A New Method for Solving Two Stage Supply Chain Fuzzy Inventory Problem

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Abstract

In this paper, we develop fuzzy integrated production-inventory-marketing model to determine the relevant profit-maximizing decision variable values. The model proposed is based on the joint total profit of both the vendor and the buyer, and it finds out the optimal ordering, pricing and shipment policies. Shortages are not allowed. Production rate, ordering and holding cost of buyer and vendor are taken as triangular fuzzy numbers. Graded mean integration representation method is used for defuzzification. The numerical example suggests that it is more beneficial for the buyer and the vendor to cooperate with each other when the demand is more price sensitive.

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1 Introduction

The goal of many research efforts related to the supply chain management is to present models to reduce operational costs. The supply chain management has enabled numerous firms to enjoy great advantages by integrating all activities associated with the flow of material, information and capital between suppliers of
raw materials and the ultimate customers. The benefits of a properly managed supply include reduced costs, faster product delivery, greater efficiency and lower costs for both the business and its customers. In the increasingly fierce competitive environment in today’s global markets, the supply chain coordination is becoming a key component. If no coordination exists, the supply chain members act independently to maximize their own profits. In traditional inventory management, the inventory and shipment polices for the vendor and the buyer in a two-echelon supply chain are managed independently. The optimal lot size for the buyer may not result in an optimal policy for the vendor, and vice versa. To overcome this difficulty, the integrated vendor-buyer model has been developed, where the joint total relevant cost for both the buyer and the vendor is minimized. Determining the ordering and shipment policies results in a reduction of the total inventory cost of the system if the determination is based on the integrated total cost function rather than the buyer’s or vendor’s individual cost function.

Goyal [7] first introduced the idea of a joint total cost for a single-vendor and a single-buyer scenario assuming an infinite production rate for the vendor and lot-for-lot policy for the shipments from the vendor to the buyer. Then Goyal [8] introduced to the efforts of generalizing the problem by relaxing the assumption of lot-for-lot. He assumed that the production lot is shipped in a number of equal-size shipment. Goyal [9] developed a model where the shipment size increases by a factor equal to the ratio of the production rate to the demand rate. He formulated the problem and developed an optimal expression for the first shipment size as a function of the number of shipments. Hill [10] generalized Goyal’s model [9] by taking the geometric growth factor as a decision variable. The basic Joint Economic Lot Sizing (JELS) models have been extended in many different directions.

Lau and Lau [11] investigated a joint pricing-inventory model (not including setup costs). Viswanathan and Wang [18] later developed a model of single-vendor, single-retailer distribution channel, where the retailers face a price-sensitive deterministic demand. Ray et al.[17] who introduced an integrated marketing-inventory model for two pricing policies, price as a decision variable and mark-up pricing. Bakal et al.[2] presented two inventory models with a price-sensitive demand. Two different pricing strategies were also investigated, where (i) the firm chooses to offer a single price in all markets selected and (ii) a different price is set for each market.

This paper is based on Mohsen S. Sajdieh, Mohammad R. Akbari Jokar [14] in which the Production rate, ordering and holding cost of buyer and vendor are taken as triangular fuzzy numbers. Here we consider a supply chain for a product which consists of a single vendor and single buyer in fuzzy Environment. The final demand for this product is assumed to be deterministic but price sensitive. The lots delivered from the vendor to the buyer are equal-sized batches. As soon as the on-hand inventory at the buyer drops to the reorder point, an order of size $\hat{Q}$ is released by the buyer. The vendor manufactures the product at the production rate $\hat{P}$ and in lot sizes which are a multiple $n$ of $\hat{Q}$. The objective is to determine the number of shipments $n$, the selling price $\hat{\delta}$ as well as the order size $\hat{Q}$, so that the total profits of the vendor and the buyer are maximized.
2. Methodology

2.1. Fuzzy Numbers

Any fuzzy subset of the real line R, whose membership function µ_A satisfied the following conditions is a generalized fuzzy number A~.

(i) µ_A is a continuous mapping from R to the closed interval [0,1],
(ii) µ_A = 0, -∞ < x ≤ a1,
(iii) µ_A = L(x) is strictly increasing on [a1,a2]
(iv) µ_A = w_A, a2 ≤ x ≤ a3
(v) µ_A = R(x) is strictly decreasing on [a3, a4]
(vi) µ_A = 0, a4 ≤ x < ∞

where 0 < w_A ≤ 1 and a1, a2, a3 and a4 are real numbers. Also this type of generalized fuzzy number be denoted as A~ = (a1, a2, a3, a4, w_A)LR; When w_A=1, it can be simplified as A~ = (a1, a2, a3, a4)L.R.

