A New Weight of

Interval Type-2 Fuzzy Rasch Model

Z. Nurnadiah* and A. Lazim

Department of Mathematics, Faculty of Science and Technology, University Malaysia Terengganu, Kuala Terengganu, Malaysia

Abstract

Due to the lots of nowadays challenging problems, there is an urgent need for an efficient method and retrieval technique of weighting part for MCDM method. Unfortunately, the currently available weighting method is less feasibility due to the fact that it is used type-1 fuzzy sets rather than type-2 fuzzy sets. Meanwhile, the fuzzy MCDM method suffers from achieving a rather consistent weighting outcome. Therefore, in this research, a new revolutionary approach for weighting part of fuzzy MCDM is proposed. Different with the original weighting process, this newly proposed method uses totally new approach; proposes a new weight method by using rasch model in interval type-2 fuzzy sets (IT2FS) concepts. This paper utilizes the nature of IT2FS concept in the evaluation process to assess the attribute weight based on the credibility of data. An example is presented to demonstrate the feasibility of the new method. Thus, this new rasch weight of interval type-2 fuzzy sets yield flexible judgment and can be applied in decision making environment.

Keywords: Rasch model, Likert scale rating, Interval type-2 fuzzy sets

Corresponding Author:
Electronic address:
*nadzlina@yahoo.co.uk
1. Introduction

Weight is the most important part and plays an important role in multi criteria decision making (MCDM). In MCDM, the weights of the criteria are crucial for measuring the importance of the criteria [1]. Many methods for solving MCDM problems require definitions of quantitative weights for the attributes. It is very important to measure attribute weights properly, because they could influence the results of the analysis such as rankings of alternatives [2]. Weight in multiple attribute decision making (MADM) can be divided into two groups which are subjective weight and objective weight. Subjective weight can reflect the subjective judgment or intuition of the decision makers (DM), and they can be obtained based on preference information of the attributes given by the DM through interviews, questionnaires or trade-off interrogation directly [3]. Objective weight can be obtained from the objective information such as decision matrix through mathematics models [4].

Since that, numerous studies have attempted to investigate and modify weight with various approaches and methods. For example, Wang and Lee [5] proposed a novel approach that involves end-user into the whole decision making process. In that proposed approach, the subjective weights assigned by decision makers (DM) were normalized into a comparable scale. In addition, they also adopt end-user ratings as an objective weight based on Shannon’s entropy theory. This method provided decision makers more information to make more subtle decisions. Then, in Kao [6] has found that one of the most difficult tasks in multiple criteria decision analysis (MCDA) was determining the weights of individual criteria so that all alternatives can be compared based on the aggregate performance of all criteria. Therefore, this problem has been transformed into the compromise programming of seeking alternatives with a shorter distance to the ideal or a longer distance to the anti-ideal despite the rankings based on the two distance measures possibly not being the same. Besides, Yue [7] has developed a method for determining weights of decision makers under group decision environment, in which the each individual decision information was expressed by a matrix in interval numbers. The positive and negative ideal solutions of group decision was defined, which were expressed by a matrix, respectively. Furthermore, Chen [8] presented SAW-based and TOPSIS-based MCDA methods and conducted a comparative study through computational experiments. Comprehensive discussions have been made on the influence of score functions and weight constraints, where the score function represents an aggregated effect of positive and negative evaluations in performance ratings and the weight constraint consists of the unbiased condition, positivity bias, and negativity bias. The correlations and contradiction rates obtained in the experiments suggest that evident similarities exist between the interval-valued fuzzy SAW and TOPSIS rankings. Huang and Peng [9] proposed a novel approach, the fuzzy rasch model, which combines item response theory (IRT) and fuzzy set theory. It determined how to obtain accurate fuzzy numbers for the criteria weights. This study has applied the fuzzy rasch model in Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
to analyse the Tourism Destination Competitiveness (TDC) of nine Asian countries. However, the inherent uncertainty and subjectivity of this method can result in weighting errors and difficulties in the criteria weight selection process.

