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Solving Bi-Level Programming Problems on Using Global Criterion Method with an Interval Approach

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Abstract

In this paper on using the global criterion method, the bi-level programming have been solved by an interval approach. We solve the bi-level programming on using Karush-Kuhn-Tucker (KKT) conditions, and global criterion method converts to a single objective and the we solve this problem. The advantage of this method can be extended solve for solving the problems with large-scale. In the end numerical example is given to illustrate the efficiency of the method.

Keywords: Global Criterion, bi-Level Programming, Interval Coefficient

1 Introduction

A bi-level programming problem is formulated for a problem in which two decision-maker make decisions successively. For example, in a decentralized firm, top management, an executive board, or headquarters makes a decision such as a budget of the firm, and then each division determines a production plane in the full knowledge of the budged. We can also site the Stackelberg duo-poly: two firms supply homogeneous goods to a market, and consequently the predominant firm first its level of supply, and then the over firm determines that of itself after it realizes that of the predominant firm. Research on multi-level mathematical programming to solve organizational planning and

decision-making problems has been conducted widely. The research and application have concentrated mainly on bi-level programming (see [1, 5, 12, 13]). In real physical world, the coefficients of objective functions are not usually a constant but they are mostly in the form of interval coefficients. Steuer [14] and Tong [15] have proposed linear programming models with interval objective functions. Ishibuchi and Tanaka [3] developed a concept for optimization of multi objective programming problems with interval objective functions. A new treatment of the interval objective in linear programming problems was developed by Inuiguchi and Kume [8] by introducing the minimax criterion as used indecision theory. Chanas and Kuchta [4] have generalized the known concept of thesolution of the linear programming problem with interval coefficients in the objective function based onpreference relations between intervals. In this paper, we solve the bi-level programming on using KKT conditions, and global criterion method converts to a single objective and the we solve this problem.

2 Formulation of the Problem

2.1 Global criterion for two objective programming

To deal with the problem of two objective programming, we utilize the method of global criterion [6] with a normalized process [7]. The essence of this approach includes the following:
(i) reference points, i.e., the concept of an ideal system.
(ii) distance, i.e., location of alternatives away from reference points.
(iii) normalization, i.e., the process to eliminate no measurability among objectives.
Note that, based on two reference points in each objective, Hwang et al [7] proposed a normalized process to obtain a better estimation. Reference points of the technique are positive ideal solution (PIS) $f_k^+(x)$ and negative ideal solution (NIS) $f_k^-(x)$ [11]. A PIS is defined, for each objective, as the maximum (the best) solution of the single-objective maximization problem, and the minimum (the best) solution of the single-objective minimization problem. NIS is defined as the opposite of PIS. In the other words, for each objective, the maximum solution is the worst to the minimization problem, and the minimum solution is the worst to the maximization problem. A total of four numbers of PISs and NISs are generated for a problem with two objectives. In addition, the distance measured is related to Minkowski's Lp metric with some modifications (see [9, 11]). The Lp metric defines distance between two points, i.e., one objective f_k and its ideal solution $f_k^+(x)$, in k-dimensional space

$$d_p = \Big(\sum_{k=1}^{2}(f_k^+(x) - f_k(x))^p\Big)^{\frac{1}{p}}, \qquad for \quad p \geqslant 1$$

.

The parameter, p, for distance setting depends on the preference of the DM is the case of manipulation. There are several popular settings:
(i) p = 1, Manhattan distance;
(ii) p = 2, Euclidean distance;
(iii) p = ∞, Tchebycheff distance.
For the convenience of processing, it is quite common to use an extreme case, i.e., p = 1 or p = ∞ so that the above representation can be kept in linear form. The method of global criterion is to minimize the distance function of the multiple objectives as

$$Min \quad d_p = \Big\{\sum_{k=1}^{2}\Big(\frac{f_k^+(x) - f_k(x)}{f_k^+(x) - f_k^-(x)}\Big)^p\Big\}^{\frac{1}{p}}$$

$$\text{s.t} \quad p \geqslant 1, \ x \in \text{X}$$

where X is the constraint set of the above MODM expression.

