Earnings, Coalitions
and the Stability of Firms

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Abstract
This paper presents an economy in which workers hired by a firm receive without cost a firm-specific training that enables them to potentially become independent producers. This specific training changes a worker's outside option according to the firm in which he works. By modelling explicitly the workers' decision to stay or to leave the firm, the paper determines a stable earning profile of the economy. Two main results are obtained. Firstly, that such a stable earning profile may allow for a vector of wages higher than the basic neoclassical wages and for wages differentials across industries even for initially homogenous workers. Secondly, that an industry equilibrium wage depends upon the relative degree of competition existing therein. Both the results seem to match labour markets empirical evidence.

Keywords: Earnings, Oligopoly, Firm Stability

1 Introduction
It is empirically and theoretically well accepted that companies enjoying larger market shares in the product market are used to pay higher wages to their workers [2], [4]. One of the most common explanations of such phenomenon is that firms with higher surpluses, usually associated with larger market shares, allow unions to grasp higher rents over workers' reservation wages. Using for instance a simple Nash bargaining model, the resulting equilibrium wage - given each group of players' bargaining power - turns out to be positively related to the level of firm's expected profit. A fact usually not accounted for by this type of analysis, however,
is that a high market profitability usually increases workers' reservation wages. This may be the case if the value of a trained worker's outside option when she or he decides to leave the firm (for instance after a break of the wage negotiation) either setting up a new venture as entrepreneur or establishing a partnership with other colleagues or, finally, becoming self-employed, becomes greater the higher is the industry expected profitability. As a consequence, it is not entirely clear which cause is responsible of the phenomenon described above. The aim of this paper is to explicitly model the workers' decision to stay or to leave the firm in which they are employed, and to observe which firm's stable earning profile raises as a result, i.e. a vector of payments received both by the employees and by the owners of a firm that respects certain stability requirements. The main assumption of the model is that at the beginning two different groups of individuals exist in the economy: the entrepreneurs, endowed with a specific knowledge of a given industry production process and the workers, endowed with just their workforce and without any knowledge about how to produce a commodity. Once an entrepreneur decides to set up a firm in a given industry, however, she or he hires a certain number of workers and these workers acquire a firm-specific training. When this happens every worker can bargain with the firm his compensation, threatening to leave whenever, given his available outside options, the wage proposed by the entrepreneur is not satisfactory. Since the model assumes oligopolistic competition within each industry, in such a strategic environment every entrepreneur is sensitive to the possibility that employees leave their workplace setting up new competing production units. This simple ingredient of the model permits to obtain some interesting results. On the one hand, an economy stable earning profile usually comprehends a vector of wages higher than the basic neoclassical (reservation) wage, also giving rise to wage differentials across industries for initially homogeneous workers; on the other hand, this vector of wages depends upon the relative degree of competition of the industry in which firms operate. In two somehow related papers [6], [7] apply an intra-firm multilateral bargaining framework with non binding agreements between a firm and its worker to yield an equilibrium level of wages and employment. By assuming complete irreplaceability of each single worker, a stable (i.e. non renegotiable) earnings profile for the firm and the workers is characterized and proved to be equal to the Shapley value of the corresponding cooperative game. More recently, [1] model the sequence of information spillover among agents as a sequential bargaining process occurring between informed and uninformed agents. Similarly, in [5] it is modelled explicitly the process of sequential entry of informed workers increasing market competition in a simple multi-stage setup. None of the above mentioned papers considers the possibility that workers leave the firm in coalitions. The next section outlines the basic structure of the model. Section 3 introduces an application of the model describing the main results of the paper. Section 4 concludes the paper.
2 Model Setup

