Soft Intuitionistic Fuzzy Sets

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Abstract

Molodtsov introduced the concept of soft sets and application of soft sets in decision making and medical diagnosis problems. This was later generalized to soft fuzzy set. In this paper we present the innovative definition of a soft intuitionistic fuzzy set. We also present the definitions of the operations of union, intersection, OR and AND operators of soft intuitionistic fuzzy sets along with illustrative examples.

Keywords: Soft fuzzy set; soft intuitionistic fuzzy set; intuitionistic fuzzy set; soft set

1. Introduction

In this paper we introduced new concepts of soft intuitionistic fuzzy set. There has been incredible interest in the subject due to its diverse applications, ranging from engineering and computer science to social behavior studies. Fuzzy set was introduced by Lotfi A. Zadeh[11]. Intuitionistic fuzzy sets was provided by Atanassov [2]. A soft set was later defined by Molodtsov [8]. Maji et al [6], came up with the reduction of the weight soft set. Aktas et al [1] introduced the concept of soft group theory. Maji et al [7] contributed towards the fuzzification of the notion of soft set. Aktas and Cagman [1] and Aktas et al. [18] introduced the basic version of soft group theory, while Maji et al. [17] described the application of soft set theory to a decision making problem using rough sets. Recently Kong et al. [12, 13] applied the soft set
theoretic approach in decision making problems. Fuzzy soft set was first mentioned
by Maji et al. [5]. Soft fuzzy set was defined by Yao et al. [10] followed by
intuitionistic fuzzy soft set defined by Xu Yong-jie et al.[9]. Alkhazaleh et al. [16]
introduced the concept of fuzzy parameterized interval-valued fuzzy soft set and gave
its application in decision making. Alkhazaleh et al. [14] also introduced soft
multisets as a generalization of Molodtsov’s soft set and Alkhazaleh et al. [15]
proposed the concept of possibility fuzzy soft set. In this paper we define a soft
intuitionistic fuzzy set and define its operations of equality, complement, subset,
union, intersection, OR and AND operators along with several examples.

2. Preliminaries

**Definition 2.1.** (see [8]). Let $U$ be an initial set and $E$ be a set of parameters. Let $P(U)$ denotes the power set of $U$, and let $A \subseteq E$. A pair $(F, A)$ is called a soft set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$.

**Definition 2.2.** (see [5]). Let $U$ be an initial set and $E$ be a set of parameters. Let $F(U)$ denotes the fuzzy power set of $U$, and let $A \subseteq E$. A pair $(F, A)$ is called a fuzzy soft set over $U$, where $F$ is a mapping given by $F : A \rightarrow F(U)$.

**Definition 2.3.** (see [10]). Let $U$ be an initial set and $E$ be a set of parameters. Let $P(U)$ denotes the power set of $U$. Let $A \subseteq E$. A pair $(F, A)$ is called a soft fuzzy set over $U$, where $F$ is a mapping given by $F : A \rightarrow P(U)$ and

$$F(x) = \{ y \in U : R_\alpha(x, y) \geq \alpha, x \in A, y \in U, \alpha \in [0,1] \} \subseteq X \times Y,$$

is defined as cut-set

$$\tilde{R} \in F(X \times Y).$$

**Definition 2.4.** (see [9]). Consider $U$ and $E$ as a universe set and a set of parameters respectively. $IFS(U)$ denotes the intuitionistic fuzzy power set of $U$. Let $A \subseteq E$. A pair $(F, A)$ is an intuitionistic fuzzy soft set over $U$ where the mapping $F$ is given by $F : A \rightarrow IFS(U)$.

