Properties of Intuitionistic Fuzzy Semi-Boundary

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Abstract

The purpose of this paper is to introduce intuitionistic fuzzy semi-boundary. We have studied the properties of intuitionistic fuzzy semi boundary. We obtain intuitionistic fuzzy semi-boundary in product related spaces.

Keywords: Intuitionistic fuzzy sets, Intuitionistic fuzzy topological spaces, Intuitionistic fuzzy boundary, Intuitionistic fuzzy semi-boundary, Intuitionistic fuzzy semi-closed sets, Intuitionistic fuzzy semi-continuous mapping.

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1. Introduction

After the introduction of fuzzy sets by Zadeh [10], there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [5] introduced the notion of intuitionistic fuzzy topological spaces. For the past few years, many researchers were going on in intuitionistic fuzzy topological spaces and many concepts in fuzzy topology were extended to intuitionistic fuzzy topology.

Fuzzy boundary and fuzzy semi boundary were introduced M. Athar and B. Ahmad in [1] in 2008. In this paper, we are extending the above concept to intuitionistic fuzzy topological space. We study some of the basic properties of intuitionistic fuzzy semi boundary with examples. Properties of intuitionistic fuzzy semi-interior, intuitionistic fuzzy semi-closure and intuitionistic fuzzy semi boundary have been obtained in product related spaces. We give necessary conditions for fuzzy continuous mapping.

2. Preliminaries

Before entering to our work, we recall the following notations, definitions and intuitionistic fuzzy sets as given by Atanassov [2], Coker [5] and Hanafy[7]. Throughout this paper, $(X,\tau)$, $(Y,\sigma)$ and $(Z,\eta)$ always means an intuitionistic fuzzy topological spaces in which no separation axioms are assumed unless otherwise mentioned.

**Definition 2.1.** [2] Let $X$ be a non-empty fixed set. An intuitionistic fuzzy set (IFS, for short), $A$ is an object having the form

$$A = \{<x, \mu_A(x), \gamma_A(x) > : x \in X\}$$

where the mapping $\mu_A : X \to I$ and $\gamma_A : X \to I$ denotes respectively the degree of membership (namely $\mu_A(x)$) and the non-membership (namely $\gamma_A(x)$) of each element $x \in X$ to a set $A$, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Obviously, every set $A$ on a non-empty set $X$ is an IFS having the form

$$A = \{<x, \mu_A(x), \gamma_A(x) > : x \in X\}$$
Definition 2.2. [2] Let \( X \) be a non-empty set and let the IFS’s \( A \) and \( B \) in the form
\[
A = \{< x, \mu_A(x), \gamma_A(x) \geq x \in X \}; \quad B = \{< x, \mu_B(x), \gamma_B(x) \geq x \in X \}
\]
Let \( \{A_j : j \in J\} \) be an arbitrary family of IFS’s in \( (X, \tau) \). Then,
\[
(i) \quad \bar{A} = \{< x, \mu_A(x), \gamma_A(x) \geq x \in X \}
\]
\[
(ii) \quad \bigcap A_j = \{< x, \min \mu_{A_j}(x), \min \gamma_{A_j}(x) \geq x \in X \}
\]
\[
(iii) \quad \bigcup A_j = \{< x, \max \mu_{A_j}(x), \max \gamma_{A_j}(x) \geq x \in X \}
\]
\[
(iv) \quad \bar{I} = \{< x, 1, 0 \geq x \in X \} \quad \text{and} \quad \bar{0} = \{< x, 0, 1 \geq x \in X \}
\]
\[
(vi) \quad \bar{A} = A, \quad \bar{1} = 0 \quad \text{and} \quad \bar{0} = 1.
\]

Definition 2.3. [5] Let \( X \) and \( Y \) be two non-empty sets and \( f : (X, \tau) \to (Y, \sigma) \) be a mapping. If \( B = \{< y, \mu_B(y), \gamma_B(y) \geq y \in Y \} \) is an IFS in \( Y \), then the pre image of \( B \) under \( f \) is denoted and defined by \( f^{-1}(B) = \{< x, \mu_B(f(x)), \gamma_B(f(x)) \geq x \in X \} \) since, \( \mu_B, \gamma_B \) are fuzzy sets, we explain that
\[
f^{-1}(\mu_B(x)) = \mu_B(f(x))
\]

