Simulation of Natural Convection in a Complicated Enclosure with Two Wavy Vertical Walls

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Abstract

Natural convection in a square enclosure having two wavy vertical walls is studied. An enclosure is filled with fluid-saturated porous media. A cavity is heated spatially on bottom wall while remaining walls are maintained at lower temperature. The objective of this study is to examine the flow field, temperature distribution and heat transfer inside the cavity when values of Darcy number, Rayleigh number and wave amplitude are changed. To analyze the performance, the governing differential equations and boundary conditions are coded into FlexPDE 6.17 Professional which is a finite element model builder and numerical solver. From the study results, decreasing Darcy and Rayleigh numbers decrease the strength of convection. In addition, the magnitudes of temperature and heat distributions become smaller. In the case of increasing wave amplitude, the flow intensity is obstructed by barricades resulting from extending the amplitude.

Keywords: Finite Element Method, Natural Convection, Porous Media

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1 Introduction

The study of natural convection in a two-dimensional enclosure with fluid-saturated porous media has received significant attention and it plays an important role in many applications. Many researchers study the different shapes of enclosure with various boundary conditions. Most of them are cylindrical, rectangular, trapezoidal and triangular. Due to a large number of technical applications such as geophysics, geothermal reservoirs, insulation of building, crude oil production, separation processes in industries, etc., the convection motion in a complicated cavity is studied to understand these systems.

In a cavity filled with porous media, Basak et al. [1] studied the natural convection flows in a trapezoidal enclosure and influence of uniform and non-uniform heating of bottom wall such that two vertical walls are maintained at constant cold temperature while the top wall is insulated. Basak et al. [2] investigated the effects of uniform and non-uniform heating of inclined wall on natural convection flows within an isosceles triangular enclosure. Their solution procedures are carried out by a penalty finite element analysis with bi-quadratic elements. Koca et al. [3] considered the triangular domain which bottom wall is heated partially whereas the temperature of inclined wall is maintained at lower uniform temperature than heated wall. In addition, triangular enclosure can be used in the application of roof structure to study heat transfer inside it. Asan and Namli [4],[5] investigated the effects of high-base ratio, Rayleigh number and heat transfer rate for the convection in a pitched roof of triangular cross-section under summer and winter conditions by using control volume integration solution technique. Pensiri and Supot [6] also analyzed the flow field and temperature distribution in an attic space of right-triangular enclosure such that the inclined wall is heated and the ceiling room is cooler.

In a complicated cavity, Dalal and Das [7],[8],[9] studied the natural convection in a square enclosure having three flat walls and a wavy right vertical wall consisting of one, two and three undulations. Furthermore, Oztop et al. [10] studied the natural convection heat transfer in a wavy-walled enclosure containing internal heat sources at different wave ratios by using finite-volume method. Interesting parameters are the internal and external Rayleigh numbers and the amplitude of wavy walls.

This paper is the study of natural convection in a square enclosure such that two vertical walls are wavy. The fluid-saturated porous media is contained inside it. Bottom wall is heated spatially while remaining walls are maintained at lower temperature. The objective is to investigate the convection motion inside the cavity when Darcy number, Rayleigh number and wave amplitude are varied by using finite element method. In section 2, the problem characteristics and notations used throughout this paper are described. Section 3 involves the
details of solution procedure. Finally, we evaluate the computational results and the conclusions are discussed.

2 Notations and Problem Characteristics

The following notations are applied throughout the paper.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>$AR$</td>
<td>aspect ratio, $AR = H/L$</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity (m s$^{-1}$)</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the enclosure (m)</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure (Pa)</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>$T_C$</td>
<td>temperature of cold wall</td>
</tr>
<tr>
<td>$T_H$</td>
<td>temperature of hot wall</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>wave amplitude</td>
</tr>
<tr>
<td>$\theta$</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$K$</td>
<td>permeability of the porous medium</td>
</tr>
<tr>
<td>$\psi$</td>
<td>stream function</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Cartesian coordinates (m)</td>
</tr>
<tr>
<td>$u, v$</td>
<td>$x, y$ components of velocity</td>
</tr>
<tr>
<td>$U, V$</td>
<td>$x, y$ components of dimensionless velocity</td>
</tr>
<tr>
<td>$X, Y$</td>
<td>dimensionless distance along $x, y$ coordinate</td>
</tr>
<tr>
<td>$f_1(y)$</td>
<td>$1 - \lambda + \lambda \cos(2\pi n(1 - y))$</td>
</tr>
<tr>
<td>$f_2(y)$</td>
<td>$1 - \lambda + \lambda \cos(2\pi ny)$</td>
</tr>
</tbody>
</table>