To deal with the uncertainties and subjectivity problems, the idea from type-2 fuzzy sets is used. Since fuzzy sets (FS) or usually known as type-1 fuzzy sets (T1FS) has found not be able to handle all kinds of uncertainties appearing in real life problem domain [10]. Therefore in 1975, Zadeh [11] has introduced a type-2 fuzzy sets (T2FS) concepts that are more realible to handle all the uncertainties. However, T2 FS is difficult to understand and explain. Hence, Mendel and Liang [12] introduced new concepts that are more easier to calculate. These new concepts are allowing the characterization of a type-2 fuzzy set with a superior membership function and an inferior membership function; these two functions can be represented each one by a type-1 fuzzy set membership function. The interval between these two functions represent the footprint of uncertainty (FOU), which is used to characterize a type-2 fuzzy set. Then in 2006, Mendel, John and Liu [13] are introduced new concepts of interval type-2 fuzzy sets (IT2FS). The simplest T2FS are interval type-2 fuzzy sets whose elements’ degree of membership are intervals with secondary membership degree of 1.0. If we can use interval type-2 fuzzy sets [13] for handling entropy weights problems, then there is room for more flexibility due to the fact that T2FS provide more flexibility to represent uncertainties than traditional T1FS [14]. Since the idea of IT2FS was built, many of literatures mentioned about the weighting part in IT2FS concept. Until now, there are lots of papers discussed on the improvement of weight in IT2FS concept. Starts in 1996, Park and Kim [15] presented the characteristic or attribute of a fuzzy weighted additive rule (FWAR). FWAR has provided a method of assessing weights for criteria based on the fuzzy opinions of experts. This method was elicited weights for aggregating the fuzzy sets of Type 1 and Type 2 by using quadratic programming. But FWAR has not expressed a specific description on the information of experts' overall evaluations for alternatives. If the style of experts' overall evaluations is characterized as linear additive weighting of the performances of alternative for criterion, then the optimal aggregated value of the given performances of alternative is given by the arithmetic mean of the experts' overall evaluations, without solving time-consuming quadratic programming in FWAR. Then in 2007, Wu and Mendel [16] focused on the linguistic weighted average (LWA), where the weights are always words modeled as interval type-2 fuzzy sets (IT2 FSs), and the attributes may also (but do not have to) be words modeled as IT2 FSs; consequently, the output of the LWA is an IT2 FS. The LWA can be viewed as a generalization of the fuzzy weighted average (FWA) where the type-1 fuzzy inputs are replaced by IT2 FSs. This paper presents the theory, algorithms, and an application of the LWA. Besides, Sevastjanov and Figat [17] proposed a new approach to MCDM in a fuzzy setting based on the aggregation of aggregating modes. The problem of appropriate common scale for representation of objective and subjective criteria was solved using the simple subset hood measure based on $\alpha$-cut representation of fuzzy values. The synthesis of the tools of Type 2 and Level 2 fuzzy sets was used to elaborate an appropriate method for such
aggregation. Moreover, Zhou et al. [18] defined a new type of OWA operator, the type-1 OWA operator that works as an uncertain OWA operator to aggregate type-1 fuzzy sets with type-1 fuzzy weights, which can be used to aggregate the linguistic opinions or preferences in human decision making with linguistic weights. The procedure for performing type-1 OWA operations was analysed and type-2 linguistic quantifiers was proposed in order to identify the linguistic weights associated to the type-1 OWA operator. Then, the problem of how to derive linguistic weights used in type-1 OWA aggregation given such type of quantifier was solved. Furthermore, Jeon et al. [19] adopted type-2 fuzzy sets concepts to design a weight evaluating approach. In that proposed method, the upper and lower fuzzy membership functions of the type-2 fuzzy logic filters were derived from the type-1 (or primary) fuzzy membership function. The weights from upper and lower membership functions were considered to be multiplied with the candidate deinterlaced pixels. Experimental results proved that this method were both objectively and subjectively to other different conventional deinterlacing methods. Then, Zhou et al. [20] proposed a new type of OWA operator, termed type-2 OWA operator, to aggregate the linguistic opinions or preferences in human decision making modeled by type-2 fuzzy sets. A Direct Approach to aggregating interval type-2 fuzzy sets by type-2 OWA operator was suggested in this paper. Some examples were provided to delineate the proposed technique. Besides, also in 2010, Lin and Roopaei [21] adjusted weights, centers and widths of proposed fuzzy neural network (FNN) based on the adaptive interval type-2 fuzzy logic. These modeling errors can be eliminated for a class of SISO time-delay nonlinear systems. The proposed scheme has the advantage that can guarantee the H1 tracking performance to attenuate the lumped uncertainties caused by the unmodelled dynamics, the approximation error and the external disturbances. Next in 2011, Akay et al. [22] presented a new concept selection methodology that extends the fuzzy information axiom (FIA) approach to incorporate IT2FSs. The proposed methodology was called interval-type-2 fuzzy information axiom (IT2-FIA). IT2-FIA method was also enriched by using ordered weighted geometric aggregation operator to include the decision maker’s attitude during the aggregation process. Besides, Chen and Chang [23] presented a new method for fuzzy rule interpolation for sparse fuzzy rule-based systems based on the ratio of fuzziness of interval type-2 fuzzy sets. This proposed method can overcome the drawbacks of the existing methods by calculating the weights of the closest fuzzy rules with respect to the observation to obtain an intermediate consequence fuzzy set. Then, it was used the ratio of fuzziness of interval type-2 fuzzy sets to infer the fuzzy interpolated result based on the intermediate consequence fuzzy set. Some examples were used to compare the fuzzy interpolated results of the proposed method with the results by the existing methods. The experimental results showed that the proposed fuzzy rule interpolation method gets more reasonable results than the existing methods. Moreover, Chen et al. [24] presented a new method to deal with fuzzy multiple attributes group decision-making problems based on ranking interval type-2 fuzzy sets. First, the authors were proposed a new method for ranking interval type-2 fuzzy sets. Then, they proposed a new method for fuzzy multiple attributes group
decision-making based on the proposed ranking method of interval type-2 fuzzy sets. They also use some examples to illustrate the fuzzy multiple attributes group decision-making process of the proposed method. The proposed method was simpler than the methods presented in Lee and Chen [25][26] for fuzzy multiple attributes group decision-making based on interval type-2 fuzzy sets. It provides us with a useful way for dealing with fuzzy multiple attributes group decision-making problems based on interval type-2 fuzzy sets. Motivated from the idea of IT2FS in weighting part, this paper develops a new weight of fuzzy rasch model. Thus, to test and evaluate the performance of the proposed method, part of the numerical examples from Huang and Peng [9] is used. Therefore, it believes that this model can rectify the inaccuracy of the fuzzy numbers assigned by the individual experts for the specific criteria. Hence, this model is more suitable to represent uncertainties than the original fuzzy sets. Indeed, the combination between two ideas which are T2FS and fuzzy rasch model is believed to overcome all the problems above.