Definition 2.4. [5] An intuitionistic fuzzy topology [3, 4] (IFT, for short) on a non-empty set \( X \) is a family \( \tau \) of IFS’s in \( X \) satisfying the following axioms:
\[
(i) \quad 1, 0 \in \tau
\]
\[
(ii) \quad A_1 \cap A_2 \in \tau \quad \text{for some} \quad A_1, A_2 \in \tau
\]
\[
(iii) \quad \bigcup A_j \in \tau \quad \text{for any} \quad \{A_j : j \in J\} \in \tau
\]
In this case, the ordered pair \( (X, \tau) \) is called intuitionistic fuzzy topological space \( (IFTS, \text{for short}) \) and each IFS in \( \tau \) is known as an intuitionistic fuzzy open set (IFOS, for short) in \( X \). The complement of an intuitionistic fuzzy open set is called intuitionistic fuzzy closed set (IFCS, for short).

Definition 2.5. [5] Let \( (X, \tau) \) be an IFTS and let \( A = \{< x, \mu_A(x), \gamma_A(x) \geq x \in X \} \) be an IFS in \( X \). Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure of \( A \) are defined by
\[
\text{int}(A) = \bigcup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}
\]
\[
\text{cl}(A) = \bigcap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}
\]
Remark 2.6. [5] For any IFS $A$ in $(X, \tau)$, we have $\text{cl}(A) = \overline{\text{int}(A)}$ and $\text{int}(A) = \overline{\text{cl}(A)}$.

Definition 2.7. [6] Let $A$ be a fuzzy set in an IFTS $(X, \tau)$. Then, $A$ is called a fuzzy semi-open set of $X$ if there exists a $B \in \tau$ such that $B \leq A \leq \text{cl}(B)$.

Definition 2.8. [6] Let $A$ be a fuzzy set in an IFTS $(X, \tau)$. Then, semi-closure (briefly $\text{scI}$) and semi-interior (briefly $\text{sI}$) are given as

$$\text{scI}(A) = \bigcap \{B / A \leq B, B \text{ is fuzzy semiclosed} \}$$

$$\text{sI}(A) = \bigcup \{B / B \leq A, B \text{ is fuzzy semiopen} \}$$

Definition 2.9. [8] A mapping $f : X \to Y$ is said to be intuitionistic fuzzy irresolute if $f^{-1}(B)$ is fuzzy semi-open in $X$, for each fuzzy semi-open set $B$ in $Y$.

Definition 2.10. [9] Let $A$ be an IFS in an IFTS $X$. Then, the intuitionistic fuzzy boundary of $A$ (IBd $A$, for short) is defined as $\text{IBd } A = \text{cl } A \cap \text{cl } A^c$.

3. Intuitionistic Fuzzy Semi-Boundary

Definition 3.1. Let $A$ be an IFS in an IFTS $(X, \tau)$. Then, the intuitionistic fuzzy semi-boundary of $A$ is defined as $\text{IsBd}(A) = \text{scI}(A) \cap \text{scI}(A^c)$. Obviously, $\text{IsBd}(A)$ is an IFSCS in $X$.

Proposition 3.2. For an IFS $A$ in an IFTS $(X, \tau)$, the following conditions hold.

1. $\text{IsBd}(A) = \overline{\text{IsBd}(A^c)}$
2. If $A$ is fuzzy semi-closed, then $\text{IsBd}(A) \leq A$.
3. If $A$ is fuzzy semi-open, then $\text{IsBd}(A) \leq A^c$.
4. Let $A \leq B$ and $B \in \text{IFSC}(X)$ (resp., $B \in \text{IFSO}(X)$).

Then, $\text{IsBd}(A) \leq B$ (resp., $\text{IsBd}(A) \leq B^c$), where $\text{IFSC}(X)$ (resp., $\text{IFSO}(X)$) denotes the class of fuzzy semi-closed (resp., fuzzy semi-open) sets in $X$.