2.2. Triangular Fuzzy Number

The fuzzy set A=(a1,a2,a3) where a1< a2< a3 and defined on R, is called the triangular fuzzy number, if the membership function of A is given by

\[ \mu_A = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \]

2.3. The Function Principle

The function principle was introduced by Chen [6] to treat fuzzy arithmetical operations. This principle is used for the operation of addition, subtraction, multiplication and division of fuzzy numbers.

Suppose A = (a1, a2, a3,) and B = (b1, b2, b3) are two triangular fuzzy numbers. Then

(i) The addition of A and B is

A + B = (a1+b1, a2+b2, a3+b3) where a1, a2, a3, b1, b2, b3 are any real numbers.

(ii) The multiplication of A and B is A x B = (c1, c2, c3)

where T = { a1b1, a1b3, a3b1, a3b3 } c1 = min T, c2 = a2b2, c3 = Max T

If a1, a2, a3, b1, b2, b3 are all non zero positive real numbers, then

A x B = (a1b1, a2b2, a3b3)

(iii) - B = (-b3, -b2, -b1) then the subtraction of A and B is

A - B = (a1-b3, a1-b2, a3-b1)

where a1,a2,a3, b1, b2, b3 are any real numbers
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(iv) \( \frac{1}{B} = \tilde{B}^{-1} = (1/b_1, 1/b_2, 1/b_3) \) where \( b_1, b_2, b_3 \) are all non zero positive real number, then the division of \( \tilde{A} \) and \( \tilde{B} \) is \( \tilde{A}/\tilde{B} = (a_i/b_1, a_i/b_2, a_i/b_3) \)

(v) For any real number \( K \), \( K\tilde{A} = (Ka_1, Ka_2, Ka_3) \) if \( K > 0 \)
\( K\tilde{A} = (Ka_3, Ka_4, Ka_3) \) if \( K < 0 \)

2.4. Graded Mean Integration Representation Method
If \( \tilde{A} = (a_1, a_2, a_3, w_A)_{LR} \) is a generalized fuzzy number then the defuzzified Value \( P(\tilde{A}) \) by graded mean integration representation method is given by
\[
p(\tilde{A}) = \frac{\int_0^W \left[ \frac{L^{-1}(h) + R^{-1}(h)}{2} \right] dh}{\int_0^W h dh}, \text{ with } 0 < h \leq w_A \text{ and } 0 < w_A \leq 1.
\]
If \( \tilde{A} = (a_1, a_2, a_3) \) is a triangular number then the graded mean integration representation of \( \tilde{A} \) by above formula is
\[
p(\tilde{A}) = 1/2 \int_0^1 \frac{[a_1 + h(a_2 - a_1) + a_1 - h(a_3 - a_2)] dh}{h dh} = \frac{a_1 + 4a_2 + a_3}{6}
\]

2.5. Notations

- \( P \) - Production rate of the vendor
- \( Q \) - Order quantity of the Buyer
- \( A_v \) - Setup cost of the vendor
- \( A_b \) - Ordering Cost of the Buyer
- \( c \) - the buyer's unit purchasing price
- \( \delta \) - Unit selling price of the buyer
- \( D \) - Demand rate as a function of unit selling price
- \( h_v \) - inventory holding cost for the vendor per year
- \( h_b \) - Inventory holding cost for the buyer per year
- \( n \) - Number of shipments
- \( \tilde{P} = (P_1, P_2, P_3) \) - Fuzzy Production rate of the vendor
- \( \tilde{Q} = (Q_1, Q_2, Q_3) \) - Fuzzy order quantity of the Buyer
- \( \tilde{A}_v = (A_{v_1}, A_{v_2}, A_{v_3}) \) - Fuzzy setup cost of the vendor
- \( \tilde{A}_b = (A_{b_1}, A_{b_2}, A_{b_3}) \) - Fuzzy ordering Cost of the Buyer
- \( \delta = (\delta_1, \delta_2, \delta_3) \) - Fuzzy unit selling price of the buyer
- \( \tilde{D} = (D_1, D_2, D_3) \) - Fuzzy demand rate as a function of unit selling price
- \( \tilde{h}_v = (h_{v_1}, h_{v_2}, h_{v_3}) \) - Fuzzy inventory holding cost for the vendor per year
- \( \tilde{h}_b = (h_{b_1}, h_{b_2}, h_{b_3}) \) - Fuzzy inventory holding cost for the buyer per year
- \( \tilde{n} = (n_1, n_2, n_3) \) - number of shipments