We recall these definitions in order to use them to introduce the concept of soft
intuitionistic fuzzy set and define some operations on soft intuitionistic fuzzy set.
Namely, subset, quality, null, complement, union, intersection, AND operation and
OR operation.
3. Soft Intuitionistic Fuzzy Set

3.1 Relation on Soft Intuitionistic Fuzzy Set

**Definition 3.1.1.** Let \( \tilde{R}_\alpha = (\tilde{R}_{\mu_\alpha}, \tilde{R}_{\nu_\alpha}) \) be an intuitionistic fuzzy subset of \( X \times Y \), and \( \tilde{R}_\alpha \) is defined as intuitionistic fuzzy relationship from \( X \) to \( Y \). We write \( X \rightarrow Y \), and \( \tilde{R}(x, y) \) denotes the degree of correspondence between \( x \) and \( y \) based on the relationship \( \tilde{R} \). Let \( F(X \times Y) \) denotes the family of an intuitionistic fuzzy relationship on \( X \rightarrow Y \). The set \( \tilde{R}_\alpha = \{(x, y) \in X \times Y : \tilde{R}_{\mu_\alpha}(x, y) \geq \alpha \text{ and } \tilde{R}_{\nu_\alpha}(x, y) \leq \alpha \} \subset X \times Y \) is defined as \( \alpha \)-cut set if \( \tilde{R} \in F(X \times Y) \) for \( \alpha \in [0,1] \).

3.2 Soft Intuitionistic Fuzzy Set

**Definition 3.2.1.** Let \( U \) be an initial set and \( E \) be a set of parameters. Let \( P(U) \) denotes the power set of \( U \). Let \( A \in E \). A pair \( (F, A) \) is called a soft intuitionistic fuzzy set over \( U \), where \( F \) is a mapping given by \( F : A \rightarrow P(U) \) and

\[
F(x) = \{ y \in U : (x, y) \in \tilde{R}_\alpha, x \in A, y \in U, \alpha \in [0,1] \}.
\]

**Example 3.2.1.** Suppose that \( U = \{m_1, m_2, m_3, m_4, m_5, m_6\} \) is the set of men and \( E = \{ \) educated \( (e_1), \) government employee \( (e_2), \) businessman \( (e_3), \) smart \( (e_4) \} \).

\[
\tilde{R}_\alpha = \begin{cases} 
(0.6,0.4)/ (m_1,e_1) + (0.7,0.3)/ (m_1,e_2) + \cdots + (0.1,0.9)/ (m_1,e_4) + (0.9,0.1)/ (m_2,e_1) + \cdots \\
+ (0.6,0.4)/ (m_2,e_4) + (0.5,0.5)/ (m_3,e_1) + \cdots + (0.8,0.2)/ (m_3,e_4) + \cdots + (0.8,0.2)/ (m_6,e_1) + \cdots \\
+ (0.3,0.5)/ (m_6,e_4) 
\end{cases}
\]

Then, a soft intuitionistic fuzzy set,
The fuzzy set \( F, E \) is a parameterized family \{ F(e_i), i=1,2,\ldots,5 \} and gives us a collection and approximate description of an object. The mapping \( F \) here is “men (.).” where dot (.) is to be filled up by a parameter \( e \in E \). Therefore \( F(e_i) \) means “men (educated)” whose functional-value is the intuitionistic fuzzy set \{(m_1 ,0.6,0.4),(m_2,0.9 ,0.1),(m_3,0.5 ,0.5),(m_4,0.3 ,0.5), (m_5,0.7 ,0.3), (m_6,0.8 ,0.2)\}. Thus, we can view the soft intuitionistic fuzzy set \((F,E)\) as a collection of intuitionistic fuzzy approximations (which are intuitionistic fuzzy sets) = \{ educated men = \{(m_1 ,0.6,0.4),(m_2,0.9 ,0.1),(m_3,0.5 ,0.5), (m_4,0.3 ,0.5)\}, government employee men = \{(m_1 ,0.7,0.3), (m_2,0.1 ), (m_3,0.7 ,0.3), (m_4,0.6 ,0.4), (m_5,0.5,0.5)\}, businessman men = \{(m_1 ,0.3 ,0.7),(m_2,0.1 ,0.9),(m_3,0.4 ,0.6),(m_4,0.2 ,0.8), (m_5,0.5 ,0.5), (m_6,0.3,0.5)\}. \}

3.3 Subset Of Soft Intuitionistic Fuzzy Sets

In this section we introduce the concept for subset of two soft intuitionistic fuzzy sets.