5. $(\text{IsBd}(A))^c = \text{sI}(A) \lor \text{sI}(A^c)$
6. \( \text{IsBd}(A) \leq \text{IBd}(A) \).
7. \( \text{scl}(\text{IsBd}(A)) \leq \text{IBd}(A) \).

**Proof:**

(1) Since \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) = \text{scl}(A^c) \wedge \text{scl}(A^c) = \text{IsBd}(A^c) \).

(2) \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) \leq \text{scl}(A) \), since \( A \) is an IFSCS in \( X \), \( \text{scl}(A) = A \), therefore \( \text{IsBd}(A) \leq A \).

(3) \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) \leq \text{scl}(A^c) \), since \( A \) is an IFSCS in \( X \), \( A^c \) is an IFSCS, therefore \( \text{IsBd}(A) \leq A^c \). Hence \( \text{IsBd}(A) \leq \text{scl}(A^c) \).

(4) Let \( A \leq B \) and \( B \) is in IFSCS in \( X \), then \( \text{scl}(A) \leq \text{scl}(B) \). Since \( B \) is an IFSCS, therefore \( \text{scl}(B) = B \). Now \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) \leq \text{scl}(A) \wedge \text{scl}(A^c) \leq \text{scl}(B) \).

(5) Since \( (\text{IsBd}(A))^\triangledown = (\text{scl}(A) \wedge \text{scl}(A^c))^\triangledown = (\text{scl}(A))^\triangledown \lor (\text{scl}(A^c))^\triangledown = \text{sin}(A^c) \lor \text{sin}(A) \).

Therefore \( (\text{IsBd}(A))^\triangledown = \text{sin}(A) \lor \text{sin}(A^c) \).

(6) \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) \leq \text{cl}(A^c) \wedge \text{cl}(A) = \text{IBd}(A) \), therefore \( \text{IsBd}(A) \leq \text{IBd}(A) \).

(7) \( \text{scl}(\text{IsBd}(A)) = \text{scl}(\text{scl}(A) \wedge \text{scl}(A^c)) \leq \text{scl}(\text{scl}(A) \wedge \text{scl}(A^c)) = \text{IsBd}(A) \leq \text{IBd}(A) \).

Therefore \( \text{scl}(\text{IsBd}(A)) \leq \text{IBd}(A) \).

The converse of (2) and (3) and reverse inequalities of (6) and (7) in Proposition 3.2 are, in general, not true as is shown by the following example.

**Example 3.3.** Let \( X = \{a, b\} \). Let \( \tau = \left\{ 0, A, 0.5 \right\} \) be an IFTS in \( X \) and 

\[
A = \left\{ a, 0.6, b, 0.4, \left[ a, 0.3, b, 0.2 \right] \right\}.
\]

Now \( \text{IsBd}(A) = \text{scl}(A) \wedge \text{scl}(A^c) = \hat{1} \wedge A^c = A^c \leq A \).

Therefore, \( \text{IsBd}(A) \leq A \). we have to show that \( A \) is not an in IFSCS in \( X \). For, \( \text{cl}(A) = 1 \), \( \text{int cl}(A) = 1 \leq A \). Therefore, \( A \) is not an IFSCS in \( X \).
Example 3.4. Let $X = \{a, b\}$. Let $\tau = \{\tilde{0}, A, \tilde{1}\}$ be an IFTS in $X$ and

$$A = \left\{ x, \left( \frac{a}{0.2}, \frac{b}{0.3} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right) \right\}.$$  
Now, $IsBd(A) = scl(A) \wedge scl(A^c) = A^c \wedge A^c = A^c$. i.e., $IsBd(A) \leq A^c$. We have to show that $A$ is not an IFSOS in $X$. For, $int(A) = \tilde{0}$, $cl(int(A)) = \tilde{0}$ and $A \not\subseteq \tilde{0} = cl(int(A))$. Therefore, $A$ is not an IFSOS in $X$.