2.6. Assumptions

i. The model deals with a single vendor and a single buyer for a single product.
ii. The buyer faces a linear Demand \( \tilde{D}(\tilde{\delta}) \) as a function of selling price \( \tilde{\delta} \).
iii. A finite production rate for the vendor is considered which is greater than the demand rate.

iv. The inventory is continuously reviewed. The buyer orders a lot of size \( \bar{Q} \), when the on-hand inventory reaches the reorder point.

v. The vendor manufactures a production batch \( n\bar{Q} \) at one setup. However, the size of shipment delivered to the buyer is \( \bar{Q} \).

vi. The inventory holding cost at the buyer is higher than that at the vendor. i.e., \( \bar{h}_b > \bar{h}_v \).

vii. Shortage is not allowed.

viii. The time horizon is infinite.

3. Fuzzy Mathematical Model

The optimal policy of the integrated system is derived. However, for comparative purposes, we first obtain the buyer and the vendor policies, if each party optimizes its profit independently. The policies and profits are then compared to the case of integrated system when they co-operate, particularly in information sharing.

We assume that the buyer faces a linear demand \( \bar{D}(\delta) = a - b\delta \) (\( a > b > 0 \)) as a function of its unit selling price. As \( \bar{D}(\delta) > 0 \), the maximum selling price is \( a/b \), i.e., \( \delta < a/b \). The buyer’s yearly profit is equal to the gross revenue minus the sum of purchasing, order processing, and inventory holding costs. The buyer wishes to maximize its yearly profit function, \( \bar{T}\bar{P}_b \), through the optimal choice of selling price and order quantity, i.e.,

\[
\max_{\delta, Q} \bar{T}\bar{P}_b(\delta, Q) = (a - b\delta)(\delta - c) - \frac{(a-b\delta)\bar{A}_b}{\bar{Q}} - \frac{\bar{h}_b \bar{Q}}{2} \quad \text{such that,} \quad \delta < a/b \quad \text{and} \quad Q > 0 \quad \ldots \ldots (3.1)
\]

The buyer’s selling price \( \bar{\delta} \) determines the annual demand \( \bar{D}(\bar{\delta}) = a - b\bar{\delta} \). The optimal order size is then the economic order quantity,

\[
\bar{Q}^* = \sqrt{2 \left( a - b\bar{\delta} \right) \bar{A}_b / \bar{h}_b}
\]

Substituting the optimal order size into Eq.(3.1)

\[
\max_{\delta, Q} \bar{T}\bar{P}_b(\delta, \bar{Q}) = (a - b\delta)(\delta - c) - \frac{(a-b\delta)\bar{A}_b}{\bar{Q}} - \frac{\bar{h}_b \bar{Q}}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} = \frac{\bar{h}_b \sqrt{2 (a - b\delta) \bar{A}_b / \bar{h}_b}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} = \frac{\bar{h}_b \sqrt{2 (a - b\delta) \bar{A}_b / \bar{h}_b}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2} \frac{2}{2} = (a - b\delta)(\delta - c) - \left( \frac{a-b\delta}{\bar{Q}} \right) \bar{A}_b - \frac{\bar{h}_b \bar{Q}}{2}
The expression above is negative if 
Taking the second derivative of TP with respect to \( \hat{\delta} \), we have,

\[
\frac{\partial^2 TP_b}{\partial \hat{\delta}^2}(\hat{\delta}) = -2b - 2\sqrt{2(A_b \hat{b}_b a^2)} (d_0)
\]

The expression above is negative if \( a^2 > 11A_b \hat{b}_b b^2 \). In practice, \( a \) is usually very large (see Qin et al., [16]) and therefore the buyer’s profit function is concave in \( \hat{\delta} \). The optimal selling price is then determined by equating the first derivative of \( TP_b \) to zero.

\[
\frac{\partial TP_b}{\partial \hat{\delta}} = a - 2b \hat{\delta} + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_0 \hat{\delta} + d_1)
\]

\[
0 = a - 2b \hat{\delta} + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_0 \hat{\delta} + d_1)
\]

\[
2b \hat{\delta} + 2\sqrt{2(A_b \hat{b}_b a^2)} (d_0) = a + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_1)
\]

\[
\hat{\delta} = \frac{a + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_1)}{2b + 2\sqrt{2(A_b \hat{b}_b a^2)} (d_0)} 
\]  

\[{}^{\ldots (3.2)}\]

