Independent Systems of Semigroup Relations and Descriptions of Robotic Systems

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Abstract

In this paper, we consider the problem of derivability of semigroup axioms. In particular, we consider an example of system of semigroup relations without independent subsystem.

Keywords: problem of derivability of axioms, semigroup, independent axiom systems, robotic system

Many algorithmic problems of robotics received a lot of attention recently (see e.g. [1] – [7]). The representation of robotic systems plays an important role in solutions of robotic tasks (see e.g. [8]).

There is a natural way to represent a robotic system by elements of some semigroup. In particular, let 

\{q[1], q[2], \ldots, q[n]\}

be the system of states of a robot,

\{a[1], a[2], \ldots, a[m]\}

be the system of actions of a robot. An action \(a[i]\) causes some state transition from the current state \(q[j]\) to the next state \(q[k]\). This transition can be described by a semigroup relation \(q[j]a[i] = q[k]\). System of such relation we can consider as a description of the robotic system.

As a simple example of such description, we can consider a switch with two states \(q[1], q[2]\) and the only action \(a\). In particular, such switch can be described by the following semigroup:

An axiom system is satisfiable if it has a model. Let \( \Sigma = \{A[1], A[2], \ldots\} \) be a satisfiable system of axioms over a given collection of primitives. An axiom \( A[i] \in \Sigma, i \in \{1, 2, \ldots\} \), is independent if the axiom system

\[
(\Sigma \setminus \{A[i]\}) \cup \{\neg A[i]\}
\]

is satisfiable. The axiom system \( \Sigma \) is independent if each of its axioms is independent.

Note that there is considerable interest in investigation of independent systems of axioms (see e.g. [9]), relations (see e.g. [10]), and identities (see e.g. [11, 12, 13]) for groups, semigroups, rings, and other algebraic systems. In this paper, we consider not independent systems of relations for semigroups.

Let \( \Sigma \) be a system of semigroup relations. Let \( A \in \Sigma \). If \( A \) can be derived from \( \Sigma \setminus \{A\} \), then it is natural to consider \( \Sigma \setminus \{A\} \) instead \( \Sigma \). In general case, we can try to construct an independent system \( \Pi \subseteq \Sigma \) such that \( A \) can be derived from \( \Pi \) for any \( A \in \Sigma \setminus \Pi \). In particular, it is easy to check that \( T \) is given by independent system of relations.

**Theorem.** There is a system of semigroup relations \( \Sigma \) such that \( \Pi \) is not independent for any \( \Pi \subseteq \Sigma \).

**Proof.** Let

\[
S = \langle a, b, 0 \mid ab^n a = 0, b^2 ab = ba, a0 = 0a = b0 = 0b = 0, n \geq 1 \rangle
\]

be a semigroup. Let

\[
\Sigma[k] = \{ab^n a = 0, b^2 ab = ba, a0 = 0a = b0 = 0b = 0, n \geq k\}.
\]

Since

\[
\Sigma[k] \models ab^k a = 0,
\]

it is clear that

\[
\Sigma[k] \models ab^k ab = 0b = 0.
\]

In view of \( b^2 ab = ba \), for any \( k > 1 \),

\[
\Sigma[k] \models ab^k ab = 0
\]

implies

\[
\Sigma[k] \models ab^{k-1} a = 0.
\]

It is easy to see that \( \Sigma[k] \models \Sigma[p] \) for any \( k \geq p \). Therefore, \( \Sigma[k] \) is not independent for any \( k \).
References


Received: September 20, 2012