The rest of this paper is organized as follows. Section 2 is briefly reviews the basic concepts of interval type-2 fuzzy sets, rasch model and likert scale. Section 3 is extended the fuzzy rasch model to propose a new weight model for handling the uncertainties problems based on interval type-2 fuzzy sets. In Section 4, an example is used to illustrate the proposed method. Finally, the paper is concluded in Section 5.

2. Basic Concepts

In the following, we recall basic notations and definitions of interval type-2 fuzzy sets, rasch model and likert scale in Sub-section 2.1, 2.2 and 2.3.

2.1 Interval type-2 fuzzy sets

This section is briefly review some definitions of type-2 fuzzy sets and interval type-2 fuzzy sets from Mendel et al. [13].

Definition 2.1 [13]

A type-2 fuzzy set \( \tilde{A} \) in the universe of discourse \( X \) can be represented by a type-2 membership function \( \mu_{\tilde{A}} \), shown as follows (Mendel et al. [13]):

\[
\tilde{A} = \left\{ (x,u), \mu_{\tilde{A}}(x,u) \right\} \forall x \in X, \forall u \in J_x \subseteq [0,1], 0 \leq \mu_{\tilde{A}}(x,u) \leq 1.
\] (1)

where \( J_x \) denotes an interval in \([0,1]\). Moreover, the type-2 fuzzy set \( \tilde{A} \) also can be represented as follows:
\[ \tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_A(x,u)/(x,u), \]  

where \( J_x \subseteq [0,1] \) and \( \int \int \) denotes the union over all admissible \( x \) and \( u \).