The optimal order quantity can be obtained as,

\[
\hat{Q}^* = \sqrt{\frac{2(a - b \hat{\delta})A_b}{\hat{b}_b}} 
\]

\[
= \sqrt{\frac{2(a - b \hat{\delta})}{\hat{b}_b} \left( \frac{a + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_1)}{2b + 2\sqrt{2(A_b \hat{b}_b a^2)} (d_0)} \right) \frac{\sqrt{2(A_b \hat{b}_b a^2)}}{\hat{b}_b}}
\]

\[
= \sqrt{\frac{2A_b}{\hat{b}_b} \left( a \left( 2b + 2\sqrt{2(A_b \hat{b}_b a^2)} (d_0) \right) - b \left( a + bc - \sqrt{2(A_b \hat{b}_b a^2)} (d_1) \right) \right)}
\]

\[
= \sqrt{\frac{\hat{b}_b 2 \left( b + \sqrt{2(A_b \hat{b}_b a^2)} (d_0) \right)}{\hat{b}_b}}
\]
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When the buyer’s order quantity and the selling price are adopted, the orders are received by the vendor at a known interval $\frac{b(a - bc) \bar{A}_b + \sqrt{2 \bar{A}_b \bar{h}_b a} (2a d_0 + b d_1)}{\bar{h}_b \left( b + \sqrt{2 \bar{A}_b \bar{h}_b a} (d_0) \right)}$.

The vendor’s average inventory can then be obtained as follows:

$$\hat{Q}^* = \sqrt{\frac{b(a - bc) \bar{A}_b + \sqrt{2 \bar{A}_b \bar{h}_b a} (2a d_0 + b d_1)}{\bar{h}_b \left( b + \sqrt{2 \bar{A}_b \bar{h}_b a} (d_0) \right)}}$$

When the buyer’s order quantity and the selling price are adopted, the orders are received by the vendor at a known interval $\hat{Q}^* / \hat{D} (\delta)$.

The vendor’s yearly profit function is,

$$\text{max} \bar{T}_v (\bar{n}) = ac - b c \delta - \frac{(a - b \delta) \bar{A}_v}{\bar{h}_v Q^2}$$

such that $n$ is integer.

It can easily be shown that $\bar{T}_v (\bar{n})$ is concave in $n$.

Optimality conditions for $\bar{n}^*$.

$$\bar{n}^* \ (\bar{n}^* - 1) \leq \frac{2(a - b \delta) \bar{p} \bar{A}_v}{\bar{h}_v Q^2 (\bar{p} - a - b \delta)} \leq \bar{n}^* \ (\bar{n}^* - 1) \ ...... (3.6)$$

If the buyer is free to choose its own marketing and ordering policies $(\delta, \bar{Q})$, and the vendor is free to choose its number of shipment $n$, then it is straightforward that the total system profit under individual optimization, $\bar{T}_b (\bar{n})$, is equal to the sum of buyer’s and the vendor’s profits.

$$\bar{T}_b (\bar{n}) = \bar{T}_b (\bar{n}^*) + \bar{T}_v (\bar{n})$$

Suppose that both parties decide to cooperate and agree to follow the jointly optimal integrated policy. The cost stemming from the purchasing price is an internal transfer of money from one supply chain member (the vendor) to another.
supply chain member (the buyer).

Therefore, it is not a cost of the whole supply chain. The total system profit under joint optimization, \( \bar{TP}_j(\delta, \bar{Q}, \bar{n}) \) is going to be maximized,

\[
\max_{\delta, \bar{Q}, \bar{n}} \bar{TP}_j(\delta, \bar{Q}, \bar{n}) = a\delta - b\delta^2 - \frac{(a - b\delta)(\bar{A}_v + \bar{n}\bar{A}_b)}{\bar{n}\bar{Q}} - \frac{\bar{h}_b\bar{Q}}{2} - \frac{\bar{h}_v\bar{Q}}{2} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right] \quad \cdots \quad (3.7)
\]