Example 3.5. Let $X = \{a, b\}$. Let $\tau = \{\tilde{0}, A, \tilde{1}\}$ be an IFTS in $X$,

$$A = \left\{ x, \left( \frac{a}{0.2}, \frac{b}{0.3} \right), \left( \frac{a}{0.4}, \frac{b}{0.6} \right) \right\} \text{ and } B = \left\{ x, \left( \frac{a}{0.2}, \frac{b}{0.3} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right) \right\}.$$  
Now, $IBd(B) = cl(B) \wedge cl(B^c) = A^c \wedge \tilde{1} = A^c$ and $IsBd(B) = scl(B) \wedge scl(B^c) = x, \left( \frac{a}{0.3}, \frac{b}{0.4} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right) \wedge \tilde{1} = x, \left( \frac{a}{0.3}, \frac{b}{0.4} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right)$. Clearly, $A^c \notin \left\{ x, \left( \frac{a}{0.3}, \frac{b}{0.4} \right), \left( \frac{a}{0.4}, \frac{b}{0.5} \right) \right\}$. Therefore, $IBd(B) \notin IsBd(B)$.

Proposition 3.6. Let $A$ be an IFS in an IFTS $X$. Then, one has

1. $IsBd(A) = scl(A) - sint(A)$
2. $IsBd sint(A) \leq IsBd(A)$
3. $IsBd scl(A) \leq IsBd(A)$
4. $sint(A) \leq A \cdot IsBd(A)$

Proof: (1) Since $A - B = A \cap B^c$ and $scl(A) - sint(A) = scl(A) \wedge [sint(A)]^c = scl(A) \wedge sint(A^c) = IsBd(A)$.

Therefore, $IsBd(A) = scl(A) - sint(A)$.

(2) Since $IsBd(A) \leq IBd(A)$ and $sint(A) \leq A$, therefore, $IsBd sint(A) \leq IsBd(A)$.

Since $IsBd(A) = scl(A) - sint(A)$

$$IsBd(sint(A)) = scl(sint(A)) - sint(sint(A)) = scl(sint(A)) - sint(A) \leq scl(A) - sint(A) = IsBd(A)$$

Therefore, $IsBd(sint(A)) \leq IsBd(A)$. 

(3) Since
\[ IsBd(scl(A)) = scl(scl(A)) - scl\left(scl(A)^c\right) \]
\[ = scl(A) \wedge scl\left(sint(A^c)\right) \]
\[ \leq scl(A) \wedge scl\left(A^c\right) \]
\[ = IsBd(A) \]
Therefore, \( IsBd(scl(A)) \leq IsBd(A) \).

(4) Since
\[ A - IsBd(A) = A \wedge (IsBd(A))^c \]
\[ = A \wedge (scl(A) \wedge scl\left(A^c\right))^c \]
\[ = A \wedge \left(scl(A)^c \vee scl\left(A^c\right)^c\right) \]
\[ = A \wedge \left((\sin t\left(A^c\right)) \vee (\sin t(A))\right) \]
\[ \geq (A \wedge \sin t\left(A^c\right)) \vee \sin t(A) \]
\[ = \sin t(A) \]
Therefore, \( \sin t(A) \leq A - IsBd(A) \).

**Theorem 3.7.** Let \( A \) and \( B \) be IFS’s in an IFTS \( X \). Then,
\( IsBd(A \vee B) \leq IsBd(A) \vee IsBd(B) \).

**Proof:**
\[ IsBd(A \vee B) = scl(A \vee B) \wedge scl\left((A \vee B)^c\right) \]
\[ = scl(A \vee B) \wedge scl\left(A^c \wedge B^c\right) \]
\[ \leq (scl(A) \vee scl(B)) \wedge scl\left(A^c \wedge scl\left(B^c\right)\right) \]
\[ = (scl(A) \wedge scl\left(A^c\right)) \vee (scl(B) \wedge scl\left(B^c\right)) \]
\[ = (IsBd(A) \wedge scl\left(B^c\right)) \vee scl\left(A^c\right) \wedge IsBd(B) \]
\[ \leq IsBd(A) \vee IsBd(B) \]
Therefore, \( IsBd(A \vee B) \leq IsBd(A) \vee IsBd(A) \)