**Definition 2.2 [13]**

Let \( \tilde{A} \) be a type-2 fuzzy set in the universe of discourse \( X \) represented by the type-2 membership function \( \mu_A \). If all \( \mu_A = 1 \), then \( A \) is called an interval type-2 fuzzy sets. An interval type-2 fuzzy set \( \tilde{A} \) can be regarded as a special case of a type-2 fuzzy set, represented as follows:

\[ \tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x,u), \]  

where \( J_x \subseteq [0,1] \)

### 2.2 Rasch Model

IRT (item response theory) models first appeared in the field of psychometry and educational sciences to quantify human behavior [27]. They are now increasingly used in medicine to study psychological traits in psychiatry and, more recently, to assess quality of life in clinical trials or epidemiology. One of the most popular IRT models is the rasch model [28]. Rasch model was originally developed for dichotomous (two-choice) items by [29]. Rasch model is a one-parameter logistic and static model within item IRT in which the amount of a given latent trait in a person and the amount of the same latent trait reflected in various items can be estimated independently yet still compared explicitly to one another [30]. Through rasch model, each person with a certain amount of a given latent trait specifies the probability of a response in one of the categories of an item [31]. Besides, it measures the relationship between a person's ability and an item difficulty, and models this as a probabilistic function [32]. Specifically, raw data from a rating scale is converted to “an equal interval scale” measured in logits (log odd units), reflecting the item difficulty and person's ability [33][34]. It is also a logistic model of probability for monotonically increasing functions.

The rasch model is based on the concept that the probability of correctly obtaining an item is a function of a latent trait or ability [35]. Notably, the rasch model is also known as the One-Parameter Logistic Model (1PL). The rasch model converts raw data from a rating scale to “an equal interval scale” measured in logits (log odd units) [36], which reflect both the difficulty of the item and individual ability [33].
Since Andrich [37] developed RSM, it has been extensively adopted by scholars to assess the values of item and person parameters, as shown in Equation 4.

\[
\log \left( \frac{P_{nij}}{P_{ni(j-1)}} \right) = \theta_n - (\delta_i + \tau_j)
\]

(4)

In Equation 1, \( P_{nij} \) and \( P_{ni(j-1)} \) represent the probability that the item \( n \) obtains \( j \) and \( j-1 \) scores from the expert \( i \). \( \theta_n \) represents the measure score (i.e., item difficulty) of the item \( n \), \( \delta_i \) represents the measure score (i.e., individual ability) of expert \( i \), \( \tau_j \) and represents the step difficulty (i.e., threshold difficulty) of category \( j \). The step difficulty of RSM is identical for all items [38]. Thus, the RSM is useful if the psychological distances between categories are identical for all items [39], as is the case for the Likert scales.

\[
\delta_j = \delta_i + \tau_j
\]

(5)

In Equation 2, \( i = 1, \ldots, E \) and \( E \) represents the number of experts. \( j = 1, \ldots, m \) and \( m \) represents the number of linguistic scales, which range from “very unimportant” to “very improtant”.

2.3 Likert Scale

The rating scale model (RSM) devised by Andrich [37] apples Rasch’s model to polytomous rating scale instruments, which include the five-point Likert scale. The rating scale model is an additive linear model that describes the probability that a specific person (\( n \)) will respond to a specific Likert-type item (\( i \)) with a specific rating scale step (\( x \)) [40]. It is important to note that the Likert scale can be modelled with either the rating scale or the partial credit model [38][41]. The partial credit model allows the item format and the number of categories to vary from item to item (e.g., some items are scored with a 5-point scale and others with a 6-point scale). When the item format is inconsistent from item to item, the partial credit model is useful in providing estimates of the psychological distance between each set of the ordinal categories [42].