**Definition 3.3.1.** For two intuitionistic fuzzy sets \((F,A)\) and \((G,B)\) over common universe \(U\), we say that \((F,A)\) is a soft intuistiontic fuzzy subset of \((G,B)\) if:

(i) \( A \subseteq B \) and

(ii) \( \forall e \in A \ , \ F(e) \) is an intuitionistic fuzzy subset of \( G(e) \),

\( ( A \subseteq B \iff \forall x \in E, \mu_A(x) \leq \mu_B(x) \ and \ \nu_A(x) \leq \nu_B(x) ) \).

denoted by \((F,A)\subseteq(G,B)\).
3.4 Equality Of Soft Intuitionistic Fuzzy Sets

In this section we introduce the equality of two soft intuitionistic fuzzy sets.

**Definition 3.4.1.** Two soft intuitionistic fuzzy sets \((F,A)\) and \((G,B)\) over common universe \(U\) are said to be soft intuitionistic fuzzy sets equal if \((F,A)\) is a soft intuitionistic subset of \((G,B)\) and \((G,B)\) is a soft intuitionistic subset of \((F,A)\).

**Example 3.4.1** Let \((F,A)\) and \((G,B)\) of two soft intuitionistic fuzzy sets in the same common universe \(U = \{m_1, m_2, m_3, m_4, m_5, m_6\}\). \(A = \{\text{educated} (e_1), \text{Government employee} (e_2), \text{Businessman} (e_3), \text{Smart} (e_4)\}\). and \(B = \{\text{Education qualification} (e_1), \text{Government employee} (e_2), \text{Businessman} (e_3), \text{Smart} (e_4)\}\). Suppose \(F(e_1) = \{(m_1, 0.4, 0.6), (m_2, 0.1, 0.7), (m_3, 0.2, 0.6), (m_4, 0.2, 0.6), (m_5, 0.4, 0.5), (m_6, 0.3, 0.6)\}\), \(F(e_2) = \{(m_1, 0.2, 0.8), (m_2, 0.1, 0.7), (m_3, 0.2, 0.7), (m_4, 0.1, 0.9), (m_5, 0.2, 0.7), (m_6, 0.2, 0.6)\}\), \(F(e_3) = \{(m_1, 0.3, 0.5), (m_2, 0.3, 0.5), (m_3, 0.3, 0.6), (m_4, 0.2, 0.5), (m_5, 0.4, 0.5), (m_6, 0.3, 0.6)\}\). and \(G(e_1) = \{(m_1, 0.4, 0.6), (m_2, 0.1, 0.7), (m_3, 0.2, 0.6), (m_4, 0.2, 0.6), (m_5, 0.4, 0.5), (m_6, 0.3, 0.6)\}\), \(G(e_2) = \{(m_1, 0.2, 0.8), (m_2, 0.1, 0.7), (m_3, 0.2, 0.7), (m_4, 0.1, 0.9), (m_5, 0.2, 0.7), (m_6, 0.2, 0.6)\}\), \(G(e_3) = \{(m_1, 0.3, 0.5), (m_2, 0.3, 0.5), (m_3, 0.3, 0.6), (m_4, 0.2, 0.5), (m_5, 0.4, 0.5), (m_6, 0.3, 0.6)\}\). then \(F(e_1) = G(e_1), F(e_2) = G(e_2), \text{ and } F(e_3) = G(e_3)\), we have \((F,A) = (G,B)\).

3.5 Complement Of Soft Intuitionistic Fuzzy Set

In this section we will provide the complement of soft intuitionistic fuzzy set.

**Definition 3.5.1.** The complement of soft intuitionistic fuzzy set \((F,A)\) is denoted by \((F,A)^c\) and defined by \((F,A)^c = (F^c, -A)\) where \(F^c : -A \rightarrow P(U)\) is a mapping given by \(F^c(A) = \text{intuitionistic fuzzy complement of } F(-e), \forall e \in -A\).

