ a_2 \leq x \leq a_3' \\ 1 & Otherwise \end{cases}$$

Where $a_1' \leq a_1 \leq a_2 \leq a_3 \leq a_3'$ and $\mu_A(\mathrm{x}), \vartheta_A(\mathrm{x}) \leq 0.5$ for $\mu_A(\mathrm{x}) = \vartheta_A(\mathrm{x})$ $\forall x \in R$. This TrIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3)(\ a_1', a_2, a_3')$



Membership and non membership functions of TrIFN

Ranking of triangular intuitionistic fuzzy numbers

The Ranking of a triangular intuitionistic fuzzy number is completely defined by its membership and non- membership as follows [5]:
Let $\tilde{A}^I$ = (a,b,c) (e,b,f)

$$x_\mu(A) = \frac{\frac{1}{6(b-a)}[2b^3 - 3b^2a + a^3] + \frac{1}{6(c-b)}[c^3 - 3b^2c + 2b^3]}{\left(\frac{c-a}{2}\right)}$$

$$x_\vartheta(A) = \frac{\frac{1}{6(b-e)}[b^3 - 3e^2 b + 2e^3] + \frac{1}{6(f-b)}[2f^3 - 3bf^2 + b^3]}{\left(\frac{f-e}{2}\right)}$$

$$y_\mu(A) = \frac{1}{3}$$

$$y_\vartheta(A) = \frac{2}{3}$$

Rank (A) = (Sqrt $((x_\mu(A))^2 + (y_\mu(A))^2)$, Sqrt $((x_\upsilon(A))^2 + (y_\upsilon(A))^2)$)

Definition 2.5 Let $\tilde{A}^I$ and $\tilde{B}^I$ be two TrIFNs. The ranking of $\tilde{A}^I$ and $\tilde{B}^I$ by the R(.) on E, the set of TrIFNs is defined as follows:

i. $R(\tilde{A}^I) > R(\tilde{B}^I)$ iff $\tilde{A}^I \succ \tilde{B}^I$
ii. $R(\tilde{A}^I) < R(\tilde{B}^I)$ iff $\tilde{A}^I \prec \tilde{B}^I$
iii. $R(\tilde{A}^I) = R(\tilde{B}^I)$ iff $\tilde{A}^I \approx \tilde{B}^I$

Definition 2.6 The ordering $\succcurlyeq$ and $\preccurlyeq$ between any two TrIFNs $\tilde{A}^I$ and $\tilde{B}^I$ are defined as follows

i. $\tilde{A}^I \succcurlyeq \tilde{B}^I$ iff $\tilde{A}^I \succ \tilde{B}^I$ or $\tilde{A}^I \approx \tilde{B}^I$ and
ii. $\tilde{A}^I \preccurlyeq \tilde{B}^I$ iff $\tilde{A}^I \prec \tilde{B}^I$ or $\tilde{A}^I \approx \tilde{B}^I$

Definition 2.7 Let $\{\tilde{A}_i^I, i = 1,2,\dots,n\}$ be a set of TrIFNs. If $R(\tilde{A}_k^I) \leq R(\tilde{A}_i^I)$for all i, then the TrIFN $\tilde{A}_k^I$ is the minimum of $\{\tilde{A}_i^I, i = 1,2,\dots,n\}$.

Definition 2.8 Let $\{\tilde{A}_i^I, i = 1,2,\dots,n\}$ be a set of TrIFNs. If $R(\tilde{A}_t^I) \geq R(\tilde{A}_i^I)$for all i, then the TrIFN $\tilde{A}_t^I$ is the maximum of $\{\tilde{A}_i^I, i = 1,2,\dots,n\}$.

Arithmetic Operations

Addition: $\tilde{A}^I \oplus \tilde{B}^I = (a_1 + b_1, a_2 + b_2, a_3 + b_3)(a_1' + b_1', a_2 + b_2, a_3' + b_3')$

Subtraction: $\tilde{A}^I \ominus \tilde{B}^I = (a_1 - b_3, a_2 - b_2, a_3 - b_1)(a_1' - b_3', a_2 - b_2, a_3' - b_1')$

Multiplication:

$$\tilde{A}^I \otimes \tilde{B}^I = (l_1, l_2, l_3)(l_1', l_2, l_3')$$

Where,

$l_1 = \min\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$
$l_2 = a_2 b_2$
$l_3 = \max\{a_1 b_1, a_1 b_3, a_3 b_1, a_3 b_3\}$
$l_1' = \min\{a_1' b_1', a_1' b_3', a_3' b_1', a_3' b_3'\}$
$l_2 = a_2 b_2$
$l_3' = \max\{a_1' b_1', a_1' b_3', a_3' b_1', a_3' b_3'\}$