We present here a very simple oligopolistic economy in which the most relevant feature is that every worker cannot autonomously produce without having first been trained as employee in a firm. Once one individual is recruited as employee by an entrepreneur she or he acquires a specific training and then can decide whether to stay or to leave the firm. Whenever the compensation is not convenient the worker can choose either to set up a new firm as entrepreneur, or to become self-employed or to participate as a member to a partnership with other workers. By imposing some stability requirements, this simple framework can thus be used to determine an equilibrium earning profile of the economy. The main assumptions of the model are listed here. i) In the economy there are two types of agents, entrepreneurs and workers. The entrepreneurs can be distinguished into m different types, according to the specific know-how they possess. Each worker is endowed with a unit of labour, without any particular knowledge of production processes; ii) Each entrepreneur needs to hire a certain number of workers to apply his knowledge and activate the specific production process; iii) the employees that decide to leave an entrepreneurial firm can be substituted without direct costs by the entrepreneur (as long as sufficient unemployed workers are available); iv) a leaving employee can set up a new firm alone as self-employed, as entrepreneur (using the old firm's employees), or as a member of an egalitarian partnership (with the other employees); v) there are no explicit credit constraints for leaving workers; vi) members of the unemployed group cannot set up a firm or be self-employed before having been trained as employees or members of a firm; vii) the workers' training is specific to the firm and is immediately obtainable (for simplicity) once a worker become employee or member of a firm of a particular industry. This means that any deviation of a coalition of workers cannot involve workers belonging to different firms; viii) every firm acts non cooperatively within a given industry, behaving as a separate coalition; ix) before being trained within a firm the workers of the economy are homogeneous; x) the market is oligopolistic. The competition is modeled in quantities. xi) in general, only symmetric equilibria are considered.

2.1 A Simple Oligopolistic Multi-sector Economy

This section describes a simple economy in which a finite set of individuals $N = \{1, 2, ..., n\}$ is initially distributed among two subsets, such that $N = \{I_K\} \cup \{I_L\}$, where $I_K$ represents the subset of entrepreneurs of the economy, while $I_L$ is the subset of workers. As anticipated above, a distinctive
feature of the economy is that, at the beginning, each entrepreneur owns a specific knowledge of a given industry production process, while workers do not.

For simplicity, it is assumed that every entrepreneur \( i \in I_k \) sets up just one firm by hiring a certain number of workers from the group \( I_L \). As a consequence of their lack of know-how, the members of \( I_L \), at least initially, cannot set up a firm without first being recruited by at least one member of \( I_K \), or by someone that worked with him before, and so on. Let us assume \( m \) different specific know-how \((l = 1, 2, ..., m)\) and, for each one, entrepreneurs disposing from the beginning of this particular knowledge. It turns out, thus, that the number of firms in the economy will be equal to \( v^T k^l \), where \( v^T \) is the transposed \( l \)-dimensional unitary vector and \( k^l \) is the vector representing the number of entrepreneurs (and then of firms) having a knowledge of each \( l \)-th sector of production \((l = 1, 2, ..., m)\).

Since not necessarily all the workers in \( I_L \) will be hired by one of the existing entrepreneurs, the potential number of coalitions in every initial coalition structure of this economy comprehends \((v^T k^l + 1)\) coalitions \((\text{firms})\) denoted \( S^j \). That is, in the economy there are \( m \) sectors, each one with a certain number of firms \((j = 1, 2, ..., k^l)\) devoted to producing a homogeneous commodity \( y^l \). Furthermore, in general there is a coalition (that can also be empty) including all unemployed people that do not belong to any firm \( S^j \). Denoting the set of all the firms of a certain industry \( l \) as \( \Omega_l = \bigcup_{j=1,...,k^l} S^j \), the set of all the unemployed people can be represented as \( U = N \setminus \bigcup_{l=1,...,m} \Omega_l \). Let the production for self-consumption be excluded, by requiring every commodities to be sold in the market. After bearing a given fixed cost \( F \), each firm \( S^j \) has a production function specific for the industry \( g_l : R_+ \rightarrow R_+ \) represented by: \( y_j = g_l(1|I_L \cap S^j|) \) where \(|I_L \cap S^j|\) indicates the number of workers hired by each firm operating in that particular industry. This means that the number of workers in each firm \( S^j \) will be decided by every entrepreneur according to the inverse function \( I^N\left(g^{-1}(y_j)\right) = I_L \cap S^j \) where the function \( I^N(\cdot) : R_+ \rightarrow I_+ \) transforms every real number into the closest natural number. Given the existing production function, every firm is assumed to compete \( a \ la \) Cournot in the \( l \)-th homogeneous good market, with a firm's profit function \( \pi_j : R_+^2 \rightarrow R \) given by:

\[
\pi_j = \left\{ p_l(y_j, y^l_j) y_j - F \right\}_{y_j=1,...,k^l}.
\]  (1)
where \( p_l : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \) indicates the inverse demand function of the \( l \)-th sector and \( y_l = \sum_{j=1}^{k_l} y_{lj} \). Note that, in order to ensure existence and uniqueness of a Cournot equilibrium in every \( l \)-th sector \( l = (1, m) \), the following assumptions will be considered to hold in the subsequent analysis:

**A.1** the payoff of each firm of a given sector is a function of its own strategy and of the sum of strategies of all existing firms in that sector;

**A.2** strategy sets \( Y_j \) are, for every firm, compact and convex and, in particular, \( Y_j \subset \mathbb{R}_+ = \{0, y_j\} \), where \( y_j \) represents every \( j \)-th firm's production boundary given its capacity constraint;

**A.3** every firm's payoff function, \( \pi_j : Y_j \times \mathbb{R}_+ \rightarrow \mathbb{R}_+ \), is twice continuously differentiable;

**A.4** \( \frac{\partial p_l(y_j)}{\partial y_j} y_j + \frac{\partial p_l(y_j)}{\partial y_l} < 0 \); 

**A.5** \( \frac{\partial p_l(y_j)}{\partial y_l} - \frac{\partial c_j(y_j)}{\partial y_j} < 0 \);

**A.6** \( \pi_j(\ldots) \) is, for every \( l \)-th firm, strictly decreasing on \( y_j \).

**Definition** A Cournot-Nash equilibrium of the multi-sector oligopolistic economy represented here is a vector of quantity \( y_j^* \) for each \( l \)-th sector such that, for every \( j = 1, \ldots, k_l \) in a given sector, the following condition holds:

\[
\pi_j(y_j^*, y_{-j}^*) \geq \pi_j(y_j, y_{-j}^*) \quad \forall y_j \in Y^j, \text{where } y_{-j} \text{ is the sum of all firms' output in a given sector minus } j.
\]

It is well known that from assumptions **A.1-A.5**, the existence of a unique Cournot-Nash equilibrium can be proved (see, for instance, [3]). Consequently, also the multi-sector oligopolistic economy presented here has a unique Nash equilibrium.

In what follows, a further simplification will be made regarding the compensation system adopted by every firm. The basic reservation wage of unemployed workers, i.e., all \( i \in \{I_l \cap U\} \), is equal to zero (because they cannot produce without first being recruited by a firm). Moreover, the entrepreneurs are assumed to pay each worker of their firm a share \( \alpha_i^l \) of the firm's surplus (2), where \( \alpha_i^l \in [0, 1] \) is such that \( \sum_{i \in S_j^l} \alpha_i^l = 1 \). Given these assumptions, we have that for each coalition \( S^l \) the surplus is given by (1) and it is distributed according to the vector \( \alpha_i^l \).

Assuming linear utility for every player, the earnings can be expressed as:
\[ u' = \alpha_i^* \pi_j(y_j, y_j), \forall i \in \{I_L \cap S^H\}, \]
\[ u' = \left(1 - \sum_{i \in I_L \cap S^H} \alpha_i^*\right) \pi_j(y_j, y_j), \forall i \in \{I_K \cap S^H\}. \] (2)

3 The Stability of an Earning Profile

Given the oligopolistic economy described above, a stable earning profile of the economy can be defined as a feasible vector of payments such that, given certain specific assumptions concerning the behavior and the constraints of the individuals of the economy, anyone cannot improve upon. In our framework, given the Nash equilibrium quantities, a stable earning profile is an income distribution within each firm such that none, individually or as a group, wants to leave the firm by establishing a new production unit.