The rasch rating scale model allows likert scale attitude data to be thought about in developmental rather than merely descriptive ways. The rasch partial credit model provides for the investigation of mathematical performances where part marks are awarded to partially correct responses. Other kinds of analyses provide for studies of markers (for severity or bias), exploration of the dimensionality of particular constructs, interview-based data, and stage-related development. More importantly, the nature of the data produced through rasch modelling processes (genuinely interval scale data) allows a wide range of further quantitative
techniques to be used felicitously [42].

3. A new weight of fuzzy rasch model with interval type-2 fuzzy sets (IT2FS)

Motivated from Huang and Peng [9] weighting method, we extend fuzzy rasch model weight to our proposed IT2FS method to propose a new weight with type-2 fuzzy sets (T2FS) concepts. The advantage of the proposed approach is believes that the model is not only feasible but can also rectify the inaccuracy of the fuzzy numbers assigned by the individual experts for the specific criteria. Besides, the model is flexible rather than the original model due to the fact that IT2FS is better than T2FS. On the other hand, this model is more suitable to represent uncertainties because it is involve end-users into the whole weighting process. Thus, this innovative approach is capable of providing a more comprehensive weight model for weighting process. Therefore, general process of this method is listed as Sub-section 3.1.

3.1 Algorithm for the interval type-2 fuzzy rasch weight (IT2FR)

The steps of interval type-2 fuzzy rasch weight algorithm are being expressed in this section. The original rasch model has been upgraded into a new weight of fuzzy rasch in interval type-2 fuzzy sets (IT2FS) concepts. Therefore, general process of the weight of fuzzy Rasch in interval type-2 fuzzy sets concepts is listed as follows:

**Step 1:** Construct the design matrix

Construct the design matrix \( Y_p \) of the \( p \)th decision-maker and construct the average decision matrix respectively. This decision matrix is in form of interval type-2 fuzzy sets. Therefore, it is shown as follows:

\[
Y_p = \left( \tilde{f}_{ij}^p \right)_{m \times n} = \begin{bmatrix}
\tilde{f}_{11}^p & \tilde{f}_{12}^p & \cdots & \tilde{f}_{1n}^p \\
\tilde{f}_{21}^p & \tilde{f}_{22}^p & \cdots & \tilde{f}_{2n}^p \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{f}_{m1}^p & \tilde{f}_{m2}^p & \cdots & \tilde{f}_{mn}^p
\end{bmatrix}
\]

(6)

\[
Y = \left( \tilde{f}_{ij} \right)_{m \times n}
\]

(7)
where

$$\tilde{f}_{ij} = \left( \tilde{f}_{ij}^1 \oplus \tilde{f}_{ij}^2 \oplus \ldots \oplus \tilde{f}_{ij}^k \right)$$

is an interval type-2 fuzzy set, \(1 \leq i \leq m, 1 \leq j \leq n, 1 \leq p \leq k\), and \(k\) denotes the number of decision-makers.

**Step 2:** Appoint the linguistic variables for the likert rating scale.

Appoint the degree of each likert rating scale using the linguistic variables in interval type-2 fuzzy sets concept. There are six experts to indicate the degree of each scale ranging from 1-5 (from “very unimportant” to “very important”) shown as Table 1. The concept of interval type-2 fuzzy sets is adapted in likert rating scale, shown as Figure 1.

<table>
<thead>
<tr>
<th>Likert rating scale</th>
<th>Appoint number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very unimportance (VU)</td>
<td>1</td>
</tr>
<tr>
<td>Unimportance (U)</td>
<td>2</td>
</tr>
<tr>
<td>General (G)</td>
<td>3</td>
</tr>
<tr>
<td>Importance (I)</td>
<td>4</td>
</tr>
<tr>
<td>Very importance (VI)</td>
<td>5</td>
</tr>
</tbody>
</table>

**Figure 1:** Interval type-2 fuzzy sets
Step 3: Determine the degree of the importance for each criteria/indices.
Six experts indicate the degree of importance of six criteria and 15 indices using Likert rating scale ranging from 1-5 (from “very unimportant” to “very important”) that has pointed in Step 2.