### 3.1 Solution Procedure for joint model

The total system profit is concave in \( \bar{Q} \) for given values of the buyer’s selling price \( \delta \) and the number of shipments \( \bar{n} \). The optimal order quantity can then be obtained as,

\[
\frac{\partial \bar{TP}_j}{\partial \bar{Q}} = 0
\]

\[
\frac{(a - b\delta)(\bar{A}_v + \bar{n}\bar{A}_b)}{\bar{n}\bar{Q}^2} - \frac{\bar{h}_b}{2} - \frac{\bar{h}_v}{2} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right] = 0
\]

\[
\frac{(a - b\delta)(\bar{A}_v + \bar{A}_b/\bar{n})}{\bar{Q}^2} - \frac{2(a - b\delta)(\bar{A}_v + \bar{A}_b/\bar{n})}{\bar{h}_b + \bar{h}_v} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right] = \bar{Q}^2
\]

\[
\bar{Q}^* = \sqrt{\frac{2(a - b\delta)(\bar{A}_b + \bar{A}_v/\bar{n})}{\bar{h}_b + \bar{h}_v} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right]} \quad \cdots \quad (3.8)
\]

Substituting expression (3.8) into the cost function (3.7), we obtain

\[
\bar{TP}_j(\delta, \bar{n}) = a\delta - b\delta^2 - \frac{(a - b\delta)(\bar{A}_b + \bar{A}_v/\bar{n})}{\bar{h}_b + \bar{h}_v} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right] - \frac{1}{2} \left[ \frac{2(a - b\delta)(\bar{A}_b + \bar{A}_v/\bar{n})}{\bar{h}_b + \bar{h}_v} \left[ \bar{n} \left( 1 - \frac{a - b\delta}{\bar{p}} \right) - 1 + \frac{2(a - b\delta)}{\bar{p}} \right] \right] \}
\]
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\[ \overline{\mathcal{P}}_j(\delta, \bar{n}) = a\delta - b\delta^2 \]

\[ = \sqrt{\frac{(a-b\delta)^2(\overline{\mathcal{A}}_b + \overline{\mathcal{A}}_v/\bar{n})^2}{2(a-b\delta)(\overline{\mathcal{A}}_b + \overline{\mathcal{A}}_v/\bar{n})} \left\{ \overline{\mathcal{h}}_b + \overline{\mathcal{h}}_v \left[ \overline{n} \left( 1 - \frac{a-b\delta}{\overline{\mathcal{P}}} \right) - 1 + \frac{2(a-b\delta)}{\overline{\mathcal{P}}} \right] \right\}} \]

\[ = a\delta - b\delta^2 \]

\[ = \sqrt{\frac{(a-b\delta)(\overline{\mathcal{A}}_b + \overline{\mathcal{A}}_v/\bar{n})}{2} \left\{ \overline{\mathcal{h}}_b + \overline{\mathcal{h}}_v \left[ \overline{n} \left( 1 - \frac{a-b\delta}{\overline{\mathcal{P}}} \right) - 1 + \frac{2(a-b\delta)}{\overline{\mathcal{P}}} \right] \right\}} \]

\[ \overline{\mathcal{P}}_j(\delta, \bar{n}) = a\delta - b\delta^2 \]

\[ = \sqrt{\frac{2(a-b\delta)(\overline{\mathcal{A}}_b + \overline{\mathcal{A}}_v/\bar{n})}{2} \left\{ \overline{\mathcal{h}}_b + \overline{\mathcal{h}}_v \left[ \overline{n} \left( 1 - \frac{a-b\delta}{\overline{\mathcal{P}}} \right) - 1 + \frac{2(a-b\delta)}{\overline{\mathcal{P}}} \right] \right\}} \] ...

(3.9)

For a given value of \( \delta \), maximizing \( \overline{\mathcal{P}}_j \) is equivalent to minimizing the following expression:

\[ \overline{\mathcal{P}}'_j = 2(a-b\delta)(\overline{\mathcal{A}}_b + \overline{\mathcal{A}}_v/\bar{n}) \]

\[ \left\{ \overline{\mathcal{h}}_b + \overline{\mathcal{h}}_v \left[ \overline{n} \left( 1 - \frac{a-b\delta}{\overline{\mathcal{P}}} \right) - 1 + \frac{2(a-b\delta)}{\overline{\mathcal{P}}} \right] \right\} \] ...

