Exact Solutions for a BBM(m,n) Equation with Generalized Evolution

Wei Li and Yun-Mei Zhao

Department of Mathematics, Honghe University
Mengzi, Yunnan, 661100, China
wellars@163.com

Abstract

In this paper, employed exp-function method and F-expansion method, we study the BBM(m,n) equation with generalized evolution, four families of exact solutions of exp-function type are obtained. Under different parametric conditions, every family of solution can be reduced to some solitary wave solutions and periodic wave solutions. The results presented in this paper improve the previous results.

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1 Introduction

The research work of nonlinear evolution equations in Applied Mathematics and Theoretical Physics has been going on for the past forty years. By dint of some new methods, many new results have been published in this area for a long time. Here, it is worth to mention that the two methods, the Exp-function method [1,2] and F-expansion method [3] can be combined to form one method [4-6].

In this paper, by using Exp-function method combined with F-expansion method, we will study the BBM(m,n) equation with generalized evolution [7]

\[(u^l)_t + a(u^l)_x + k(u^m)_x - b(u^n)_{xxt} = 0,\]  

(1)

where \(a, b, k\) are arbitrary constants and \(l, m, n\) are integers. In [7], using a solitary wave ansatz in the form of \(sech^p\) and \(tanh^p\) functions, respectively, Triki and Ismail obtained exact bright and dark soliton solutions for Eq. (1).

In this work, we will explore more types of exact solutions of Eq.(1).
2 Description of the method

In this section, we review the combining the Exp-function method with F-expansion method \([5,6]\) at first.

Given a nonlinear partial differential equation, for instance, in two variables, as follows:

\[
P(u, u_x, u_t, u_{xx}, u_{xt}, \cdots) = 0,
\]

where \(P\) is in general a nonlinear function of its variables.

We firstly use the Exp-function method to obtain new exact solutions of the following Riccati equation

\[
\phi'\!(\xi) = \frac{d}{d\xi}\phi(\xi) = A + \gamma\phi^2(\xi),
\]

where \(A\) and \(\gamma\) are arbitrary constants, then using the Riccati equation (3) as auxiliary equation and its exact solutions, we obtain exact solutions of the nonlinear partial differential equation (2).

Seeking for the exact solutions of Eq. (3), we introducing a complex variable \(\eta\), defined by

\[
\eta = \mu\xi + \xi_0,
\]

where \(\mu\) is a constant to be determined later, \(\xi_0\) is an arbitrary constant, Riccati equation (3) converts to

\[
\mu\phi' - A - \gamma\phi^2 = 0,
\]

where prime denotes the derivative with respect to \(\eta\).

According to the Exp-function method, we assume that the solution of Eq. (5) can be expressed in the following form

\[
\phi(\eta) = \frac{a_e \exp(\epsilon \eta) + \cdots + a_{-d} \exp(-d\eta)}{b_g \exp(g\eta) + \cdots + b_{-f} \exp(-f\eta)},
\]

where \(e, d, g\) and \(f\) are positive integers which are given by the homogeneous balance principle, \(a_e, \ldots, a_{-d}, b_g, \ldots, b_{-f}\) are unknown constants to be determined. To determine the values of \(e\) and \(g\), we usually balance the linear term of the highest order in Eq. (5) with the highest order nonlinear term. Similarly, we can determine \(d\) and \(f\) by balancing the linear term of the lowest order in Eq. (5) with the lowest order nonlinear term, we obtain \(e = g, d = f\). For simplicity, we set \(e = g = 1\) and \(d = f = 1\), then Eq. (6) becomes

\[
\phi(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1} \exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1} \exp(-\eta)}.
\]