The objective of this paper is to investigate the flow field, temperature distribution and heat transfer due to natural convection inside a square enclosure having two wavy vertical walls. The fluid-saturated porous media considered as incompressible and Newtonian is contained inside an enclosure. Here, fluid is air, Prandtl number is kept constant at 0.71, and it is assumed to be laminar. Because the enclosure has two wavy vertical walls, the expression of left and right wavy walls are given by (1) and (2), respectively.

$$f_1(y) = 1 - \lambda + \lambda \cos(2\pi n(1 - y))$$
$$f_2(y) = 1 - \lambda + \lambda \cos(2\pi ny).$$

Different values of temperature are assigned to each boundary. Bottom wall is heated spatially, $T_H$, while remaining walls are maintained at lower temperature, $T_C$. The temperature expression on the bottom is written as

$$T(x) = T_C + \frac{T_H - T_C}{2} \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right].$$

Velocity on all solid walls are zero ($u = v = 0$). Aspect ratio illustrates proportion of domain size. It can be defined as the ratio of the height of
Figure 1: A cavity with two wavy vertical walls and boundary conditions.

vertical wall to the length of bottom wall \((AR = H/L)\). A physical model of an enclosure discussed above can be shown in Fig.1.

3 Governing Equation

Natural convection can be expressed by differential equations of conservation of mass, momentum and energy ([11]). The flow is simulated on two dimensional and the physical properties of fluid within an enclosure are assumed to be constant except the density in the buoyancy force. The governing equations for steady two-dimensional natural convection flow in the porous cavity are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (4)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\nu}{K} u, \quad (5)
\]

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\nu}{K} v + g\beta (T - T_C), \quad (6)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right), \quad (7)
\]

with following boundary conditions:

\(u(f_1(y), y) = u(f_2(y), y) = u(x, 0) = u(x, y) = 0,\)

\(v(f_1(y), y) = v(f_2(y), y) = v(x, 0) = v(x, y) = 0,\)

\(T(f_1(y), y) = T(f_2(y), y) = T(x, y) = T_C = 0,\)

\(T(x, 0) = T_H = T_C + \frac{T_H - T_C}{2} \left[ 1 - \cos \left( \frac{2\pi x}{L} \right) \right].\)
The above governing equations are transformed to non-dimensional form by using the following change of variables:

\[ X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad \theta = \frac{T - T_C}{T_H - T_C} \]

\[ P = \frac{pL^2}{\rho\alpha^2}, \quad Pr = \frac{\nu}{\alpha}, \quad Ra = \frac{g\beta (T_H - T_C) L^3 Pr}{\nu^2}, \quad Da = \frac{K}{L^2}. \]

The equation (4)-(7) in terms of dimensionless are:

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (8) \]

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{Pr}{Da} U, \quad (9) \]

\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{Pr}{Da} V + RaPr \theta, \quad (10) \]

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right), \quad (11) \]

Since the present work is to study the flow field, the fluid motion is displayed by using the stream function which is defined as \( U = \frac{\partial \psi}{\partial Y} \) and \( V = -\frac{\partial \psi}{\partial X} \). Thus, Eq.(8) is changed to Eq.(12)

\[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = \frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}. \quad (12) \]

To eliminate the pressure \( P \), we use the penalty finite element method with a penalty parameter \( \gamma \) such that \( P = -\gamma \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) \) ([12]). Substituting \( P \) into Eq.(9) and (10) yield Eq.(13) and (14).

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \gamma \frac{\partial}{\partial X} \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{Pr}{Da} U, \quad (13) \]

\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \gamma \frac{\partial}{\partial Y} \left( \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} \right) + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{Pr}{Da} V + RaPr \theta. \quad (14) \]

The transformed boundary conditions are:

\[ U(f_1(Y), Y) = U(f_2(Y), Y) = U(X, 0) = U(X, Y) = 0, \]

\[ V(f_1(Y), Y) = V(f_2(Y), Y) = V(X, 0) = V(X, Y) = 0, \quad (15) \]
\[ \theta(f_1(Y), Y) = \theta(f_2(Y), Y) = \theta(X, Y) = 0, \]
\[ \theta(X, 0) = \frac{1}{2} (1 - \cos(2\pi X)). \]

To visualize the heat transfer by fluid flow, heatfunction for a two-dimensional convection problem in dimensionless form can be defined as [13]
\[ \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} = \frac{\partial(U\theta)}{\partial Y} - \frac{\partial(V\theta)}{\partial X}. \tag{16} \]

With following boundary conditions:
\[ \phi(X, 0) = \pi \cos(\pi X) \]
\[ \phi(f_1(Y), Y) = \phi(f_2(Y), Y) = \phi(X, Y) = 0. \tag{17} \]

4 Results and Discussion

To analyze the numerical solutions, the governing equations mentioned previously are code into FlexPDE 6.17 Professional. FlexPDE is a software package which performs the operations necessary to turn a description of a partial differential equations system into a finite element model. Then, it solves the system and present graphical and tabular outputs of the results. In this experiment, the interested parameters are Darcy number, Rayleigh number and wave amplitude. The results obtained from varying value of these parameters are displayed by streamlines, isotherms and heatlines in which graphical outputs from FlexPDE are modified to show numerical values.