The reverse inequality in Theorem 3.7 is in general, not true as is shown by the following example.
Example 3.8. Let $X = \{a,b\}$. Let $\tau = \{0, A, 1\}$ be an IFTS in $X$ ,

$$A = \left\{ a, \left(\frac{a}{0.6}, \frac{b}{0.7}\right) \left(\frac{a}{0.4}, \frac{b}{0.3}\right) \right\}.$$ Let $B = \left\{ a, \left(\frac{a}{0.7}, \frac{b}{0.8}\right) \left(\frac{a}{0.3}, \frac{b}{0.2}\right) \right\}$ and

$$C = \left\{ a, \left(\frac{a}{0.6}, \frac{b}{0.9}\right) \left(\frac{a}{0.4}, \frac{b}{0.1}\right) \right\}$$ be IFSs in $X$.

Now, $B \lor C = \left\{ a, \left(\frac{a}{0.3}, \frac{b}{0.9}\right) \left(\frac{a}{0.7}, \frac{b}{0.1}\right) \right\}$, $scl(B) = \tilde{1}$ and $scl(B^c) = B^c$, $scl(C) = \tilde{1}$ and $scl(C^c) = C^c$. $IsBd(B) = B^c$ and $IsBd(C) = C^c$.

$scl(B \lor C) = \tilde{1}$, $scl((B \lor C)^c) = (B \lor C)^c$.

Then $IsBd(B \lor C) = (B \lor C)^c = \left\{ a, \left(\frac{a}{0.3}, \frac{b}{0.7}\right) \left(\frac{a}{0.1}, \frac{b}{0.9}\right) \right\}$.

Therefore, $IsBd(B) \lor IsBd(C) \subseteq IsBd(B \lor C)$.

Theorem 3.9. For any IFSs $A$ and $B$ in an IFTS $X$, then,

$$IsBd(A \land B) \leq (IsBd(A) \land scl(B)) \lor (IsBd(B) \land scl(A))$$

Proof:

$$IsBd(A \land B) = scl(A \land B) \land scl((A \land B)^c)$$

$$= scl(A \land B) \land scl(A^c \lor B^c)$$

$$\leq (scl(A) \land scl(B)) \lor (scl(A^c) \lor scl(B^c))$$

$$= (IsBd(A) \land scl(B)) \lor (scl(A) \land scl(B^c)$$

$$\leq (IsBd(A) \land scl(B)) \lor (scl(A) \land IsBd(B))$$

Therefore, $IsBd(A \land B) \leq (IsBd(A) \land scl(B)) \lor (scl(A) \land IsBd(B))$

Corollary 3.10. For any fuzzy sets $A$ and $B$ in an IFTS $X$, then,

$$IsBd(A \land B) \leq IsBd(A) \lor IsBd(B)$$

Proof:

$$IsBd(A \land B) = scl(A \land B) \land scl((A \land B)^c)$$

$$= scl(A \land B) \land scl(A^c \lor B^c)$$
\[
\begin{align*}
&\leq (scl(A) \land scl(B)) \land \left(scl(A^c) \lor scl(B^c)\right) \\
&= (scl(A) \land scl(B) \land scl(A^c)) \lor (scl(A) \land scl(B) \land scl(B^c)) \\
&= (IsBd(A) \land scl(B)) \lor (scl(A) \land IsBd(B)) \\
&\leq IsBd(A) \lor IsBd(B)
\end{align*}
\]

Therefore, \(IsBd(A \land B) \leq IsBd(A) \lor IsBd(B)\)

**Proposition 3.11.** Let \(A\) be an IFS in an IFTS \(X\), Then,

(i) \(IsBdIsBd(A) \leq IsBd(A)\)

(ii) \(IsBd IsBd IsBd(A) \leq IsBd IsBd(A)\)

**Proof:** (i) Since \(IsBd(IsBd(A)) = scl(IsBd(A)) \land \left(scl(IsBd(A))^c\right)\)

\[
= scl\left(scl(A) \land scl(A^c)\right) \land scl\left(scl(A) \land scl(A^c)^c\right)
\]

\[
\leq (scl scl(A) \land scl scl(A^c)) \land scl \left(scl(A) \land scl(A^c)^c\right)
\]

\[
= (scl(A) \land scl(A^c)) \land scl IsBd(A) \lor scl IsBd(A)
\]

\[
\leq scl(A) \land scl(A^c)
\]

\[
= IsBd(A)
\]

Therefore, \(IsBd[IsBd(A)] \leq IsBd(A)\).

