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Financial Transparency and Bank Runns

Debora Di Caprio and Francisco J. Santos-Arteaga

GRINEI, Universidad Complutense de Madrid
Campus de Somosaguas, 28223 Madrid, Spain
dicaper@mathstat.yorku.ca, fransant@estumail.ucm.es

Abstract

The current note illustrates how the exogenous imposition of financial transparency on the optimal incentive compatible demand deposit contract designed by a bank, allowing depositors to observe the balance sheet of the bank before withdrawing, may trigger a run with probability one despite the application of the revelation principle to the initial post-deposit game defined by the contract mechanism offered by the bank.

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1 Motivation

Define financial transparency as the public display of the information necessary for depositors to observe either the length of the withdrawing line or the balance sheet of a bank before making their withdrawing decision. The third generation currency crises literature, see for example Aghion et al. [1], does not generally consider information asymmetries between the bank and its depositors, while, if considered, the reliance on the revelation principle derived from the incentive compatibility constraint imposed on the contract designed by the bank eliminates any run incentives from the strategies of depositors, see Chang and Velasco [2]. At the same time, recent banking and currency crises have originated after the request for and imposition of transparency on the *existing* financial system of several countries, see Furman and Stiglitz [4] for a detailed treatment of this issue regarding the Asian crises. In particular, the current financial crises gave initially place to several bank runs, one of which, that on Northern Rock, was caused by a display of information regarding its balance sheet to the public, see Di Caprio and Santos-Arteaga [3] and O'Connor and

Santos-Arteaga [5] for a formal analysis of the run and a summary of the main events that triggered it.

The current note highlights the fragility of financial systems if modifications to the initial information structure employed to design optimal demand deposit contracts are exogenously imposed after agents have deposited funds and a financial intermediary has been created.

2 Model and Result

The basic model follows Peck and Shell [6], who define a Diamond-Dybvig economic environment with a finite set of identical agents, N. This assumption would allow for monitoring by depositors through a countable withdrawing line or simple observation of the balance sheet of the bank. In the latter case, bank runs can be triggered even if a continuous set of agents is assumed, as a biyection delimiting the withdrawing intervals can be easily defined on the balance sheet of the bank.

There are three time periods, $t = 0, 1, 2$. Each agent is endowed with one unit of an homogeneous good in period zero, which can be costlessly stored among periods. There exists a production technology that delivers one unit of output per unit invested if interrupted after one period, and $R > 1$ units of output if the invested unit is kept for two periods. Agents are subject to an exogenous shock in period one defined by the set of possible realizations of a given random variable, Λ^b, with an associated probability function $f(\Lambda^b)$. To simplify notation we refer to $f(\Lambda^b) = f(\lambda_i^b | \lambda_i^b \in \Lambda^b)$.

The agents affected by the shock become impatient, or type 1, and value consumption in period one only. The remaining agents, which we refer to as patient or type 2, value consumption in both periods. Denote by c_{tk}^i the amount of goods received by a type k agent in period t given state i. The state dependent utility of each agent is given by

$$U(c_{11}^i, c_{12}^i, c_{22}^i; \lambda_i^b) = \begin{cases} u(c_{11}^i(\lambda_i^b)) & \text{if the agent is impatient} \\ \rho u(c_{12}^i(\lambda_i^b) + c_{22}^i(\lambda_i^b)) & \text{if the agent is patient} \end{cases}$$

where ρ is the rate of time preference[1], $\lambda_i^b \in \Lambda^b$ represents a given relative state of the economy, defining the utility dependence on the realization of the random variable (as well as the sequential nature of the set of allocations offered by the bank), and $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing, twice continuously differentiable, and satisfies the Inada conditions, $u'(0) = \infty$ and $u'(\infty) = 0$.