In the economy, under the assumptions made before, the Nash equilibrium surplus of every firm in an industry \( l \) is given by:

\[ \pi_j^* = \left\{ \pi_j(y_j, y_j^*) \right\}_{j=1,2,\ldots,k^l} \]

where, as said before, \( y_j^* \) is the Nash equilibrium quantity vector in every \( l \)-th industry. This vector also determines the partition of the workers belonging to \( I_L \) into two different groups: one group, denoted by \( D^* \), is the set of all workers employed by all \( k^l \) entrepreneurs in each industry at the Nash equilibrium, i.e.,

\[ D^* = \sum_{I_L} \sum_{j \in I^N} \left\{ g^{i^j} \left(y_j^* \right) \right\}; \] the other, conversely, is made of all unemployed workers \( U = I_L \setminus D^* \). Assuming the existence at the Nash equilibrium of a sufficiently large and nonempty set \( U \), in the economy there are many different roles to be potentially undertaken by each individual active in a firm. For each employee the choice is between:

(1a) staying in the firm as employee and negotiating with the entrepreneur a share \( \alpha_i^l \) of the firm's equilibrium profit (1);

(2a) leaving the firm (after being trained) alone (or with other workers), becoming entrepreneur(s) of a new firm in the industry, by recruiting workers either from the same or from another firm (in the same industry), knowing that the entrepreneur in \( S^H \) will recruit other workers from the set \( U \);

(3a) leaving individually the firm and becoming self-employed, producing a certain amount of the product (with only one unit of work), given that the existing \( k^l \) entrepreneurial firms will continue to produce according to their best-reply;
(4a) leaving the firm (after being trained) with other firm's workers and becoming member of a partnership, given that the existing \( k^i \) entrepreneurial firms will continue to produce according to their best-reply;
(5a) entering the unemployed set, obtaining the basic reservation wage of the economy equal to zero. For each entrepreneur, apart from the choice to stay in the firm as entrepreneur, choices (1a)-(5a) described above are similarly feasible. Therefore, in equilibrium, the entrepreneur will never earn less than her or his workers, otherwise she or he would prefer to deviate setting up another firm.

Thus, an earning profile of the economy (a vector \( z \) of remunerations for all the players) can be viewed as stable if, given the partition of the \( N \) players into \((\sum_{k}^T k^i + 1)\) coalitions (\( k^i \) firms \( S^k \) in each industry plus the unemployment group \( U \)) according to the Nash equilibrium quantity vector of every \( l \)-th industry \( y^i_l \), no individual or group of individuals within each firm \( S^k \) can improve upon \( z^i \in \mathbb{E}^{S^k} \) by deviating, where \( \mathbb{E}^{S^k} \) describes the vector of remunerations obtained in equilibrium within every \( S^k \). Note that the allowed deviations are as in (1a)-(5a) above. Moreover, the behaviour of individuals within the complementary coalitions, i.e. all the coalitions \( S^k \setminus T \), (where \( T \) indicates a deviating coalition) is supposed to be that of continuing to produce as before the deviation occurred, according to their best-reply. This is possible through the recruitment of a certain number of workers from the unemployment set \( U \). Moreover, note that only consistent deviations are considered, that is, deviations that cannot, in turn, be objected.

It is now time to describe the formal conditions required to an earning profile \( z \) to be stable. The Nash equilibrium quantity (and consequently the number of \( I_L \) employed in each \( S^k \)) is, by the symmetry of the equilibrium considered, identical for all existing firms in an industry \( l \). Thus, the set of conditions making each firm's earning profile \( z^S^k \) stable can be characterized through a level of share \( \alpha^i \) of the profits (equal by symmetry for all firms of an industry so the superscript \( j \) can be dropped) paid to all employees of a firm of that given industry, i.e., \( \alpha^i = \sum_{e \in I_L \cap S^k} \alpha^i_e \). The level of \( \alpha^i \) can be determined through the respect of the following constraints:
a) No employee of every firm has to find convenient to become entrepreneur and setting up a new firm by hiring an optimal number of workers and paying them a share of the profit sufficient for them to stay. This condition holds when:

\[
\frac{\alpha^i \left( k^i \right) \cdot \pi^*_{ij} \left( k^i \right)}{\ell^*_{ij} \left( k^i \right)} \geq \left( 1 - \alpha^i \left( k^i + s^i + 1 \right) \right) \pi^*_{ij} \left( k^i + s^i + 1 \right)
\]
where $s' = k' \cdot (\Delta \ell^*_\mu (k')) = k' \left( \ell^*_\mu (k') - \ell^*_\mu (k' + 1) \right)$ is the number of firms (positive by assumption A.4 and A.5) created by all workers dismissed from the $k'$ firms as a result of the entry of the new firm; $\alpha' \left(k' + s + 1\right)$ indicates the share of the new firm's profit that the worker become entrepreneur earns in the new firm; $\pi^*_\mu \left(k' + s + 1\right)$ represents the equilibrium profit of the new firm (now the market includes $\left(k' + s + 1\right)$ firms); $\pi^*_\mu \left(k'\right)$ represents each initial firm's equilibrium payoff and $\ell^*_\mu \left(k'\right) = \left\{ I^N \left(y^*_\mu \left(k'\right)\right) \right\}_{\nu \in k'}$ is the number of employees hired by every firm given the initial number of firms in the market. Note that, by symmetry, within each firm, $\alpha' \left(k\right) = \frac{\alpha' \left(k'\right)}{\ell^*_\mu \left(k'\right)}$. Moreover, note that when condition (3) holds, leaving the firm and becoming entrepreneurs it is not even profitable for a group of workers (that thus need to pay the other recruited unemployed workers their equilibrium share);

b) No employee of a firm has to find convenient to become a member of a newly created partnership with some other firm's employees. This condition holds when:

$$\frac{\alpha' \left(k'\right) \cdot \pi^*_\mu \left(k'\right)}{\ell^*_\mu \left(k'\right)} \geq \frac{\pi^*_\mu \left(k' + s + 1\right)}{\ell^*_\mu \left(k' + s + 1\right)}$$

where the RHS represents each partnership member's equilibrium payoff when a new firm of this type (besides the $s'$ induced new entries) enters the market;

c) No employee of a firm has to find convenient to become self-employed:

$$\frac{\alpha' \left(k'\right) \cdot \pi^*_\mu \left(k'\right)}{\ell^*_\mu \left(k'\right)} \geq E^* \left(k' + s'' + 1\right)$$

where $E^*$ is a self-employed equilibrium payoff when a new firm of this type (besides the $s''$ induced new entries) enters the market;

d) None of $k'$ entrepreneurs in each market has to earn less than an employee. This corresponds to the condition:

$$\left(1 - \alpha' \left(k'\right)\right) \pi^*_\mu \left(k'\right) \geq \frac{\alpha' \left(k'\right) \cdot \pi^*_\mu \left(k'\right)}{\ell^*_\mu \left(k'\right)}$$

e) internal consistency:

$$\left(1 - \alpha' \left(k'\right)\right) \pi^*_\mu \left(k'\right) \geq \left(1 - \alpha' \left(k' + s + 1\right)\right) \pi^*_\mu \left(k' + s + 1\right),$$
that expresses the fact that each entrepreneur has to find convenient to pay her or his employees the equilibrium wage (characterized by the share $\alpha' (k')$) rather than let one (or more than one) employee leaving, establishing a new firm and then paying workers the new equilibrium wage. Note that when condition (3) holds with equality, expression (7) implies (6). Therefore, constraint (6) is required just when condition (4) or (5) imply shares $\alpha' (k')$ higher than that respecting condition (3). Moreover, due to the recursive nature of condition (3), a further assumption is needed. In fact, the solution of condition (3) implies that, in each round of entry, the most profitable deviation for employees is always that expressed by the choice (2a). In order to avoid that a choice that is unprofitable in a given round becomes profitable in successive rounds of entry, thus making impossible to solve expression (3), the following condition is imposed in what follows.

**A.7 (No-crossing condition)** Let $\alpha_1, \alpha_2$ and $\alpha_3$ be the share $\alpha' (k')$ that respect condition (3), (4) and (5), respectively, with equal sign. When, for a given number of existing firms $k^l$, $\alpha'(k') = \max \{\alpha_1, \alpha_2, \alpha_3\}$ then the same condition holds for every $t > k'$.

Hence, when the vector $\alpha' (k') = (\alpha_1', \alpha_2', \ldots, \alpha_n')$ respect all conditions listed above, the corresponding earning profile of the economy $\mathbf{z} (\alpha' (k'))$ can be considered to possess some properties of stability. Using (3)-(7) and given an arbitrary initial number $k^l$ of firms existing in each industry, the proposition that follows characterizes the vector $\alpha'$ associated to the stable earning profile $\mathbf{z} (\alpha')$ of the economy. Let us, for ease of notation, denote $I^N (\ell_{x} (k'))$ as $\ell_{x} (k')$.