Substituting Eq. (7) into Eq. (5), equating to zero the coefficients of all powers of \(\exp(n\eta)\) \((n = -2, -1, 0, 1, 2)\) yields a set of algebraic equations for \(a_1, a_0,\)
\(a_{-1}, b_1, b_0, b_{-1} \) and \( \mu \). Solving the system of algebraic equations by using Maple, we obtain the new exact solution of Eq. (3), which read

\[
\phi_1 = \frac{-\sqrt{-\frac{4}{\gamma}}b_1 \exp(\gamma \sqrt{-\frac{4}{\gamma}} \xi + \xi_0) + a_{-1} \exp(-\gamma \sqrt{-\frac{4}{\gamma}} \xi - \xi_0)}{b_1 \exp(\gamma \sqrt{-\frac{4}{\gamma}} \xi + \xi_0) + a_{-1} \sqrt{-\frac{4}{\gamma}} \exp(-\gamma \sqrt{-\frac{4}{\gamma}} \xi - \xi_0)},
\]

where \( a_{-1} \) and \( b_1 \) are free parameters;

\[
\phi_2 = \frac{(\gamma a_0^2 + Ab_0^2)}{4\gamma - \frac{4b_{-1}}{\xi}} \exp(2\gamma \sqrt{-\frac{4}{\gamma}} \xi + \xi_0) + a_0 + \sqrt{-\frac{4}{\gamma}} b_{-1} \exp(-2\gamma \sqrt{-\frac{4}{\gamma}} \xi - \xi_0)
\]

where \( a_0, b_0 \) and \( b_{-1} \) are free parameters.

By choosing properly values of \( a_0, a_{-1}, b_0, b_{-1} \), we find many kinds of hyperbolic function solutions and triangular periodic solutions of Eq. (3), which are listed as follows:

(i) When \( \xi_0 = 0, b_1 = 1, a_{-1} = \pm \sqrt{-\frac{4}{\gamma}}, \frac{A}{\gamma} < 0 \), the solution (8) becomes

\[
\phi = -\sqrt{-\frac{A}{\gamma}} \tanh(\gamma \sqrt{-\frac{A}{\gamma}} \xi),
\]

and

\[
\phi = -\sqrt{-\frac{A}{\gamma}} \coth(\gamma \sqrt{-\frac{A}{\gamma}} \xi).
\]

(ii) When \( \xi_0 = 0, b_1 = i, a_{-1} = \mp \sqrt{\frac{4}{\gamma}}, \frac{A}{\gamma} > 0 \), the solution (8) becomes

\[
\phi = \sqrt{\frac{A}{\gamma}} \tan(\gamma \sqrt{\frac{A}{\gamma}} \xi),
\]

and

\[
\phi = -\sqrt{\frac{A}{\gamma}} \cot(\gamma \sqrt{\frac{A}{\gamma}} \xi).
\]

(iii) When \( \xi_0 = 0, b_0 = 0, b_{-1} = 1, a_0 = \pm 2\sqrt{-\frac{4}{\gamma}}, \frac{A}{\gamma} < 0 \), the solution (9) becomes

\[
\phi = -\sqrt{-\frac{A}{\gamma}} \left[ \coth(2\gamma \sqrt{-\frac{A}{\gamma}} \xi) \pm \csc(2\gamma \sqrt{-\frac{A}{\gamma}} \xi) \right].
\]

(iv) When \( \xi_0 = 0, b_0 = 0, b_{-1} = i, a_0 = \pm 2\sqrt{-\frac{4}{\gamma}}, \frac{A}{\gamma} < 0 \), the solution (9) becomes

\[
\phi = -\sqrt{-\frac{A}{\gamma}} \left[ \tanh(2\gamma \sqrt{-\frac{A}{\gamma}} \xi) \pm \text{sech}(2\gamma \sqrt{-\frac{A}{\gamma}} \xi) \right].
\]
(v) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = 1$, $a_0 = \pm 2\sqrt{\frac{A}{\gamma}}$, $\frac{A}{\gamma} > 0$, the solution (9) becomes
\[
\phi = \sqrt{\frac{A}{\gamma}}[\tan(2\gamma\sqrt{\frac{A}{\gamma}\xi}) \pm \sec(2\gamma\sqrt{\frac{A}{\gamma}\xi})]. \tag{16}
\]
(vi) When $\xi_0 = 0$, $b_0 = 0$, $b_{-1} = i$, $a_0 = \pm 2\sqrt{\frac{A}{\gamma}}$, $\frac{A}{\gamma} > 0$, the solution (9) becomes
\[
\phi = -\sqrt{\frac{A}{\gamma}}[\cot(2\gamma\sqrt{\frac{A}{\gamma}\xi}) \mp \csc(2\gamma\sqrt{\frac{A}{\gamma}\xi})]. \tag{17}
\]
For simplicity, in the rest of the paper, we consider $\xi_0 = 0$.