Definition 2.1. *Let $* \in \{+, -, ., \%\}$ be a binaryoperation on the set of real numbers. If A and B are closed intervals, then $A * B = \{a * b : a \in A, b \in B\}$ defines a binary operation on the set of closed intervals. In the case of division, it is assumed that zero does not belong to B. The operations on intervals used in this paper may be explicitly calculated from Definition1 as*

$$A + B = [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R]$$

$$kA = k[a_L, a_R] = \begin{cases} [ka_L, ka_R] & k \geqslant 0 \\ [ka_R, ka_L] & k \leqslant 0 \end{cases}$$

where k is a real number.

2.2 Global Criterion and K.K.T Conditions for Bi-level Programming

The bi level programming problem with interval coefficients, where the upper level DM has control over the vector x_1 while the lower level DM controls the vector x_2.

$$\underset{x_1}{\text{Max}} \quad f_1(x_1, x_2) = [C_{11_L}, C_{11_R}]x_1 + [C_{12_L}, C_{12_R}]x_2 \qquad (upperlevel)$$

where x_2 solve

$$\text{Max} \quad f_2(x_1, x_2) = [C_{21_L}, C_{21_R}]x_1 + [C_{22_L}, C_{22_R}]x_2 \qquad (lowerlevel)$$

$$s.t$$

$$A_1x_1 + A_2x_2 \leqslant b$$

$$x_1 \geqslant 0, x_2 \geqslant 0$$

Letting the performance functions of $f_1$and f_2 for the two planners are linear and bounded, then the BLP problem can be represented as where $C_{ij_L}, C_{ij_R}(i = 1, 2, j = 1, 2)$ and b are vectors, A_1, and $A_2$are matrices.
The above-nested optimization model has been proven to be NP-hard by Ben-Ayed and Blair [2] and is difficult to solve.
The most popular approach to solve the nested bi-level optimization problems is using the KKT conditions and transforms the original problem to its first level auxiliary problem [11]. In this way, the problem is reduced to a regular mathematical programming problem. However, the transformed problem or the auxiliary problem is difficult to solve due to nonlinearity, which was introduced through the complementary slackness conditions (CSCs). Many researchers have devised approaches to solve this nonlinear problem [10]. In this method, a zero-one variable, η and ξ, is added for each constraint wu = 0 and xν = 0, respectively. In addition, each of these constraints is replaced by two linear inequalities involving η and ξ and M, a large positive constant[3]. The auxiliary formulation now becomes

$$\text{Max} \quad f_1(x_1, x_2) = [C_{11_L}, C_{11_R}]x_1 + [C_{12_L}, C_{12_R}]x_2$$

subject to

$$A_1x_1 + A_2x_1 + u = b$$

$$w^T A_2 - \nu = [C_{22_L}, C_{22_R}]$$

$$w \leqslant M\eta, \ u \leqslant M(1 - \eta)$$

$$x_2 \leqslant M\xi, \ \nu \leqslant M(1 - \xi)$$

$$\eta, \ \xi \in \{0, 1\}$$

$$x_1, x_2, \ w, \ u, \ \nu \geqslant 0$$

3 Numerical Example

A bi level decentralized system with a center at the upper level and two independent divisions at the lower level .

$$\underset{x_1}{\text{Max}} \quad f_1(x_1, x_2) = [1,4]x_1 + [2,3]x_2 - [1,5]x_3$$

where x_2 and $x_3$solves

$$\underset{x_2}{\text{Max}} \quad f_{21}(x_1, x_2) = [1,3]x_1 + [3,8]x_2 - [2,5]x_3$$

$$\underset{x_3}{\text{Max}} \quad f_{22}(x_1, x_2) = -[1,4]x_1 + [1,6]x_2 + [1,4]x_3$$

$$\begin{aligned} & s.t \\ & \quad x_1 + 3x_2 + x_3 \leqslant 5 \qquad\qquad 3x_1 + x_2 \leqslant 9 \\ & \quad 2x_1 + x_2 + 3x_3 \leqslant 8 \qquad\qquad 4x_1 - 2x_2 + x_3 \geqslant 2 \\ & \quad x_i \geqslant 0, \quad i = 1,2,3 \end{aligned}$$