Consider the optimization problem faced by a bank, given the previous framework and conditional on its information set, $\Gamma_b = \{N, \Lambda^b, f(\Lambda^b)\}$, assumed identical to the one of its depositors except for the privately observed

[1] For simplicity, it is generally assumed that $\rho = 1$.

type realization of the latter ones. Jointly with a sequential service constraint, the altruistic bank maximizes the following value function $V(\Lambda^b, f(\Lambda^b))$

$$\max_{\substack{c_{11}^i, c_{12}^i, c_{22}^i \\ i=1,...,\sharp\Lambda^b}} \sum_{\substack{i=1 \\ \lambda_i^b \in \Lambda^b}}^{\sharp\Lambda^b} f(\lambda_i^b) \left[\sum_{j=1}^{i} (\lambda_j^b - \lambda_{j-1}^b) u(c_{11}^j(\lambda_j^b)) + (N - \lambda_i^b)\rho u(c_{12}^i(\lambda_i^b) + c_{22}^i(\lambda_i^b)) \right]$$

where $\sharp\Lambda^b$ denotes the cardinality of the set of possible realizations, $\lambda_0^b = 0$, and $(\lambda_j^b - \lambda_{j-1}^b)$ represents the number of agents contained between two consecutive realizations within Λ^b, subject to the set of budget constraints, which must be satisfied $\forall \lambda_i^b \in \Lambda^b$

$$\sum_{j=1}^{i} (\lambda_j^b - \lambda_{j-1}^b) c_{11}^j(\lambda_j^b) + (N - \lambda_i^b) \left(c_{12}^i(\lambda_i^b) + \frac{c_{22}^i(\lambda_i^b)}{R} \right) = N, \quad i = 1, ..., \sharp\Lambda^b$$

and a corresponding incentive compatibility condition, *icc* henceforth, for the mechanism

$$\sum_{i=1}^{\sharp\Lambda^b} f_b(\lambda_i^b) \left[u(c_{22}^i(\lambda_i^b)) \right] \geq$$

$$\sum_{i=1}^{\sharp\Lambda^b} f_b(\lambda_i^b) \sum_{j=1}^{i} \left[\frac{(\lambda_j^b - \lambda_{j-1}^b)}{\lambda_i^b + 1} u(c_{11}^j(\lambda_j^b)) + \frac{1}{\lambda_i^b + 1} u(c_{11}^{i+1}(\lambda_i^b + 1)) \right],$$

such that

$$f_b(\lambda_i^b) = \frac{[1 - (\lambda_i^b/N)] f(\lambda_i^b)}{\sum_{\lambda'=0}^{N-1} [1 - (\lambda'/N)] f(\lambda')}, \quad \forall \lambda_i^b, \lambda' \in \Lambda^b$$

where $f(\lambda_i^b)$ stands for the ex-ante (prior) probability assigned to the number of impatient agents λ_i^b, with $\lambda_i^b \in \Lambda^b \subseteq \{1, ..., N\}$. This function is generally assumed to be common knowledge among all agents, and therefore, also between depositors and the bank, in period zero. *Patient* agents update this probability in period one, after receiving the type-determining signal, using Bayes' rule. The updating process delivers the subjective probability, as of each agent, of having λ_i^b impatient depositors in the economy, conditional on the agent being patient, $f_b(\lambda_i^b)$. These probabilities are used by depositors to calculate the expected payoffs on which to base their strategic behavior in the post-deposit game generated by the bank contract structure that solves the previous optimization problem. Beliefs are a direct function of $f(\lambda_i^b)$, assumed to be known by all agents. This homogeneity assumption implies that depositors share the beliefs of the bank, denoted by $f_b(\lambda_i^b)$, the subscript b representing the bank, when defining their optimal strategies.

The *icc* condition is based on the expected payoffs the bank assumes its depositors calculate if they are not able to observe the length of the withdrawing line or its balance sheet before deciding whether or not to withdraw, leading to a static (defined and fixed as of period zero) inequality in expected payoffs. Therefore, the *icc* equilibrium condition differs from the set of constraints required to guarantee the existence of stable (no-run) equilibria through the set of post-deposit subgames that would be generated for each $\lambda^w \leq \lambda^b_{\sharp\Lambda^b}$ if transparency is imposed, where λ^w stands for the number of withdrawing agents that are either observed in line or define a corresponding balance sheet for the bank through its budget constraint, and $\lambda^b_{\sharp\Lambda^b}$ is the supremum of Λ^b. At the same time, the revelation principle states that runs on the bank can be prevented, except if caused by a sunspot, as long as the post-deposit game played by patient depositors has a no-run Nash equilibrium where agents can coordinate their withdrawing strategies. If this game is unique, the existence of such an equilibrium is imposed on the demand deposit mechanism through the *icc* condition. However, for the revelation principle to apply to the entire set of post-deposit subgames we need a no-run equilibrium to exists for each and every one of these subgames.