Scalar multiplication:

i. $k\tilde{A}^I = (ka_1, ka_2, ka_3)(k a_1', ka_2, ka_3')$, for $K > 0$
ii. $k\tilde{A}^I = (ka_3, ka_2, ka_1)(k a_3', ka_2, ka_1')$, $for\ K < 0$

3. Intuitionistic Fuzzy Transportation Problem and its Mathematical Formulation

Consider a transportation with m IF origins (rows) and n IF destinations (columns). Let c_{ij} be the cost of transporting one unit of the product from ith IF (Intuitionistic Fuzzy) origin to jth IF destination. $\tilde{a}_i^I = (a_i^1, a_i^2, a_i^3)(a_i^{1'}, a_i^2, a_i^{3'})$ be the quantity of commodity available at IF origin i. $\tilde{b}_j^I = \left(b_j^1, b_j^2, b_j^3\right)\left(b_j^{1'}, b_j^2, b_j^{3'}\right)$ the quantity of commodity needed at intuitionistic fuzzy destination j.

$\tilde{x}_{ij}^I = (x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'})$ is the quantity transported from ith IF origin to jth IF destination, so as to minimize the IF transportation cost.

(IFTP) Minimize $\tilde{Z}^I = \sum_{i=i}^{m}\sum_{j=1}^{n} c_{ij} \otimes (x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'})$

Subject to,

$$\sum_{j=1}^{n}(x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'}) \approx (a_i^1, a_i^2, a_i^3)\left(a_i^{1'}, a_i^2, a_i^{3'}\right), for\ i = 1,2,\dots,m$$

$$\sum_{i=1}^{m}(x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'}) \approx \left(b_j^1, b_j^2, b_j^3\right)\left(b_j^{1'}, b_j^2, b_j^{3'}\right), for\ j = 1,2,\dots,n$$

$$(x_{ij}^1, x_{ij}^2, x_{ij}^3)(x_{ij}^{1'}, x_{ij}^2, x_{ij}^{3'}) \succcurlyeq \tilde{0}^I, \qquad for\ i = 1,2,\dots,m \quad and \quad j = 1,2,\dots,n$$

Where m = the number of supply points

n = the number of demand points

The above IFTP can be stated in the below tabular form

	1	2…	n	IF Supply
1	$\tilde{x}_{11}^{I}$ c_{11}	$\tilde{x}_{12}^{I}\cdots$ $c_{12}\cdots$	$\tilde{x}_{1n}^{I}$ c_{1n}	$\tilde{a}_{1}^{I}$
2 . . .	$\tilde{x}_{21}^{I}$ c_{21} . . .	$\tilde{x}_{22}^{I}\cdots$ $c_{22}\cdots$ . . .	$\tilde{x}_{2n}^{I}$ c_{2n} . . .	$\tilde{a}_{2}^{I}$ . . .
m	$\tilde{x}_{m1}^{I}$ c_{m1}	$\tilde{x}_{m2}^{I}\cdots$ $c_{m2}\cdots$	$\tilde{x}_{mn}^{I}$ c_{mn}	$\tilde{a}_{m}^{I}$
IF Demand	$\tilde{b}_{1}^{I}$	$\tilde{b}_{2}^{I}\ \ldots$	$\tilde{b}_{n}^{I}$	$\sum_{i=1}^{m}\tilde{a}_{i}^{I} = \sum_{j=1}^{n}\tilde{b}_{j}^{I}$

4. The Computational Procedure for Intuitionistic Fuzzy Zero Point Method

This proposed method is used for finding the optimal basic feasible solution in an intuitionistic fuzzy environment and the following step by step procedure is utilized to find out the same.

Step 1. Construct the transportation table whose cost matrix is crisp value as well as supplies and demands are intuitionistic fuzzy numbers. Convert the given problem into a balanced one, if it is not, by ranking method.

Step 2. In the cost matrix subtract the smallest element in each row from every element of that row.

Step 3. In the reduced matrix that is after using the step 2, subtract the smallest element in each column from every element of that column.

Step 4. Check if each row intuitionistic fuzzy supply is less than to sum of the column Intuitionistic fuzzy demands whose reduced costs in that row are zero. Also, check if each column intuitionistic fuzzy demand is less than to the sum of the intuitionistic fuzzy supplies whose reduced costs in that column are zero. If so, go to step 7. Otherwise, go to step 5.

Step 5. Draw the minimum number of vertical lines and horizontal lines to cover all the zeros of the reduced cost matrix such that some entries of row(s) or / and column(s) which do not satisfy the condition of the step 4 are not covered.

Step 6. Develop the new revised reduced cost matrix table as follows:

i. Select the smallest element among all the uncovered elements in the cost matrix.
ii. Subtract this least element from all the uncovered elements and add it to the element which lies at the intersection of any two lines. Thus, we obtain the modified cost matrix and then go to step 4.

Step 7. Select a cell in the reduced cost matrix whose reduced cost is the maximum cost say (α, β). If there are more than one occur then select arbitrarily.