Step 4: Calculate the step parameters \( \delta_{ij} \) to generate the weight of interval type-2 fuzzy Rasch (IT2FR) model.
Adapting the concept of triangular interval type-2 fuzzy sets in fuzzy Rasch model weight, then, this new concept is used to substitute the values of importance calculated in Step 3.

Step 5: Calculate the arithmetic average
Use an arithmetic average to integrate the new weight of each expert. This study uses an arithmetic average to integrate the fuzzy weight of each expert. The concept of triangular interval type-2 fuzzy sets is adapted as follows:

\[
\tilde{w}_j = \frac{1}{FOU(\tilde{w}_j)} = \left( \frac{1}{E} \sum_{j=1}^{E} \tilde{W}_{ijc} \right) \left( \frac{1}{E} \sum_{j=1}^{E} \tilde{W}_{ijc} \right)
\]

Adapting the concept of triangular interval type-2 fuzzy sets in fuzzy Rasch model weight, then, this new concept is used to substitute the values of importance calculated in Step 3.

Step 6: Weight
Finally, calculate the weight of attributes by using the weight formula. We use \( w_j \) to represent the outcome of weight value of attribute \( j \), and it can be defined as:

\[
\tilde{w}_j = 1 / FOU(\tilde{w}_j) = \left[ \tilde{w}_j, \tilde{w}_j \right]
\]

\[
\left( \tilde{w}_j, \tilde{w}_j \right) = \left[ \delta_{ijc}, \delta_{ijc}, \delta_{ijc} ; 1 \right] \left( \delta_{ijc}, \delta_{ijc}, \delta_{ijc} ; 1 \right)
\]

\( \tilde{w}_j \) represents the interval type-2 fuzzy weight of expert \( i \) to criteria \( c \) for step \( j \).

4. Numerical parts

In this section, we give a numerical example to test the ability of proposed method to handle weight problems. Example in this section refers to weight problem used in [9].

This study applies new weight model to analyse the weight of TDC of nine Asian countries. The evaluation procedure used in this study comprises several steps.
First, the evaluation criteria for the TDC were determined. Second, determine the weights of the evaluation criteria/indices with new Rasch model. Then, determine the degree of the importance for each criteria/indices. Third, calculate the step parameters \( \delta_i \) to generate the new weight of interval type-2 Rasch weight. Fourth use an arithmetic average to integrate the new weight of each expert.

**Step 1:** Determine the evaluation criteria/indices and the alternatives.
This study analyses the competitiveness of the tourist industry for each of the selected Asian countries during the year 2009 using six criteria and 15 indices. Table 2 lists the six assessment criteria and 15 indices.

**Table 2:** Lists of six assessment criteria and 15 indices

<table>
<thead>
<tr>
<th>Criteria/Indices</th>
<th>International tourism arrivals ( C_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>International tourism receipts ( C_2 )</td>
</tr>
<tr>
<td>Availability of attractions</td>
<td>Hotel room ( C_3 )</td>
</tr>
<tr>
<td></td>
<td>Number of operating airlines ( C_4 )</td>
</tr>
<tr>
<td></td>
<td>Air transportation ( C_5 )</td>
</tr>
<tr>
<td>Availability of service</td>
<td>Transportation costs ( C_6 )</td>
</tr>
<tr>
<td></td>
<td>Hotel price ( C_7 )</td>
</tr>
<tr>
<td>Affordability</td>
<td>Cost-of-living index ( C_8 )</td>
</tr>
<tr>
<td>Positive market image</td>
<td>Quality of life ( C_9 )</td>
</tr>
<tr>
<td></td>
<td>Quality of the natural environment ( C_{10} )</td>
</tr>
<tr>
<td>Peace and stability</td>
<td>Safety and security ( C_{11} )</td>
</tr>
<tr>
<td></td>
<td>Business costs of crime and violence ( C_{12} )</td>
</tr>
<tr>
<td></td>
<td>Business costs of terrorism ( C_{13} )</td>
</tr>
<tr>
<td>Cultural links</td>
<td>Nation culture ( C_{14} )</td>
</tr>
<tr>
<td></td>
<td>Discrimination (race, gender) ( C_{15} )</td>
</tr>
</tbody>
</table>