3 Solutions for the BBM(m,n) equation

In order to obtain new exact travelling wave solutions for Eq. (1), we use
\[
u(x,t) = u(\xi), \quad \xi = B(x - \omega t), \tag{18}
\]
where $B$ and $\omega$ are constants, and substituting the (18) into Eq. (1), integrating two sides of the equation with respect to $\xi$ and setting the constants of integration equal to zero, we obtain
\[
(a - \omega)u' + ku^m + b\omega B^2(u^n)'' = 0. \tag{19}
\]

3.1. Case I: $l = n$, $m \neq n$

In this case, balancing the order of the nonlinear term $u^m$ with the term $(u^n)''$ in (19), we obtain
\[
mP = nP + 2, \tag{20}
\]
so that
\[
P = \frac{2}{m - n}. \tag{21}
\]
To get a closed form solution, it is natural to use the transformation
\[
u = v^{\frac{1}{m-n}}, \tag{22}
\]
and Eq. (19) becomes
\[
(a - \omega)(m - n)^2v^2 + k(m - n)^2v^3 + \omega b B^2 \left[n(2n - m)(v')^2 + n(m - n)vv'' \right] = 0. \tag{23}
\]

Now, we assume that the solution of Eq. (23) can be expressed in the following form
\[
v = v(\xi) = \sum_{j=0}^{N} \alpha_j \phi^j(\xi), \tag{24}
\]
where $N$ is positive integers which are given by the homogeneous balance principle, $\phi(\xi)$ is a solution of Eq. (3). Balancing $vv''$ term with $v^3$ term in (23) gives $N = 2$. Therefore, we obtain

$$v = \alpha_0 + \alpha_1 \phi(\xi) + \alpha_2 \phi^2(\xi).$$

(25)

Substituting Eq. (25) into (23) and using the Riccati equation (3), collecting the coefficients of $\phi(\xi)$, we have

$$\frac{1}{D} \left[ C_0 + C_1 \phi(\xi) + C_2 \phi^2(\xi) + \cdots + C_5 \phi^5(\xi) + C_6 \phi^6(\xi) \right] = 0.$$  

(26)

Because the expresses to these coefficients $D, C_0 = 0, C_1 = 0, C_2 = 0, C_3 = 0, C_4 = 0, C_5 = 0, C_6 = 0$ of $\phi(\xi)$ in Eq. (26) are too lengthiness, so we omit them. But we can directly use the command "solve" in mathematical software Maple to solve the following set of algebraic equations

$$C_0 = 0, \quad C_1 = 0, \quad C_2 = 0, \quad C_3 = 0, \quad C_4 = 0, \quad C_5 = 0, \quad C_6 = 0.$$  

(27)

Solved the above algebraic equations, we obtain

$$\alpha_0 = \frac{(m+n)(\omega-a)}{2kn}, \quad \alpha_1 = 0, \quad \alpha_2 = \frac{(m+n)(\omega-a)\gamma}{2knA}, \quad B = \frac{(m-n)\sqrt{\frac{a-\omega}{\omega b A}}} {2n}.$$  

(28)

Thus from Eqs. (25) and (28) we obtain the solution of Eq. (23) in the form:

$$v = \frac{(m+n)(\omega-a)}{2kn} + \frac{(m+n)(\omega-a)\gamma}{2knA} \phi^2(\xi),$$

(29)

where $\phi(\xi)$ is a solution of Eq. (3).

Substituting new solutions (8) and (9) of Riccati equation into solution (29), using the transformation (22), we have the following two families of solutions to Eq. (1).