After applying the KKT conditions to the original problem, a new auxiliary problem with only one level will be obtained.

$$\begin{aligned} & \text{Max} \quad f_1(x_1, x_2) = [1,4]x_1 + [2,3]x_2 - [1,5]x_3 \\ & \quad s.t \\ & \quad x_1 + 3x_2 + x_3 + u_1 = 5 \qquad\qquad 3x_1 + x_2 + u_3 = 9 \\ & \quad 2x_1 + x_2 + 3x_3 + u_2 = 8 \qquad\qquad 4x_1 - 2x_2 + x_3 + u_4 = 2 \\ & \quad 3w_1 + w_2 + w_3 - 2w_4 - \nu_1 = [3,8] \qquad\qquad w_1 + 3w_2 + w_4 - \nu_2 = [1,4] \\ & \quad w_i \leqslant M\eta_i, \ u_i \leqslant (1-\eta_i)M \qquad\qquad x_j \leqslant M\xi_j, \ \nu_j \leqslant M(1-\xi_j) \\ & \quad \eta_i, \xi_j \in \{0,1\} \\ & \quad x_1, x_2, x_3, w_i, \nu_j \geqslant 0, \quad i = 1,\ldots,4, j = 1,2 \end{aligned}$$

for solving the single-objective maximization problem with interval coefficients as above , we convert it the following problem.

$$\begin{aligned}
&\text{Max} \quad z_L(x) = x_1 + 2x_2 - 5x_3 \\
&\text{Max} \quad z_R(x) = 4x_1 + 3x_2 - x_3 \\
&\quad s.t \\
&\qquad x_1 + 3x_2 + x_3 + u_1 = 5 \qquad\qquad 3x_1 + x_2 + u_3 = 9 \\
&\qquad 2x_1 + x_2 + 3x_3 + u_2 = 8 \qquad\qquad 4x_1 - 2x_2 + x_3 + u_4 = 2 \\
&\qquad 3w_1 + w_2 + w_3 - 2w_4 - \nu_1 \leqslant 3 \qquad w_1 + 3w_2 + w_4 - \nu_2 \leqslant 1 \\
&\qquad 3w_1 + w_2 + w_3 - 2w_4 - \nu_1 \geqslant 8 \qquad w_1 + 3w_2 + w_4 - \nu_2 \geqslant 4 \\
&\qquad w_i \leqslant M\eta_i,\ u_i \leqslant (1-\eta_i)M \qquad\qquad x_j \leqslant M\xi_j,\ \nu_j \leqslant M(1-\xi_j) \\
&\qquad \eta_i, \xi_j \in \{0,1\} \\
&\qquad x_1, x_2, x_3, w_i, \nu_j \geqslant 0, \quad i = 1,\ldots,4, j = 1,2
\end{aligned}$$

The above constraint will be expressed as Z.
After obtaining the (PIS)$z^+(x) = (z_L^+, z_R^+) = (4.25, 13.25)$ and (NIS) $z^-(x) = (z_L^-, z_R^-) = (-10.97, -0.41)$, We solve the following problem for p=1,

$$\begin{aligned}
&\text{Min} \quad d_p = \Big\{\Big(\frac{4.25 - z_L(x)}{4.25 - (-10.97)}\Big)^p\Big\}^{\frac{1}{p}} + \Big\{\Big(\frac{13.25 - z_R(x)}{13.25 - (-0.41)}\Big)^p\Big\}^{\frac{1}{p}} \\
&\quad s.t \\
&\qquad \text{x}\in \text{Z}
\end{aligned}$$

The minimal Manhattan distance $d_p = 0$ with $x^* = (x_1, x_2) = (2.75, 0.75, 0)$

$$f_1^*(x) = [4.25, 13.25] \quad \& \quad f_{21}^*(x) = [5, 14.25] \quad \& \quad f_{22}^*(x) = [-10.25, 1.75].$$

4 Conclusion

In this paper bi-level programming problems have been solved by global criterion method in which we found it much easier and simpler solution compare to the other methods.To be sure of the effectiveness of the method , one numerical example is solved.

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