Proposition 2.1 *If the icc condition binds in equilibrium, the exogenous imposition of financial transparency on the optimal demand deposit contract designed by a bank triggers a run with probability one, despite the application of the revelation principle to the initial post-deposit game defined by the contract mechanism offered by the bank.*

Proof The envelope theorem is applied through the proof, since it is a known result that the optimal demand deposit contract assigns $c^i_{21}(\lambda^b_i) = c^i_{12}(\lambda^b_i) = 0$, $\forall \lambda^b_i \in \Lambda^b$. Without loss of generality, assume that $\Lambda^b = \{\lambda^b_1, \lambda^b_2\}$. An optimal mechanism designed by the bank for a given set $\Lambda^b = \{\lambda^b_1, \lambda^b_2\}$, and an associated probability function $f(\Lambda^b)$, is defined by the vector

$$\mathbf{m}(f(\Lambda^b)) = (c^1_1(\lambda^b_1), c^2_1(\lambda^b_2), c^1_2(\lambda^b_1), c^2_2(\lambda^b_2)),$$

where $c^1_1(\lambda^b_1)$ denotes the consumption given to the first λ^b_1 agents in line who declare being impatient, while $c^2_1(\lambda^b_2)$ is allocated to each of the remaining impatient agents in line. The corresponding consumption allocations offered to the patient depositors withdrawing in the second period are given by $c^1_2(\lambda^b_1)$, if only λ^b_1 agents withdraw in the first period, and $c^2_2(\lambda^b_2)$ if λ^b_2 impatient agents withdraw. The *icc* employed by the bank to design the mechanism is given by

$$\sum_{i=1}^{2} f_b(\lambda^b_i) u\left(\frac{[N - \sum_{j=1}^{i}(\lambda^b_j - \lambda^b_{j-1})c^j_1(\lambda^b_j)]R}{N - \lambda^b_i}\right) \geq$$

$$\sum_{i=1}^{2} f_b(\lambda_i^b) \sum_{j=1}^{i} \left[\frac{(\lambda_j^b - \lambda_{j-1}^b)}{\lambda_i^b + 1} u(c_1^j(\lambda_j^b)) + \frac{1}{\lambda_i^b + 1} u(c_1^{i+1}(\lambda_i^b + 1)) \right]$$

Assume that the *icc* holds with equality, see Peck and Shell [6].[2] To simplify notation assume that $c_1^{2+1}(\lambda_2^b+1) = c_1^2(\lambda_2^b)$. A binding *icc* leads to the following first order condition defining the values of $c_1^2(\lambda_2^b)$ and $c_2^2(\lambda_2^b)$

$$u'(c_1^2(\lambda_2^b)) = \left[\frac{f(\lambda_2^b)(\lambda_2^b - \lambda_1^b) + \delta f(\lambda_2^b) \frac{\lambda_2^b - \lambda_1^b}{N - \lambda_2^b}}{f(\lambda_2^b)(\lambda_2^b - \lambda_1^b) - \delta f(\lambda_2^b) \frac{(\lambda_2^b + 1) - \lambda_1^b}{\lambda_2^b + 1} - \delta f(\lambda_1^b) \frac{1}{\lambda_1^b + 1}} \right] R u'(c_2^2(\lambda_2^b))$$

where δ is the multiplier associated with the *icc*, and the term in brackets is clearly larger than one. It follows directly from this condition that $c_2^2(\lambda_2^b) > c_1^2(\lambda_2^b)$. Given a binding *icc*, the following cases are possible (though described in terms of consumption allocations the analysis remains unchanged if utility values are employed).

Case (i). The value of $c_2^2(\lambda_2^b)$ is equal to the weighted average defined by $c_1^1(\lambda_1^b)$ and $c_1^2(\lambda_2^b)$. If this is the case, $c_2^1(\lambda_1^b)$ must also be equal to the corresponding weighted average of $c_1^1(\lambda_1^b)$ and $c_1^2(\lambda_2^b)$. Therefore, $c_2^1(\lambda_1^b) < c_1^1(\lambda_1^b)$ leading to a run on the bank for all values of $\lambda^w < \lambda_1^b$. The run stops after λ_1^b is reached, since $c_2^2(\lambda_2^b) > c_1^2(\lambda_2^b)$, eliminating any incentive for patient agents to misrepresent their types (unless the *expected* consumption remaining after the initial run is smaller than $c_1^2(\lambda_2^b)$, in which case the run extends to the entire set of patient depositors).