**Proposition 1** Under no-crossing condition and for a given number of entrepreneurs (and firms) existing in every industry, a stable earnings profile of the economy is characterized by the following share $\alpha'^*$ of profit in every $S^h$ of a given industry:

(i) $\alpha'^* = \frac{\sum_{i=1}^{S} \pi_{\ell_{x}} (k', \ell_{x} (k')) \sum_{j=1}^{S} \Delta t_{j} (\ell_{x} (k') + 1)}{\sum_{j=1}^{S} \Delta t_{j} (\ell_{x} (k') + h)}$

if $\alpha_1 = \max \{\alpha_1, \alpha_2, \alpha_3\}$ and $\alpha'^* = \frac{\pi_{\ell_{x}} (k', \ell_{x} (k'))}{\pi'_{\ell_{x}} (k')}$

(ii) $\min \left\{ \frac{\pi_{\ell_{x}} (k') - E_{\ell_{x}} (k')}{\pi'_{\ell_{x}} (k')} \right\} \geq \alpha'^* = \frac{\pi_{\ell_{x}} (k', \ell_{x} (k') + 1)}{\pi'_{\ell_{x}} (k', \ell_{x} (k') + 1)}$

if $\alpha_2 = \max \{\alpha_1, \alpha_2, \alpha_3\}$ and
their departure and an increase in the existing market competition. Finally, the
reasoning must be done when the best workers' outside option is to be
ensuring, consequently, a lower earning. When the best workers' outside option is
given.

\[ \alpha_i = \frac{\pi^x_1(k^t l^t)\pi^x_1(l^t+1)}{\pi^x_1(l^t)} - \frac{\pi^x_1(k^t l^t+1)\pi^x_1(l^t)}{\pi^x_1(l^t+1)} \]

Proof When every \( \alpha^k \) is equal to the maximum value amongst the three expressions in brackets above in (i), (ii) and (iii), the constraints (3), (4) and (5) are respected for each coalition \( S^j \) of a given industry. When \( \alpha_1 = \max \{ \alpha_1, \alpha_2, \alpha_3 \} \), the first expression for \( \alpha^k \) is obtained by iteratively solving the differential equation in (3) by assuming a finite number of potential entrants \( t = T \). This is the maximum number of available entrants, given that the set of unemployed people is finite. Note that, for \( t = 1, \ldots, T-1 \), we indicate with
\[ \Delta t^x(k^t l^t+1) = \Delta t^x(k^t l^t) - \Delta t^x(k^t l^t+1) \]
the number of workers dismissed by every firm at each round of entry as a consequence of the reduction of output. However, the expression can still be solved assuming an infinite number of potential entrants \( t \). We will consider these different possibilities in the model application. It has to be noticed that when the constraint (3) is satisfied, no coalition of workers would like to become entrepreneur of a new firm. The share
\[ 1 - \alpha^x(k^t+1) \]
ensuring, consequently, a lower earning. When the best workers' outside option is characterized by \( \alpha_2 \) (corresponding to the option to create a partnership), the entire problem must be modified to account for the fact that, in the iterative solution of (5), the best outside option for the employees working in the new entrepreneurial firm created at the first round of entry is now represented by the share \( \alpha_2 \).

Provided this, \( \alpha_1 \) must be modified accordingly, thus yielding \( \alpha_3 \) as expressed above. The share \( \alpha_2 \) is thus nothing but the solution of constraint (4). A similar reasoning must be done when the best workers' outside option is to be self-employed (corresponding to the share \( \alpha_3 \)), which is then used to derive \( \alpha_4 \).