**Example 3.5.1** Let \((F,A)\) of a soft intuitionistic fuzzy sets in the common universe \(U = \{m_1, m_2, m_3, m_4, m_5\}\). \(A = \{\text{educated} (e_1), \text{Government employee} (e_2), \text{Businessman} (e_3), \text{Smart} (e_4)\}\).
Businessman \( (e_3) \), Smart \( (e_4) \). Then \( \neg A = \{ \text{not educated} (\neg e_1), \text{not Government employee} (\neg e_2), \text{not Businessman} (\neg e_3) \} \). Suppose that

\[
F(e_1) = \{(m_1,0.6,0.4),(m_2,0.9,0.1),(m_3,0.4,0.6),(m_4,0.7,0.3),(m_5,0.5,0.5)\},
\]

\[
F(e_2) = \{(m_1,0.2,0.8),(m_2,0.7,0.3),(m_3,0.9,0.1),(m_4,0.5,0.5),(m_5,0.4,0.6)\},
\]

\[
F(e_3) = \{(m_1,0.3,0.7),(m_2,0.5,0.5),(m_3,0.6,0.4),(m_4,0.2,0.8),(m_5,0.1,0.9)\}. \]

For the element \( F(\neg e_1) \) has

\[
\{(m_1,0.4,0.6),(m_2,0.1,0.9),(m_3,0.6,0.4),(m_4,0.3,0.7),(m_5,0.5,0.5)\}. \]

For the element \( F(\neg e_2) \) has

\[
\{(m_1,0.8,0.2),(m_2,0.3,0.7),(m_3,0.1,0.9),(m_4,0.5,0.5),(m_5,0.6,0.4)\}. \]

For the element \( F(\neg e_3) \) has

\[
\{(m_1,0.7,0.3),(m_2,0.5,0.5),(m_3,0.4,0.6),(m_4,0.8,0.2),(m_5,0.9,0.1)\}. \]

### 3.6 Null Soft Intuitionistic Fuzzy Set

In this section we will introduce the null of soft intuitionistic fuzzy set.

**Definition 3.6.1.** Let \( (F,A) \) a soft intuitionistic fuzzy set over \( U \) is said to be a null soft intuitionistic fuzzy set denoted by \( \Phi \), if \( \forall e \in A, \ F(e) = \text{null intuitionistic fuzzy of } U \) (null-set).

**Example 3.6.1** Let \( (F,A) \) of a soft intuitionistic fuzzy set in the common universe \( U = \{m_1,m_2,m_3,m_4,m_5\} \). \( A = \{ \text{educated} (e_1), \text{Government employee} (e_2), \text{Businessman} (e_3) \} \). Then

The soft intuitionistic fuzzy set \( (F,A) \) is the collection of approximation as below

\[
(F,A) = \{ \text{educated man} = \phi, \text{Government employee man} = \phi, \text{Businessman} = \phi \}.
\]

Here \( (F,A) \) is the null soft intuitionistic fuzzy set.

### 3.7 Union Of Soft Intuitionistic Fuzzy Set

In this section we introduce the union of soft intuitionistic fuzzy set.

**Definition 3.7.1.** The union of two soft intuitionistic fuzzy sets \( (F,A) \) and \( (G,B) \) over a common universe \( U \) is the soft intuitionistic fuzzy set \( (P,C) \) where \( C = A \cup B \) and \( \forall e \in C \)
Soft intuitionistic fuzzy sets

\[
H(e) = \begin{cases} 
F(e) & \text{if } e \in A - B \\
G(e) & \text{if } e \in B - A \\
F(e) \cup G(e) & \text{if } e \in A \cap B.
\end{cases}
\]

we write \((F,A) \bigcup (G,B) = (P,C)\).

**Example 3.7.1** Assume that \(U = \{m_1, m_2, m_3\}\) is the set of men under consideration and \(E = \{e_1, e_2, e_3, e_4\}\) is the set of parameters where each parameter is an intuitionistic fuzzy word or a sentence involving intuitionistic fuzzy words \(A, B \subset E\) such that \(A = \{\text{educated (}e_1\text{), government employee (}e_2\text{), businessman (}e_4\text{)}\}\), and \(B = \{\text{educated (}e_1\text{), smart (}e_2\text{), businessman (}e_4\text{)}\}\). The soft intuitionistic fuzzy set \((F,A)\) and \((G,B)\) describe the “attractiveness of the man” thought by person X and person Y respectively.