**Step 2:** Appoint the linguistic variables for the likert rating scale.
The degree of each likert rating scale is appointed using the linguistic variables in interval type-2 fuzzy sets concept. Table 3 lists the linguistic variables from “very
unimportant” to “very important” obtained from the six experts. Based on Table 3, this study finds 
$$\tilde{w}_j = \left[ \tilde{w}_j^L, \tilde{w}_j^M, \tilde{w}_j^U \right] = \left[ \tilde{w}_j^L, \tilde{w}_j^M, \tilde{w}_j^U \right]^T$$.

Table 3: The linguistic variables

<table>
<thead>
<tr>
<th>Fuzzy Number</th>
<th>Very Unimportance</th>
<th>Unimportance</th>
<th>General</th>
<th>Importance</th>
<th>Very Importance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert 1</td>
<td>-</td>
<td>(-7,-1.7,1;1)</td>
<td>(-7,-1.3,1;1)</td>
<td>(-4.2,0.7,1;1)</td>
<td>(1.5,9,9;1)</td>
</tr>
<tr>
<td>Expert 2</td>
<td>(-8.3,-8.3,0.5;1), (-8.3,-2.1,2.2;1)</td>
<td>(-8.3,-8.3,0.5;1), (-8.3,-2.1,2.2;1)</td>
<td>(-4.2,0.4,2;1), (-4.2,0.4,2;1)</td>
<td>(-2.1,2.8;1), (-2.1,2.8;1)</td>
<td>(-0.4,8,8,1), (-0.4,8,8,1)</td>
</tr>
<tr>
<td>Expert 3</td>
<td>-</td>
<td>(-8.6,-8.6,0.1;1), (-8.6,-8.6,0.1;1)</td>
<td>(-8.6,-8.6,0.1;1), (-8.6,-8.6,0.1;1)</td>
<td>(-3.4,1.7,4,1), (-3.4,1.7,4,1)</td>
<td>(-1.7,4,7,4), (-1.7,4,7,4)</td>
</tr>
<tr>
<td>Expert 4</td>
<td>(-7.3,-7.3,1;1), (-7.3,-1.5,2;1)</td>
<td>(-7.3,-7.3,1;1), (-7.3,-1.5,2;1)</td>
<td>(-3.8,-0.1,3,8;1), (-3.8,-0.1,3,8;1)</td>
<td>(-2.5,1.3,7,8;1), (-2.5,1.3,7,8;1)</td>
<td>(-1.7,8,7,8), (-1.7,8,7,8)</td>
</tr>
<tr>
<td>Expert 5</td>
<td>-</td>
<td>(-8.3,-8.3,1;1), (-8.3,-8.3,1;1)</td>
<td>(-8.3,-8.3,1;1), (-8.3,-8.3,1;1)</td>
<td>(-5.8,-5.8,2,7;1), (-5.8,-5.8,2,7;1)</td>
<td>(5.8,0,5,8,1), (5.8,0,5,8,1)</td>
</tr>
<tr>
<td>Expert 6</td>
<td>-</td>
<td>(-7.8,-7.8,1;1), (-7.8,-7.8,1;1)</td>
<td>(-7.8,-7.8,1;1), (-7.8,-7.8,1;1)</td>
<td>(-3.8,1.2,7,5,1), (-3.8,1.2,7,5,1)</td>
<td>(-1.7,5,7,5), (-1.7,5,7,5)</td>
</tr>
</tbody>
</table>

Step 3: Determine the degree of the importance for each criteria/indexes.
This study assesses TDC by instructing six experts to indicate the degree of importance of six criteria and 15 indices on a likert rating scale ranging from 1-5 (from “very unimportant” to “very important”), as listed in Table 4.