We already said above that the condition is always respected when \( \alpha^k \) respects all constraints (3)-(5). Thus the initial entrepreneurs will always find convenient to pay their employees the equilibrium share of the profit \( \alpha^k(k^t) \) rather than risk their departure and an increase in the existing market competition. Finally, the
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LHS of the expressions above represents the solution of constraints (6) and (7), ensuring that every entrepreneur does not have incentive to deviate or give rise to a self-employed equilibrium. Since \( \alpha' \) is equal to the maximum among the three expressions presented above, by satisfying the tightest of the constraints, the entrepreneur will automatically respect the others. Furthermore, straightforward calculations show that the solution of difference equation (7) is simply given by \( \alpha' \left( k^i \right) \), that is always respected by definition. Thus for the entrepreneurs is always convenient to pay their employees the equilibrium share of the profit \( \alpha' \left( k^i \right) \) rather than risk their departure and an increasing in the existing market competition. When these inequality hold, provided that under the equilibrium share \( \alpha' \) the constraints (3)-(5) are satisfied, the entrepreneurs will never find convenient either to set up a new firm as entrepreneur, partner or self-employed, or to be self-employed from the beginning. In general then, given the partition of the economy determined by the Nash equilibrium, when the vector \( \alpha^* = (\alpha'^1, ..., \alpha'^m) \) respects all the expression represented above, the corresponding earning profile of the economy:

\[
\begin{align*}
\mathbb{Z}(\alpha^*) &= \left\{ \left( 1 - \alpha'^i \right) \pi^{1,i} \cdot \cdots \left( 1 - \alpha'^i \right) \pi^{k,i} \right\} \forall i \in (I_L \cap S^j) \text{ and } l = 1, m; \\
\left\{ \alpha'^i \pi^{1,i} \right\} \forall i \in (I_L \cap S^j) \cdot \cdots \cdot \left\{ \alpha'^i \pi^{k,i} \right\} \forall i \in (I_L \cap S^j) \text{ and } l = 1, m; \left\{ 0 \right\} \forall i \in U. \text{ is stable. } \left\{ 0 \right\} \forall i \in U.
\end{align*}
\]

The proposition just indicates the way to find the stable earning profile of the oligopolistic economy described above. What it is basically stated is that, under simple stability conditions, each firm has to offer a wage sufficiently high to keep its workers inside the firm. This wage, expressed as a share of the firm's profit, has to be high enough to prevent that even the finest subcoalition of the firm, represented by each single worker, can setup a new firm in the form of an entrepreneurial firm, a partnership or a self-employed unit by using the workforce available in the economy.

4 A Linear Application

4.1 A One-sector Economy

In order to show that the workers' equilibrium share of the profits (and then their remuneration) possess the features described in the introduction, let us introduce a simple single-industry example. For simplicity, let the production function of each firm of the industry be a linear function of the number of workers within the firm, that is, \( y^j = 1 \cdot S^j \cap I_L \cdot 1 \). In order to make calculations even simpler, let \( \rho(Y) = a - Y \),
be the linear inverse demand for the homogeneous good, where \( Y = \sum_{j=1}^{k} y^j \) is the total quantity of the good delivered in the market, with \( a > Y \). Straightforward manipulation of the best-reply functions obtained from the maximization of (4), (5) and (6) yield the Nash equilibrium values of the main variables and which of every firm's payoff, as shown in the table below.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>EQUILIBRIUM VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y^i, t^i )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} )</td>
</tr>
<tr>
<td>( Y^* )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} )</td>
</tr>
<tr>
<td>( p(Y) )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} )</td>
</tr>
<tr>
<td>( \pi^i )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} - F )</td>
</tr>
<tr>
<td>( \pi^i + 1 )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} - F )</td>
</tr>
<tr>
<td>( V^{i+1} )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} - F )</td>
</tr>
<tr>
<td>( E^{i+1} )</td>
<td>( \frac{\alpha}{(\pi + \eta_y)} - F )</td>
</tr>
</tbody>
</table>

By proposition 1, using the results of tab 1, the value of \( \alpha^* \) that makes \( z \) core-stable is given by the maximum between:

\[
\alpha_1 = \frac{\pi^i + 1}{\pi^i} \cdot \frac{\eta_y}{(\pi + \eta_y)} \cdot \frac{\alpha}{(\pi + \eta_y)} F \left( \frac{\alpha}{(\pi + \eta_y)} \right), \quad \alpha_2 = \frac{\pi^i + 1}{\pi^i} \cdot \frac{\eta_y}{(\pi + \eta_y)} \cdot \frac{\alpha}{(\pi + \eta_y)} F \left( \frac{\alpha}{(\pi + \eta_y)} \right), \quad \alpha_3 = \frac{\pi^i + 1}{\pi^i} \cdot \frac{\eta_y}{(\pi + \eta_y)} \cdot \frac{\alpha}{(\pi + \eta_y)} F \left( \frac{\alpha}{(\pi + \eta_y)} \right)
\]

subject to the fact that: \( \max \{ \alpha_1, \alpha_2, \alpha_3 \} \leq \alpha_4 = \frac{\frac{\alpha}{(\pi + \eta_y)} F \left( \frac{\alpha}{(\pi + \eta_y)} \right)}{1 + \frac{\alpha}{(\pi + \eta_y)}} \).