Family 1

$$u_1(x, t) = \left\{ \frac{(m+n)(\omega-a)}{2kn} + \frac{(m+n)(\omega-a)\gamma}{2knA} \right\} \left[ \frac{\frac{\sqrt{\frac{a-\omega}{\omega b A}}}{\sqrt{\frac{a-\omega}{\omega b A}}} \exp\left(\gamma \sqrt{-\frac{a}{\omega b A}}\right) + a_{-1} \exp\left(-\gamma \sqrt{-\frac{a}{\omega b A}}\right)}{b_1 \exp\left(\gamma \sqrt{-\frac{a}{\omega b A}}\right) + a_{-1} \sqrt{\frac{a}{\omega b A}}} \right]^{\frac{1}{m-n}},$$

(30)

where $\xi = \frac{(m-n)\sqrt{\frac{a-\omega}{2n}}}{2n}(x - \omega t), \quad l = n.$
If we set \( b_1 = 1, \ a_{-1} = \pm \sqrt{-\frac{4}{\gamma}}, \) and \( \frac{4}{\gamma} < 0 \) in Eq. (30), we obtain

\[
\begin{align*}
  u_{1(1)}(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} \right\} \text{sech}^2 \left( \gamma \sqrt{-\frac{A}{\gamma}} \xi \right) \frac{1}{m-n}, \\
  u_{1(2)}(x, t) &= \left\{ -\frac{(m + n)(\omega - a)}{2kn} \right\} \text{csch}^2 \left( \gamma \sqrt{-\frac{A}{\gamma}} \xi \right) \frac{1}{m-n}, \\
  u_{1(3)}(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} \right\} \sec^2 \left( \gamma \sqrt{-\frac{A}{\gamma}} \xi \right) \frac{1}{m-n}, \\
  u_{1(4)}(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} \right\} \csc^2 \left( \gamma \sqrt{-\frac{A}{\gamma}} \xi \right) \frac{1}{m-n}.
\end{align*}
\]

Family 2

\[
\begin{align*}
  u_2(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} + \frac{(m + n)(\omega - a)\gamma}{2knA} \right\} \times \left[ \frac{(\gamma \sqrt{-\frac{A}{\gamma}})^{\frac{1}{n-1}} + b_{-1} \exp(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) + b_{0} + b_{-1} \exp(-2\gamma \sqrt{-\frac{4}{\gamma}} \xi)}{\frac{(\gamma \sqrt{-\frac{A}{\gamma}})^{\frac{1}{n-1}} \exp(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) + b_{0} + b_{-1} \exp(-2\gamma \sqrt{-\frac{4}{\gamma}} \xi)}} \right]^2 \frac{1}{m-n},
\end{align*}
\]

where \( \xi = \frac{(m-n)}{2n} \sqrt{-\frac{4}{\gamma}}(x - \omega t), \quad i = n. \)

If we set \( b_0 = 0, \ b_{-1} = 1, \ a_0 = \pm 2\sqrt{-\frac{4}{\gamma}}, \) and \( \frac{4}{\gamma} < 0 \) in Eq. (35), we obtain

\[
\begin{align*}
  u_{2(1)}(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} - \frac{(m + n)(\omega - a)}{2kn} \right\} \times \left[ \coth(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) \pm \text{csch}(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) \right]^2 \frac{1}{m-n} \\
  &= \left( \frac{(m + n)(\omega - a)}{kn \left[ 1 \mp \cosh(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) \right]} \right) \frac{1}{m-n}. \quad (36)
\end{align*}
\]

Setting \( b_0 = 0, \ b_{-1} = i, \ a_0 = \pm 2\sqrt{-\frac{4}{\gamma}}, \) and \( \frac{4}{\gamma} < 0 \) in Eq. (35), we get

\[
\begin{align*}
  u_{2(2)}(x, t) &= \left\{ \frac{(m + n)(\omega - a)}{2kn} - \frac{(m + n)(\omega - a)}{2kn} \right\} \times \left[ \tanh(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) \pm \text{sech}(2\gamma \sqrt{-\frac{4}{\gamma}} \xi) \right]^2 \frac{1}{m-n} \quad (37)
\end{align*}
\]
Setting \( b_0 = 0, b_{-1} = 1, a_0 = \pm 2\sqrt{\frac{A}{\gamma}}, \) and \( \frac{A}{\gamma} > 0 \) in Eq. (35), we have

\[
\begin{align*}
\frac{u^{(3)}_2(x, t)}{2kn} &= \left\{ \frac{(m + n)(\omega - a)}{2kn} + \frac{(m + n)(\omega - a)}{2kn} \right. \\
& \times \left[ \tan(2\gamma \sqrt{\frac{A}{\gamma}} \xi) \pm \sec(2\gamma \sqrt{\frac{A}{\gamma}} \xi) \right]^2 \right\}^{\frac{1}{m-n}}.
\end{align*}
\] (39)