Table 4: Degree of importance of 15 indices (scale ranges from 1 to 5)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$C_2$</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$C_3$</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$C_4$</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$C_5$</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$C_6$</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$C_7$</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$C_8$</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$C_9$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$C_{10}$</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$C_{11}$</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
A new weight of interval type-2 fuzzy Rasch model

\[ C_{ij} \]

\[ C_{ij} \]

\[ C_{ij} \]

\[ C_{ij} \]

Step 4: Calculate the step parameters \( \delta_{ij} \) to generate the weight of interval type-2 fuzzy Rasch (IT2FR) model.
The concept of triangular interval type-2 fuzzy sets in fuzzy Rasch model weight is adapted. Table 5 stated the linguistic of triangular type-2 fuzzy sets for the criteria of \( C_1 \).

**Table 5: Linguistic of triangular interval type-2 fuzzy sets**

<table>
<thead>
<tr>
<th>Criteria/ Indexes</th>
<th>( C_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expert 1</td>
<td>((-4.2,07,9;1),(-4.2,0.7,9;1))</td>
</tr>
<tr>
<td>Expert 2</td>
<td>((-2.1,2,8;1),(-2.1,2,8;1))</td>
</tr>
<tr>
<td>Expert 3</td>
<td>((-1.7,4,7;4;1),(-1.7,4,7;4;1))</td>
</tr>
<tr>
<td>Expert 4</td>
<td>((-2.5,1.3,7;8;1),(-2.5,1.4,7;8))</td>
</tr>
<tr>
<td>Expert 5</td>
<td>((-2.7,5.8,5.8;1),(-2.7,5.8,5.8;1))</td>
</tr>
<tr>
<td>Expert 6</td>
<td>((-1.7,5.7,5;1),(-1.7,5.7,5;1))</td>
</tr>
</tbody>
</table>

Step 5: Calculate the arithmetic average
An arithmetic average is used to integrate the new weight of each expert.

Step 6: Weight
The weight of attributes is calculated by using the weight formula. Thus, the results for weight of interval type-2 Rasch model is listed in Table 6

**Table 6: Weight of interval type-2 Rasch model**

<table>
<thead>
<tr>
<th>Criteria/ Indexes</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>((-2.250,4.117,7.583;1),(-2.250,4.117,7.583;1))</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>((-2.183,5.150,7.583;1),(-2.183,5.150,7.583;1))</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>((-5.167,0.217,6.417;1),(-5.167,0.217,6.417;1))</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>((-4.05,1.067,6.517;1),(-4.05,1.067,6.517;1))</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>((-4.883,1.133,5.617;1),(-4.883,1.133,5.617;1))</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>((-6.433,-1.533,4.117;1),(-6.433,-1.533,4.117;1))</td>
</tr>
<tr>
<td>( C_7 )</td>
<td>((-6.200,-1.433,4.933;1),(-6.200,-1.433,4.933;1))</td>
</tr>
</tbody>
</table>
The analytical results presented in this study demonstrate that the weight of attributes is in the concept of interval type-2 rasch model. Perhaps, this result will be useful in determining the MCDM problems.

5. Conclusions

In this paper we developed a new weight of interval type-2 fuzzy rasch model. The concept of evaluation of two different methods which are interval type-2 fuzzy set and fuzzy rasch were implemented to develop a new weight of interval type-2 fuzzy rasch model. A simple modification on properties of fuzzy rasch has been made into interval type-2 fuzzy sets concepts. Numerical example has been given to demonstrate the proposed method. Hence, the proposed method provides us with a useful way to handle the weight problems in a more flexible and more intelligent manner due to the fact that it uses interval type-2 fuzzy sets rather than traditional type-1 fuzzy sets to represent the evaluating values and the weights of attributes. Besides, this approach is believes that the model is not only feasible but can also rectify the inaccuracy of the fuzzy numbers assigned by the individual experts for the specific criteria. On the other hand, this model is more suitable to represent uncertainties because it is involved-users into the whole weighting process. Thus, this innovative approach is capable of providing a more comprehensive weight model for weighting process. For further research, results from the new weight will be used into the decision making approach. Later, the proposed method will provides us with a useful way, more flexible, less uncertainty and subjectivity to handle the fuzzy multiple attribute group decision-making problems.

Acknowledgement

This research is supported by MyBrain15 scholarship (MyPhD). This support is gratefully acknowledged.
References


Received: February, 2012