The picture below plots the values of \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_4 \) (for given values of parameters) over a variable number of initial existing firms in the economy \((k = 1, \ldots, 100)\). It is immediately visible that, for the values of the parameters selected, \( \alpha_4 \) is greater than the other alphas over all the range of \( k \). This implies that condition (6) is respected and every entrepreneur earns more than her or his employees. Moreover, for a low number of firms in the economy, \( \alpha_2 \) is the maximum among \( \alpha_1, \alpha_2, \alpha_3 \) and then it represents the equilibrium workers' share of the profits in every firm. For a larger number of firms, however, \( \alpha_4 \) becomes the equilibrium share. The values of \( \alpha_4 \) are positively related to \( k \) for a relatively low number of firms.
This depends on the fact that the leaving employee has to pay a lower share to convince the other employees to follow him (in order to keep their remuneration unaffected), the higher the number of firms in the market. However, for a high number of firms \( k \), the negative effect of the number of firms on the equilibrium \( \alpha \) (needed to keep each employee within the firm) prevails on the effect of \( k \). Note that when every firm's profit tends to zero the equilibrium earning share tends to zero as well. This feature ensures that, since the workers' outside option varies with the given partition of the economy, the economy equilibrium earning profile changes according to the monetary value of every firm's market share. When either the dimension of the market (represented by the parameter \( a \)) increases, or the fixed cost \( F \) necessary to set up a firm decreases, or the number of firms in the economy rises, the equilibrium workers compensation decreases as well. This appears to be a basic feature of the model presented.

### 4.2 Two-sector Economy

It is straightforward to extend the results obtained above to an economy in which there is more than one industry. An interesting feature of this extension is that, since the workers' training is specific to every firm belonging to a certain industry, each worker can negotiate a share of his firm's profit by using the threat of setting up a new firm only in that particular industry. This feature makes possible the
existence of an economy earning profile characterized by different levels of compensation for individuals that, although initially homogeneous, operate in different industries. The Figure 2 below depicts the equilibrium $\alpha^{l*}$ in two different industries designed to represent, for instance, a heavy industry ($l = 1$) characterized by a large market size and high fixed costs and a light industry ($l = 2$), with a small market size and low fixed costs. In this example, $\alpha^{l*} = \alpha_3$, that means that the light industry workers' compensation depends on a self-employment leaving threat, while the heavy industry equilibrium employees' share is $\alpha^{2*} = \alpha_2$, that indicates that the partnership-type of deviation is the active threat for the workers of this industry. A result of this extreme specificity of the workers' training across industries is the very high discrepancy between workers' equilibrium compensations between the two industries, even when the number of firms is the same in each industry.

Many other cases are possible in the framework presented. The economy vector of workers' remuneration can vary across industries due to the different number of firms operating in those industries even with the same size of market and equal fixed costs. A certain industry workers' remuneration can be for instance be lower (due to the large number of firms), even in presence of a level of fixed investment higher than that of the other sectors of the economy, and so on.

5 Concluding Remarks

The paper has described an oligopolistic economy in which each worker employed by a firm receives a firm-specific training. This training enables him to
potentially leave the firm in which is employed whenever - given the structure of the sector he belongs to and given his specific competence - he finds the compensation received not satisfactory. By carefully defining the conditions needed to ensure that anyone in the economy, given his payoff, does not want to change his situation through a feasible deviation, a stable earning profile for the economy has been characterized. The wages associated to this earning profile present two features: they are higher than neoclassical reservation wages (that in the model are equal to zero); they turn out to be sensitive to the size of firms' market shares and then in turn to the dimension of fixed costs, that of the market and to the number of firms operating in each industry.

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References


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