(ii) Proof is similar to (i).

**Remark 3.12.** The reverse inequality of (1) of proposition 3.11 is in general, not true as is shown by the following example.

**Example 3.13.** Let \(X = \{a, b\}\). Let \(\tau = \{0, 1\}\) be an IFTS in \(X\),

\[
A = \left\{ x, \begin{bmatrix} a & b \\ 0.2 & 0.3 \end{bmatrix} \left\{ a & b \\ 0.4 & 0.6 \end{bmatrix} \right\}
\]

Let \(B = \left\{ x, \begin{bmatrix} a & b \\ 0.4 & 0.7 \end{bmatrix} \left\{ a & b \\ 0.6 & 0.3 \end{bmatrix} \right\}\) be an IFS in \(X\),

\(scl(B) = 1\) and \(scl(B^c) = 1\), \(IsBd(B) = 1\) and \(IsBd(IsBd(B)) = IsBd(\tilde{1}) = 0\). Therefore \(IsBd(B) \not\leq IsBd(IsBd(B))\).

**Theorem 3.14.** Let \(f : X \to Y\) be an intuitionistic fuzzy semi-continuous mapping. Then, we have,

\(IsBd(f^{-1}(A)) \leq f^{-1}(IsBd(A))\) for any IFS \(A\) in \(Y\).
Proof: Let \( f : X \to Y \) be an intuitionistic fuzzy semi-continuous mapping, and \( A \) be an IFS in \( Y \). Clearly, \( f^{-1}(\text{cl}(A)) \) is an IFSCS in \( X \). Now,

\[
\text{IsBD}(f^{-1}(A)) = \text{scl}(f^{-1}(A)) \cup \text{scl}(f^{-1}(A^c)) \\
= \text{scl}(f^{-1}(A)) \cup \text{scl}(f^{-1}(A^c)) \\
\leq \text{scl}(f^{-1}(\text{cl}(A))) \cup \text{scl}(f^{-1}(\text{cl}(A^c))) \\
= f^{-1}(\text{cl}(A)) \cup f^{-1}(\text{cl}(A^c)) \\
= f^{-1}(\text{cl}(A)) \cup f^{-1}(\text{cl}(A^c)) \\
= f^{-1}(\text{IsBD}(A))
\]

Therefore, \( \text{IsBD}(f^{-1}(A)) \leq f^{-1}(\text{IsBD}(A)) \).

Theorem 3.15. Let \( f : X \to Y \) be an intuitionistic fuzzy irresolute mapping and \( A \) be an

IFS in \( Y \), \( \text{IsBD}(f^{-1}(A)) \leq f^{-1}(\text{IsBD}(A)) \).

Proof: Let \( f : X \to Y \) be an intuitionistic fuzzy irresolute mapping and \( A \) be an

IFS in \( Y \). Now,

\[
\text{IsBD}(f^{-1}(A)) = \text{scl}(f^{-1}(A)) \cup \text{scl}(f^{-1}(A^c)) \\
\leq \text{scl}(f^{-1}(\text{scl}(A))) \cup \text{scl}(f^{-1}(\text{scl}(A^c))) \\
= f^{-1}(\text{scl}(A)) \cup f^{-1}(\text{scl}(A^c)) \\
= f^{-1}(\text{scl}(A)) \cup f^{-1}(\text{scl}(A^c)) \\
= f^{-1}(\text{IsBD}(A))
\]

Therefore, \( \text{IsBD}(f^{-1}(A)) \leq f^{-1}(\text{IsBD}(A)) \)
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