(3.10)

Taking the first and second partial derivatives of \( \overline{\mathcal{P}}'_j \) with respect to \( \bar{n} \), we obtain

\[ \frac{\partial \overline{\mathcal{P}}'_j}{\partial \bar{n}} = 2(a-b\delta) \left[ \overline{\mathcal{A}}_b \overline{\mathcal{h}}_v \left( 1 - \frac{a-b\delta}{\overline{\mathcal{P}}} \right) - \frac{2\overline{\mathcal{A}}_v \overline{\mathcal{h}}_b \overline{\mathcal{h}}_v}{\overline{n}^2} \right] \]

\[ = \frac{2(a-b\delta) [\overline{\mathcal{A}}_b \overline{\mathcal{h}}_v (\overline{\mathcal{P}} - a-b\delta)^2 n^2 - \overline{\mathcal{A}}_v \overline{\mathcal{h}}_b \overline{\mathcal{P}} + \overline{\mathcal{A}}_v \overline{\mathcal{h}}_v \overline{\mathcal{P}} - 2\overline{\mathcal{A}}_v \overline{\mathcal{h}}_v (a-b\delta)]}{\overline{n}^2 \overline{\mathcal{P}}} \]
\[
\frac{\partial TP_j}{\partial \bar{n}} = \frac{2(a - b\delta)[A_b \bar{h}_v (\bar{p} - a - b\delta)n^2 - 2A_v \bar{h}_v (a - b\delta) - \bar{A}_v (\bar{h}_b - \bar{h}_v)\bar{p}]}{\bar{n}^2 \bar{p}}
\]

\[
\frac{\partial^2 TP_j}{\partial \bar{n}^2} = 2(a - b\delta) \left[ \frac{2A_v \bar{h}_b - 2A_v \bar{h}_v}{\bar{n}^3} + \frac{4A_v \bar{h}_v (a - b\delta)}{\bar{n}^3 \bar{p}} \right]
\]

\[
\frac{\partial^2 TP_j}{\partial \bar{n}^2} = \frac{4(a - b\delta)[A_v \bar{h}_v \bar{p} - A_v \bar{h}_v \bar{p} + 4A_v \bar{h}_v (a - b\delta)]}{\bar{n}^3 \bar{p}}
\]

\[
\frac{\partial^2 TP_j}{\partial \bar{n}^2} = \frac{4(a - b\delta)A_v [2 \bar{h}_v (a - b\delta) + \bar{p} (\bar{h}_b - \bar{h}_v)]}{\bar{n}^3 \bar{p}}
\]

As \( \bar{h}_b > \bar{h}_v \) and \( \delta < a/b \), the second derivative is positive. Consequently, \( TP_j \) is strictly convex \( \bar{n} \), solving \( \partial TP_j/\partial \bar{n} = 0 \).

\[
2(a - b\delta)[A_b \bar{h}_v (\bar{p} - a - b\delta)n^2 - 2A_v \bar{h}_v (a - b\delta) - \bar{A}_v (\bar{h}_b - \bar{h}_v)\bar{p}] = 0
\]

\[
\bar{n}^2 = \frac{A_v \bar{h}_v (\bar{p} - a - b\delta)}{A_b \bar{h}_v (\bar{p} - a - b\delta)}
\]

\[
\bar{n} = \sqrt{\frac{A_v \bar{h}_v (\bar{p} - a - b\delta)}{A_b \bar{h}_v (\bar{p} - a - b\delta)}}
\]

As can be seen, the number of shipments is a non-increasing function of \( \delta \).

The minimum number of shipments can then be established when \( \delta < a/b \),

\[
\bar{n}_{min} = \max \left\{ \sqrt{\frac{A_v (\bar{h}_b - \bar{h}_v)}{A_b \bar{h}_v}}, 1 \right\} \quad \ldots (3.11)
\]

The maximum number of shipments can also be obtained when \( \delta = 0 \),

\[
\bar{n}_{max} = \frac{A_v \bar{h}_v (\bar{p} - a - b\delta)}{A_b \bar{h}_v (\bar{p} - a)} \quad \ldots (3.12)
\]

These two values are used in the solution algorithm as the upper and the lower bounds of the number of shipments.

### 3.2 Defuzzification of this model

Using Graded mean integration representation method, We will get the crisp value of Selling price (\( \delta \)), Order size (Q), number of shipments (n) for both individual and Joint model, Total profit function for vendor (TP\(_v\)), Total profit function for buyer (TP\(_b\)), Total system profit under individual optimization (TP\(_I\)), the joint total profit of vendor (TP\(_{vJ}\)), the joint total profit of buyer (TP\(_{bJ}\)), Total system profit under joint optimization (TP\(_J\)).