Suppose that intuitionistic fuzzy relationship

\[
\hat{R} = \left\{ \begin{array}{c}
(0.3,0.4) + (0.5,0.3) + (0.1,0.4) \\
(m_1,e_1) + (m_1,e_2) + (m_1,e_4) \\
(m_2,e_1) + (m_2,e_2) + (m_2,e_4) \\
(m_3,e_1) + (m_3,e_2) + (m_3,e_4) \\
(0.8,0) + (0.1,0.5) + (0.3,0.6)
\end{array} \right\}.
\]

Thus, we can view the soft intuitionistic fuzzy set \((F,A)\) as a collection of intuitionistic fuzzy approximations (which are under the fuzzy relationship \(\hat{R}\)) as below:

\[
(F,A) = \left\{ \begin{array}{c}
\text{educated man} = \left\{ \begin{array}{c}
(0.3,0.4) \\
(0.8,0) \\
(0.4,0)
\end{array} \right\}, \\
\text{government employee man} = \left\{ \begin{array}{c}
(0.5,0.3) \\
(0.1,0.5) \\
(0.5,0.5)
\end{array} \right\}, \\
\text{businessman} = \left\{ \begin{array}{c}
(0.1,0.4) \\
(0.3,0.6) \\
(0.3,0.4)
\end{array} \right\}
\end{array} \right\}.
\]
And suppose that intuitionistic fuzzy relationship
\[
\tilde{R} = \left\{ \frac{(0.1,0.5)}{(m_1,e_1)} + \frac{(0.4,0.5)}{(m_1,e_3)} + \frac{(0.1,0.6)}{(m_1,e_4)} + \frac{(0.4,0.3)}{(m_2,e_1)} + \frac{(0.1,0.7)}{(m_2,e_3)} + \frac{(0.5,0.3)}{(m_2,e_4)} + \frac{(0.4,0.3)}{(m_3,e_1)} + \frac{(0.2,0.5)}{(m_3,e_3)} + \frac{(0.3,0.4)}{(m_3,e_4)} \right\}.
\]

Thus, we can view the soft intuitionistic fuzzy set \((G,B)_{\tilde{R}}\) as a collection of fuzzy approximations (which are under the fuzzy relationship \(R\)) as below:

\[
(G,B)_{\tilde{R}} = \left\{ \right. \text{educated man} = \left\{ \frac{(0.1,0.5)}{m_1}, \frac{(0.4,0.3)}{m_2}, \frac{(0.4,0.3)}{m_3} \right\}, \right. \text{smart man} = \left\{ \frac{(0.4,0.5)}{m_1}, \frac{(0.1,0.7)}{m_2}, \frac{(0.2,0.5)}{m_3} \right\}, \right. \text{businessman} = \left\{ \frac{(0.1,0.6)}{m_1}, \frac{(0.5,0.3)}{m_2}, \frac{(0.3,0.4)}{m_3} \right\}. \right.
\]

To find the \((F,A)_{\tilde{R}} \cup (G,B)_{\tilde{R}} = (H,C)_{\tilde{R}}\)

We have

\[
C = \{ \text{educated} (e_1), \text{smart} (e_2), \text{government employee} (e_3), \text{businessman}(e_4) \}, \text{ then } \]

\[
H(e_1) = F(e_1) \cup G(e_1)
\]

\[
= \left\{ \right. \max(0.3,0.1), \min(0.4,0.5), \max(0.8,0.4), \min(0.0,0.3) \right\}
\]

\[
= \left\{ \right. \max(0.0,0.4), \min(0.4,0.3) \right\}
\]

\[
= \left\{ \right. (0.3,0.4), (0.8,0), (0.4,0.3) \right\},
\]

\[
H(e_2) = F(e_2)
\]

\[
= \left\{ \right. (0.5,0.3), (0.1,0.5), (0.5,0.5) \right\},
\]
\[ H(e_3) = G(e_3) \]
\[ = \left\{ \frac{(0.4, 0.5), (0.1, 0.7), (0.2, 0.5)}{m_1, m_2, m_3} \right\}, \text{ and} \]

\[ H(e_4) = F(e_4) \cup G(e_4) \]
\[ = \left\{ \frac{\max(0.2, 0.1), \min(0.8, 0.6), \max(0.4, 0.5), \min(0.5, 0.3)}{m_1, m_2}, \frac{\max(0.6, 0.3), \min(0.2, 0.4)}{m_3} \right\} \]
\[ = \left\{ \frac{(0.2, 0.6), (0.5, 0.3), (0.6, 0.2)}{m_1, m_2, m_3} \right\}. \]