Setting \( b_0 = 0, b_{-1} = i, a_0 = \pm 2\sqrt{\frac{A}{\gamma}}, \) and \( \frac{A}{\gamma} > 0 \) in Eq. (35), we have

\[
\begin{align*}
\frac{u^{(4)}_2(x, t)}{2kn} &= \left\{ \frac{(m + n)(\omega - a)}{2kn} + \frac{(m + n)(\omega - a)}{2kn} \right. \\
& \times \left[ \cot(2\gamma \sqrt{\frac{A}{\gamma}} \xi) \mp \csc(2\gamma \sqrt{\frac{A}{\gamma}} \xi) \right]^2 \right\}^{\frac{1}{m-n}} \\
&= \left( \frac{(m + n)(\omega - a)}{kn \left[ 1 \pm \cos(2\gamma \sqrt{\frac{A}{\gamma}} \xi) \right]} \right)^{\frac{1}{m-n}}. 
\end{align*}
\] (40)

### 3.2. Case II: \( m = n, l \neq n \)

In this case, balancing the order of the nonlinear term \( u^l \) with the term \( (u^n)^\prime\prime \) in (19), we obtain

\[
LP = nP + 2,
\] (42)

so that

\[
P = \frac{2}{l - n}.
\] (43)

To get a closed form solution, it is natural to use the transformation

\[
u = \frac{1}{l-n},
\] (44)

and Eq. (19) becomes

\[
(a - \omega)(l - n)^2 v^3 + k(l - n)^2 v^2 + \omega bB^2 \left[ n(2n - l)(v')^2 + n(l - n)v v'' \right] = 0. \] (45)

By the same manipulation as is illustrated in Case I, we obtain the solution of Eq. (45) in the form:

\[
v = \frac{k(l + n)}{2n(\omega - a)} + \frac{k(l + n)\gamma}{2n(\omega - a)A} \phi^2(\xi), \quad \xi = \frac{(n - l)\sqrt{\frac{k}{\omega b A^2}}}{2n}(x - \omega t),
\] (46)

where \( \phi(\xi) \) is a solution of Eq. (3).

Substituting new solutions (8) and (9) of Riccati equation into solution (46), using the transformation (44), we have the following two families of solutions
to Eq. (1).

**Family 3**

\[ u_3(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} + \frac{k(l + n)\gamma}{2n(\omega - a)A} \\
 \times \left[ -\sqrt{-\frac{\gamma}{A}b_1 \exp(\gamma \sqrt{\frac{\gamma}{A}}x)} + a_{-1} \exp(\gamma \sqrt{\frac{\gamma}{A}}x) \right] \right. \\
\left. \left. \frac{b_1 \exp(\gamma \sqrt{\frac{\gamma}{A}}x) + a_{-1} \exp(\gamma \sqrt{\frac{\gamma}{A}}x)}{\sqrt{\frac{\gamma}{A}}} \right] \right\}^{\frac{1}{1-n}}, \tag{47} \]

where \( \xi = \frac{(n-t)}{2n}(x - \omega t), \quad m = n. \)

If we set \( b_1 = 1, \quad a_{-1} = \pm \sqrt{-\frac{\gamma}{A}}, \quad \text{and} \quad \frac{\gamma}{A} < 0 \) in Eq. (47), we obtain

\[ u_{3(1)}(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} \sech^2(\gamma \sqrt{-\frac{A}{\gamma}} x) \right. \\
\left. \left. \left\{ \frac{1}{1-n} \right. \right\} \right\}, \tag{48} \]

and

\[ u_{3(2)}(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} \csch^2(\gamma \sqrt{-\frac{A}{\gamma}} x) \right. \\
\left. \left. \left\{ \frac{1}{1-n} \right. \right\} \right\}, \tag{49} \]

Setting \( b_1 = i, \quad a_{-1} = \mp \sqrt{\frac{\gamma}{A}}, \quad \text{and} \quad \frac{\gamma}{A} > 0 \) in Eq. (47), we get

\[ u_{3(3)}(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} \sec^2(\gamma \sqrt{-\frac{A}{\gamma}} x) \right. \\
\left. \left. \left\{ \frac{1}{1-n} \right. \right\} \right\}, \tag{50} \]

and

\[ u_{3(4)}(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} \csc^2(\gamma \sqrt{-\frac{A}{\gamma}} x) \right. \\
\left. \left. \left\{ \frac{1}{1-n} \right. \right\} \right\}. \tag{51} \]