For a given value of \( n \), TP\(_J\) can be rewritten as

\[
TP_J(D) = m_1 D + m_2 D^2 - \sqrt{m_3 D + m_4 D^2}
\]

Where, \( m_1 = a/b \quad m_2 = -1/b \).
There is a one-to-one relationship between price and demand. Therefore, we base our analysis on the identification of the optimal value of demand, rather than the optimal value of price. The first and second Partial derivative of \( TP_J(D) \), with respect to \( D \) are as follows.

\[
\frac{\partial TP_J(D)}{\partial D} = m_1 + 2m_2D - \frac{m_3 + 2m_4D}{2\sqrt{m_3D + m_4D^2}}
\]

\[
\frac{\partial^2 TP_J(D)}{\partial D^2} = 2m_2 + \frac{m_3^2}{4}(m_3D + m_4D^2)^{-3/2}
\]

Three cases can occur for \( TP_J(D) \), depending on the number of shipments.

**Case 1: \( n=1 \)**

Hence \( m_4 > 0 \), and therefore there are two saddle points, \( SP_1 \) and \( SP_2 \). The total profit function is convex when \( SP_1 < D < SP_2 \), and is concave when \( D \leq SP_1 \) or \( D \geq SP_2 \). The optimal value of the demand is then \( D^* = LO_1 \) if \( LO_1 < a \), and It is \( D^* = a \) if \( LO_1 \geq a \).

**Case 2: \( n=2 \)**

Hence, \( m_4 = 0 \), and therefore there is a saddle point, \( SP = b^{2/3}m_3^{1/3}/4 \). The total profit function is convex when \( D < SP \), and is concave when \( D \geq SP \). Because the total profit function is zero at \( D = 0 \), there is no more than one local optimal amount for the demand. The optimal value of the demand is then \( D^* = LO_2 \) if \( LO_2 < a \), and It is \( D^* = a \) if \( LO_2 \geq a \).

**Case 3: \( n \geq 3 \)**

Hence, \( m_4 < 0 \), and therefore there are two saddle points, \( SP_1 \) and \( SP_2 \). The total profit function is concave when \( SP_2 < D < SP_1 \) and it is convex when \( D \leq SP_2 \) or \( D \geq SP_1 \). Moreover, \( m_3 \leq 0 \) and thus \( SP_1 > 0 \) and \( SP_2 > 0 \). The optimal value of the demand is then, \( D^* = LO_3 \) if \( LO_3 < a \), and It is \( D^* = a \) if \( LO_3 \geq a \).

As no closed form solution exists for the local-optimal values of the demand, we use a numerical method to find \( LO_i \), i=1,2,3.

### 4 Solution Algorithms

We establish the following algorithm to find the optimal values of the three decision variables for the integrated model.

1. Initialize by setting \( TP_J(D)^{opt} = 0 \).

   Calculate \( n_{min} \) & \( n_{max} \) equ(11) and equ(12) respectively. Set \( n = n_{min} \).

2. If \( n = 1 \) , then determine \( LO_1 \). If \( LO_1 < a \), then set \( D = LO_1 \). Otherwise, \( D = a \).

3. If \( n = 2 \) , then determine \( LO_2 \). If \( LO_2 < a \), then set \( D = LO_2 \). Otherwise, \( D = a \).

4. If \( n \geq 3 \), then determine \( LO_3 \). If \( LO_3 < a \), then set \( D = LO_3 \). Otherwise, \( D = a \).

5. Set \( \delta = (a - D)/b \)

6. Compute the value of order quantity using equation (8).

7. Calculate \( TP_J \) using equation (7).

8. If \( TP_J > TP_J^{opt} \), then set \( TP_J^{opt} = TP_J(\delta, Q, n) \), \( n_{opt} = n \), \( \delta_{opt} = \delta \), \( Q_{opt} = Q \).
9. Increment n by 1. If $n \leq n_{\text{max}}$ then goto step 3.
10. The current solution is globally optimal.