Thus, we can view the soft intuitionistic fuzzy set \((H, C)\) as a collection of fuzzy approximations (which are under the fuzzy relationship \(R\)) as below:

\[(H, C) = \left\{ \begin{array}{l}
   \text{educated man} = \left\{ \frac{(0.3, 0.4), (0.8, 0), (0.4, 0.3)}{m_1, m_2, m_3} \right\}, \\
   \text{smart man} = \left\{ \frac{(0.5, 0.3), (0.1, 0.5), (0.5, 0.5)}{m_1, m_2, m_3} \right\}, \\
   \text{government employee man} = \left\{ \frac{(0.4, 0.5), (0.1, 0.7), (0.2, 0.5)}{m_1, m_2, m_3} \right\}, \\
   \text{businessman} = \left\{ \frac{(0.2, 0.6), (0.5, 0.3), (0.6, 0.2)}{m_1, m_2, m_3} \right\}. 
\end{array} \right\} \]

3.8 Intersection Of Soft Intuitionistic Fuzzy Set

In this section we will introduce the intersection of soft intuitionistic fuzzy set to state the definition intersection of soft intuitionistic fuzzy set.

**Definition 3.8.1.** The intersection of two soft intuitionistic fuzzy sets \((F, A)\) and \((G, B)\) over a common universe \(U\) is the soft intuitionistic fuzzy set \((H, C)\) where \(C = A \cup B\) and \(\forall e \in C\).
\[ H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases} \]

we write \((F, A)_{\tilde{g}} \cap (G, B)_{\tilde{g}} = (H)_{\tilde{g}}\).

### 3.9 AND Operation On Soft Intuitionistic Fuzzy Set

In this section we introduce AND operator of soft intuitionistic fuzzy set to state the definition AND operator on soft intuitionistic fuzzy set.

**Definition 3.9.1. (AND operator).** If \((F, A)_{\tilde{g}}\) and \((G, B)_{\tilde{g}}\) be two soft intuitionistic fuzzy sets then, "\((F, A)_{\tilde{g}} \text{ AND (G, B)}_{\tilde{g}}\)" is an soft intuitionistic fuzzy set denoted by \((F, A)_{\tilde{g}} \cap (G, B)_{\tilde{g}} = (H, A \cap B)_{\tilde{g}}\), where \(H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall (\alpha, \beta) \in A \times B\).

### 3.10 OR Operation On Intuitionistic Fuzzy Soft Set

In this section we provide the OR operator on soft intuitionistic fuzzy set to state the definition OR operator of soft intuitionistic fuzzy set.

**Definition 3.10.1. (OR operator).** If \((F, A)_{\tilde{g}}\) and \((G, B)_{\tilde{g}}\) be two soft intuitionistic fuzzy sets then, "\((F, A)_{\tilde{g}} \text{ OR (G, B)}_{\tilde{g}}\)" is a soft intuitionistic fuzzy set denoted by \((F, A)_{\tilde{g}} \cup (G, B)_{\tilde{g}} = (O, A \cup B)_{\tilde{g}}\), where \(O(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall (\alpha, \beta) \in A \times B\).
4. Conclusion

In this paper the basic concept of a vague soft set is recalled. We have introduced the concept of soft intuitionistic fuzzy set as an extension to the intuitionistic fuzzy set. The basic properties on soft intuitionistic fuzzy set are also presented. The complement, null, equality, union, intersection, subset, AND operator and OR operator

As far as future direction are concerned. It is hoped that our findings will help enhancing this study on fuzzy soft sets for the researchers.

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