Step 8. Select a cell in the α- row or / and β- column of the reduced cost matrix which is the only cell whose reduced cost is zero and then allot the maximum possible value to that cell. If such cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced cost matrix whose reduced cost is zero.

Step 9. Reform the reduced intuitionistic fuzzy transportation table after deleting the fully used intuitionistic fuzzy supply points and the fully received intuitionistic fuzzy demand points and also, modify it to include the not fully used intuitionistic fuzzy supply points and the not fully received intuitionistic fuzzy demand points.

Step 10. Repeat step 7 to the step 9 until all intuitionistic fuzzy supply points are fully used and all intuitionistic fuzzy demand points are fully received. This allotment yields an optimal solution.

5. Numerical Example:

Consider the 4× 4 IFTP

	IFD1	IFD2	IFD3	IFD4	IF supply
IFO1	16	1	8	13	(2,4,5)(1,4,6)
IFO2	11	4	7	10	(4,6,8)(3,6,9)
IFO3	8	15	9	2	(3,7,12)(2,7,13)
IFO4	6	12	5	14	(8,10,13)(5,10,16)
IF demand	(3,4,6)(1,4,8)	(2,5,7)(1,5,8)	(10,15,20)(8,15,22)	(2,3,5)(1,3,6)	

Since $\sum_{i=i}^{m} \tilde{a}_i^I = \sum_{j=1}^{n} \tilde{b}_j^I$ = (17, 27, 38) (11, 27, 44), the problem is balanced IFTP.

Now, using the step 2 to the step 3 of the intuitionistic fuzzy zero point method, we have the following reduced intuitionistic fuzzy transportation table.

	IFD1	IFD2	IFD3	IFD4	IF supply
IFO1	14	0	7	12	(2,4,5)(1,4,6)
IFO2	6	0	3	6	(4,6,8)(3,6,9)
IFO3	5	13	7	0	(3,7,12)(2,7,13)
IFO4	0	7	0	9	(8,10,13)(5,10,16)
IF demand	(3,4,6)(1,4,8)	(2,5,7)(1,5,8)	(10,15,20)(8,15,22)	(2,3,5)(1,3,6)	

Now, using the step 4 to the step 6 of the intuitionistic fuzzy zero point method, we have the following allotment table.

	IFD1	IFD2	IFD3	IFD4	IF supply
IFO1	11	0	4	14	(2,4,5)(1,4,6)
IFO2	3	0	0	8	(4,6,8)(3,6,9)
IFO3	0	11	2	0	(3,7,12)(2,7,13)
IFO4	0	10	0	14	(8,10,13)(5,10,16)
IF demand	(3,4,6)(1,4,8)	(2,5,7)(1,5,8)	(10,15,20)(8,15,22)	(2,3,5)(1,3,6)	

Now, using the allotment rules of the intuitionistic fuzzy zero point method, we have the allotment

	IFD1	IFD2	IFD3	IFD4	IF supply
IFO1		(2,4,5)(1,4,6)			(2,4,5)(1,4,6)
IFO2		(-3,1,5)(-5,1,7)	(-1,5,11)(-4,5,14)		(4,6,8)(3,6,9)
IFO3	(-2,4,10) (-4,4,12)			(2,3,5)(1,3,6)	(3,7,12)(2,7,13)
IFO4	(-7,0,8) (-11,0,12)		(-1,10,21)(-6,10,26)		(8,10,13)(5,10,16)
IF demand	(3,4,6)(1,4,8)	(2,5,7)(1,5,8)	(10,15,20)(8,15,22)	(2,3,5)(1,3,6)	

The intuitionistic fuzzy optimal solution in terms of triangular intuitionistic fuzzy numbers:

$\tilde{x}^{I}{}_{12}$ = (2,4,5)(1,4,6), $\tilde{x}^{I}{}_{22}$= (-3,1,5)(-5,1,7), $\tilde{x}^{I}{}_{23}$= (-1,5,11)(-4,5,14),

$\tilde{x}^{I}{}_{31}$= (-2,4,10)(-4,4,12), $\tilde{x}^{I}{}_{34}$= (2,3,5)(1,3,6), $\tilde{x}^{I}{}_{41}$= (-7, 0,8)(-11,0,12),

$\tilde{x}^{I}{}_{43}$=(-1,10,21)(-6,10,26)

Hence, the total intuitionistic fuzzy transportation minimum cost is

Min $\tilde{Z}^{\mathrm{I}}$ = (-76,131,345)(-173,131,442)

6. Conclusion

Mathematical formulation of intuitionistic fuzzy transportation problem and procedure for finding an intuitionistic fuzzy optimal solution are discussed with relevant numerical example. The new arithmetic operations of triangular intuitionistic fuzzy numbers are employed to get the optimal solution in terms of triangular intuitionistic fuzzy numbers. The same approach of solving the intuitionistic fuzzy problems may also be utilized in future studies of operational research.

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