5. Numerical Example

We consider an example with the following data:

$\bar{p} = (3100, 3200, 3300) / \text{year}$  \hspace{1cm} $\bar{a}_v = (\text{Rs}.300, \text{Rs}.400, \text{Rs}.500) / \text{setup}$

$\bar{a}_b = (\text{Rs}.20, \text{Rs}.25, \text{Rs}.30) / \text{order}$  \hspace{1cm} $\bar{h}_v = (\text{Rs}.3, \text{Rs}.4, \text{Rs}.5) / \text{unit} / \text{year}$

$\bar{h}_b = (\text{Rs}.4, \text{Rs}.5, \text{Rs}.6) / \text{unit} / \text{year}$  \hspace{1cm} $a=1500, b=\{10,20,30,\ldots,100\}, c=\text{Rs}.5/\text{unit}.$

We analyze the effect of demand’s price-sensitivity. The effect is evaluated by the impact on the benefits of vendor-buyer coordination, as well as the impact on the decision variables. In order to gain insight into this effect, different levels of $b$ have been considered, i.e., $b=\{10,20,30,\ldots,100\}$. The percentage improvement obtained by joint optimization compared to individual optimization is defined in order to clarify the benefits of coordination.

\[\text{TP}_J \text{ and TP}_I \text{ represent the total system profit under joint and individual optimization, respectively. The net benefit under joint optimization should be shared by both parties in some equitable fashion. In order to encourage the buyer to cooperate with the vendor, a judicious method is essential for allocating profit. A proposed way is that the joint total profit/cost be allocated to the buyer and the vendor as follows (see Ouyang et al., [15]; Wu and Ouyang, [18]; Goyal, [7])}

\[TP_{VJ} = \frac{TP_{V(n)}}{TP_J(\delta, Q, n)} \text{ TP}_J(\delta, Q, n) \text{ & } TP_{BJ} = \frac{TP_{B(n)}}{TP_J(\delta, Q, n)} \text{ TP}_J(\delta, Q, n)\]

Where $TP_{VJ}$ and $TP_{BJ}$ are the profit of the vendor and the buyer under a coordinated supply chain, respectively.

<table>
<thead>
<tr>
<th>$b$</th>
<th>$\delta$</th>
<th>$Q$</th>
<th>$n$</th>
<th>$TP_v$</th>
<th>$TP_B$</th>
<th>$TP_I$</th>
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<tbody>
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<td>10</td>
<td>(77.3647, 77.5463, 77.7284)</td>
<td>(69.3534, 85.1196, 104.3512)</td>
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<td>(1686.1606, 2376.3957, 2899.1850)</td>
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<td>(53473.929, 54513.2756, 55385.6257)</td>
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<td>(39.8763, 40.0620, 40.2486)</td>
<td>(67.7799, 83.5918, 102.8448)</td>
<td>(5,5,5)</td>
<td>(1564.5032, 2265.8689, 2772.5776)</td>
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<tr>
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<td>(5,5,5)</td>
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<td>(14436.9866, 14777.2382, 15118.9846)</td>
<td>(15880.8619, 16931.7259, 17780.2816)</td>
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</tbody>
</table>
Two stage supply chain fuzzy inventory problem

Table 2. Decision variables under JOINT optimization (Fuzzy environment)

<table>
<thead>
<tr>
<th>b</th>
<th>δ</th>
<th>Q</th>
<th>n</th>
<th>TP_VI</th>
<th>TP_HI</th>
<th>TP_I</th>
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<td>(86.4734, 110.9304, 142.0357)</td>
<td>(4,4,4)</td>
<td>(1623.9883, 2378.7981, 3007.9717)</td>
<td>(49878.244, 52189.587, 54455.8999)</td>
<td>(53343.4407, 54568.3851, 55480.4419)</td>
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<tr>
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<td>(84.8850, 110.5327, 143.8255)</td>
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<tr>
<td>30</td>
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<td>(4,4,4)</td>
<td>(1239.4374, 2172.5076, 3114.5141)</td>
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<td>(4,4,4)</td>
<td>(678.1471, 1772.0791, 3268.602)</td>
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Table 3 Decision variables under individual optimization (after defuzzification)

<table>
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<th>B</th>
<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>$T_{P_V}$</th>
<th>$T_{P_B}$</th>
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Table 4 Decision variables under JOINT optimization (after defuzzification)

<table>
<thead>
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<th>$\delta$</th>
<th>Q</th>
<th>n</th>
<th>$T_{P_VJ}$</th>
<th>$T_{P_BJ}$</th>
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6. Conclusion

In this paper, we developed a fuzzy integrated production-inventory-marketing model for two-stage supply chains. Here we find out the optimal
ordering, pricing and shipment policies. We found the crisp value of number of shipments $n$, the selling price $\delta$ and order size $Q$ by using graded mean integration method.

References


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