In this study, the experiments are categorized according to the different values of \( Da, Ra \) and \( \lambda \). Computations have been carried out for various \( Da = 10^{-4} - 10^{-2} \) and \( Ra = 10 - 10^5 \) covering wide range of applications ([1],[9]). Also, the range of wave amplitude from 0.01 – 0.1 have been studied ([9]). The aspect ratio, \( AR \), and numbers of undulations are fixed at 1 and 2, respectively. Boundary conditions of walls and heatfunction are given by Eq.(15),(16) and (17).

Streamlines, isotherms and heatlines obtained from varying interested parameters are shown for some cases to illustrate fluid circulation and heat transfer inside the cavity. Fig.2. shows the results of decreasing \( Da \) in which Fig.2(a)-2(c) are the results for \( Da = 10^{-2} \) and Fig.2(d)-2(f) are for \( Da = 10^{-4} \). Streamlines illustrate that fluid circulations are symmetric with respect to the vertical center line \( \psi = 0.0 \) such that the left half is positive value and the right half is negative. The opposite sign indicates a direction motion in such a way that positive and negative signs give anti-clockwise and clockwise circulation patterns, respectively. The main flow moves down to the bottom and the flow strength decreases when \( Da \) is decreased. The maximum value of stream function in Fig.2(a) is \( \psi_{max} = 7.5 \) and continually decreases to \( \psi_{max} = 0.16 \) as
shown in Fig.2(d). The temperature distribution can be seen from isotherm pattern. In Fig.2(b), the isotherms disperse the entire enclosure while the magnitude of distribution becomes smaller at low $Da$ as shown in Fig.2(e). Decreasing $Da$ also plays a role on heat transfer. Heatlines in Fig.2 shows that heat flows from the bottom portion to the top wall and heat distribution becomes smaller similar to isotherm pattern.

![Figure 2: Streamlines (left), isotherms (middle) and heatlines (right) for $Ra = 10^5$, $\lambda = 0.05$, $Da = 10^{-2}$ (above), $Da = 10^{-4}$ (bottom).](image)

As $Ra$ is decreased to $10^3$ (Fig.3), it is observed that the intensity of circulation is less than the previous case with $Ra = 10^5$ (Fig.2(a)). Similar to the case of decreasing $Da$, the magnitudes of temperature and heat distributions become smaller when $Ra$ is reduced.

The results obtained from varying wave amplitude are displayed by streamline (Fig.4). It is noted that the flow field near two vertical wavy walls is distorted due to the shape of the enclosure. Moreover, the flow strength decreases with increasing values of wave amplitude. For $\lambda = 0.02$, the maximum value of stream function is $\psi_{max} = 9.0$ and $\psi_{max} = 5.5$ when $\lambda$ is increased to 1.0.
5 Conclusion

The present work investigates the natural convection in a square enclosure such that two vertical walls are wavy. The fluid-saturated porous media is contained inside it. Here, the considered fluid is air. The aim of this study is to examine the effects of Darcy number, Rayleigh number and wave amplitude on natural convection expressed by differential equation of conservation of mass.
momentum and energy. To analyze the performance, the governing equations are coded into a software package FlexPDE 6.17 Professional. Interesting results are obtained and displayed by streamlines, isotherms and heatlines.

The values of Rayleigh and Darcy numbers have been chosen based on wide range of applicability. The $Ra$ has been varied from $10^{-10}$ to $10^5$. In the enclosure filled with porous media, $Da$ is used to describe the flow inside it. In order to observe permeability, the range of $Da$ is chosen between $10^{-4} - 10^{-2}$. Streamlines shown in previous section illustrate that the decrease of $Da$ decreased the strength of convection. In addition, the magnitudes of isotherms and heatlines patterns become smaller with low $Da$. In the case of increasing $Ra$, it is equivalent to increase buoyancy driven flow. It is observed that adding $Ra$ enhances the strength of heat and fluid flow. The increase of wave amplitude affects the flow intensity inside the cavity, that is, the flow is obstructed by barricades resulting from extending the amplitude. The wavy wall has no a big effect for temperature and heat distribution.

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