**Family 4**

\[ u_4(x, t) = \left\{ \begin{array}{l} \frac{k(l + n)}{2n(\omega - a)} + \frac{k(l + n)\gamma}{2n(\omega - a)A} \\
\times \left[ \frac{(\gamma a^2 + A \delta^2)}{4\gamma \sqrt{-\frac{\gamma}{A}b_1}} \exp(2\gamma \sqrt{-\frac{\gamma}{A}} x) + a_0 + \frac{1}{4\gamma \sqrt{-\frac{\gamma}{A}b_1}} \exp(2\gamma \sqrt{-\frac{\gamma}{A}} x) \right] \right. \\
\left. \left. \left\{ \frac{1}{1-n} \right. \right\} \right\}, \tag{52} \]

where \( \xi = \frac{(n-t)}{2n}(x - \omega t), \quad m = n. \)
If we set $b_0 = 0, b_{-1} = 1, a_0 = \pm 2\sqrt{-\frac{A}{\gamma}},$ and $\frac{A}{\gamma} < 0$ in Eq. (52), we obtain

$$u_{4(1)}(x, t) = \left\{ \frac{k(l + n)}{2n(\omega - a)} - \frac{k(l + n)}{2n(\omega - a)} \right\} \left[ \frac{1 + \cosh(2\gamma\sqrt{-\frac{A}{\gamma}})}{1 - \cosh(2\gamma\sqrt{-\frac{A}{\gamma}})} \right]^\frac{1}{n} .$$

Setting $b_0 = 0, b_{-1} = i, a_0 = \pm 2\sqrt{-\frac{A}{\gamma}},$ and $\frac{A}{\gamma} < 0$ in Eq. (52), we get

$$u_{4(2)}(x, t) = \left\{ \frac{k(l + n)}{2n(\omega - a)} - \frac{k(l + n)}{2n(\omega - a)} \right\} \left[ \tanh(2\gamma\sqrt{-\frac{A}{\gamma}}) \pm isech(2\gamma\sqrt{-\frac{A}{\gamma}}) \right]^\frac{1}{n} .$$

Setting $b_0 = 0, b_{-1} = 1, a_0 = \pm 2\sqrt{\frac{A}{\gamma}},$ and $\frac{A}{\gamma} > 0$ in Eq. (52), we have

$$u_{4(3)}(x, t) = \left\{ \frac{k(l + n)}{2n(\omega - a)} + \frac{k(l + n)}{2n(\omega - a)} \right\} \left[ \tan(2\gamma\sqrt{\frac{A}{\gamma}}) \pm sec(2\gamma\sqrt{\frac{A}{\gamma}}) \right]^\frac{1}{n} .$$

Setting $b_0 = 0, b_{-1} = i, a_0 = \pm 2\sqrt{\frac{A}{\gamma}},$ and $\frac{A}{\gamma} > 0$ in Eq. (52), we have

$$u_{4(4)}(x, t) = \left\{ \frac{k(l + n)}{2n(\omega - a)} + \frac{k(l + n)}{2n(\omega - a)} \right\} \left[ \cot(2\gamma\sqrt{\frac{A}{\gamma}}) \pm \csc(2\gamma\sqrt{\frac{A}{\gamma}}) \right]^\frac{1}{n} .$$

$$= \left( \frac{k(l + n)}{n(\omega - a) [1 \pm \cos(2\gamma\sqrt{\frac{A}{\gamma}})]} \right)^\frac{1}{n} .$$

4 Conclusions

In this paper, by using Exp-function method combined with F-expansion method, we studied the BBM(m,n) equation with generalized evolution, four families of exact solutions of exp-function type are obtained. Under different parametric conditions, every family of solution can be reduced to some solitary wave solutions and periodic wave solutions, the majority of these results are very different to those in Ref. [7]. This shows that some solutions with different wave forms can be expressed by the same one solution of exp-function type. From these abundant results, it is easy to know that the Exp-function method combined F-expansion method is useful to many nonlinear partial equations.
References


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