Case (ii). The value of $c_2^2(\lambda_2^b)$ is smaller than the weighted average defined by $c_1^1(\lambda_1^b)$ and $c_1^2(\lambda_2^b)$. In this case, $c_1^2(\lambda_2^b) < c_2^2(\lambda_2^b) < c_1^1(\lambda_1^b)$ and $c_2^1(\lambda_1^b)$ can either be larger or smaller than $c_1^1(\lambda_1^b)$. Hence, a binding *icc* implies that $c_1^1(\lambda_1^b)$ is larger than the weighted average defined by $c_2^1(\lambda_1^b)$ and $c_2^2(\lambda_2^b)$, leading to an attack on the bank for all values of $\lambda^w < \lambda_1^b$.

Case (iii). The value of $c_2^2(\lambda_2^b)$ is larger than the weighted average defined by $c_1^1(\lambda_1^b)$ and $c_1^2(\lambda_2^b)$. This inequality implies that $c_2^1(\lambda_1^b) < c_1^1(\lambda_1^b)$, leading to a run on the bank for all values of $\lambda^w < \lambda_1^b$ if $c_2^2(\lambda_2^b) < c_1^1(\lambda_1^b)$. On the other hand, if $c_2^2(\lambda_2^b) > c_1^1(\lambda_1^b)$, we have $c_2^2(\lambda_2^b) > c_1^1(\lambda_1^b) > c_2^1(\lambda_1^b)$, defining a monotonically increasing set of allocations for patient depositors in λ_i^b. Following the same reasoning used in (ii), a binding *icc* leads to a self-contained run if $c_1^1(\lambda_1^b) > c_1^2(\lambda_2^b)$. A bank run could be prevented if the payoff sequence satisfies $c_2^2(\lambda_2^b) >$

[2]Peck and Shell assume a utility function for impatient depositors given by a monotonically increasing transformation of that of patient depositors, i.e. $Au(c)$, with $A > 1$. The consumption allocation inequality derived from the first order condition remains valid if $\left[\frac{f(\lambda_2^b)(\lambda_2^b - \lambda_1^b) + \delta f(\lambda_2^b) \frac{\lambda_2^b - \lambda_1^b}{N - \lambda_2^b}}{f(\lambda_2^b)(\lambda_2^b - \lambda_1^b) - \delta f(\lambda_2^b) \frac{(\lambda_2^b + 1) - \lambda_1^b}{\lambda_2^b + 1} - \delta f(\lambda_1^b) \frac{1}{\lambda_1^b + 1}} \right] R > A$.

$c_1^2(\lambda_2^b) > c_1^1(\lambda_1^b) > c_2^1(\lambda_1^b)$, which is an impossibility. To see why, note that in order to have $c_1^1(\lambda_1^b) > c_2^1(\lambda_1^b)$, the following inequality must be satisfied

$$c_1^1(\lambda_1^b) > \frac{[N - \lambda_1^b c_1^1(\lambda_1^b)]R}{N - \lambda_1^b}$$

whose right hand side defines $c_2^1(\lambda_1^b)$ based on the value of $c_1^1(\lambda_1^b)$. The same procedure applies to $c_2^2(\lambda_2^b) > c_1^2(\lambda_2^b)$, in which case we require that

$$c_1^2(\lambda_2^b) < \frac{[N - \lambda_1^b c_1^1(\lambda_1^b)]R}{N - \lambda_2^b + (\lambda_2^b - \lambda_1^b)R}$$

Substituting the first inequality into the second we obtain

$$c_1^2(\lambda_2^b) < c_1^1(\lambda_1^b) \left[\frac{(N - \lambda_1^b)}{N - \lambda_2^b + (\lambda_2^b - \lambda_1^b)R} \right]$$

where the term in brackets is clearly smaller than one. Thus, given $\lambda_2^b > \lambda_1^b$ and $c_2^2(\lambda_2^b) > c_2^1(\lambda_1^b)$, we must have that $c_1^2(\lambda_2^b) < c_1^1(\lambda_1^b)$, contradicting the required sequence. ∎

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