Melting Effect on Unsteady Heat and Mass Transfer by MHD Mixed Convection Flow over an Impulsively Stretched Vertical Surface in a Quiescent Fluid

S. M. M. EL-Kabeir\textsuperscript{a,b} and A. M. Rashad\textsuperscript{a}

\textsuperscript{a}Department of Mathematics, Aswan University, Faculty of Science, Egypt
am_rashad@yahoo.com

\textsuperscript{b}Department of Mathematics, Salman Bin Abdul Aziz University, College of Science and Humanity Studies, Al-Kharj, KSA

Abstract

The problem of unsteady, laminar, coupled heat and mass transfer by MHD mixed convective boundary-layer flow of electrically conducting fluid over impulsively stretched vertical surface in an unbounded quiescent fluid with aiding external flow in the presence of a transverse magnetic field and melting effects is considered. The flow is impulsively set into motion rest and both of the temperature and concentration at the surface are also suddenly changed from that of the ambient fluid. The governing partial differential equations are transformed into a set of non-similar equations and solved numerically by an efficient implicit, iterative, finite-difference method. A parametric study illustrating the influence of various physical parameters is performed. Numerical results for the steady-state velocity, temperature and concentration profiles as well as the time histories of the skin-friction coefficient, local Nusselt number and the local Sherwood number are presented graphically and discussed.

Keywords: Unsteady boundary-layer, melting effect, magnetohydrodynamics, Heat and Mass Transfer

1. INTRODUCTION

Coupled heat and mass transfer accompanied by melting effect has received much attention in recent years because of its important applications in permafrost
melting, frozen ground thawing, casting and welding processes as well as phase change material (PCM). In manufacturing processes such as hot extrusion, a material such as metals, polymers, ceramics and others is pushed or drawn through a die of the desired cross-section to produce different types of objects. The melting effect is important in hot extrusion or hot working process since it is done above the material's recrystallization temperature to keep the material from work hardening and to make it easier to push the material through the die. Also, in the permafrost research, the melting effect plays an important role in the problems of permafrost melting and frozen ground thawing. According to the analysis of Walker [1], the phenomenon of permafrost degradation in Arctic Alaska is very critical due to global warming and this result accelerates the greenhouse effect. Many studies have been reported concerning the melting process by heat convection mechanism. For example, Kazmierczak et al. [2] presented similarity solutions to analyze the melting phenomenon from a vertical plate in porous medium induced by forced convection of a dissimilar fluid. Hassanien and Bakier [3] studied the melting effect in mixed convection flow from a horizontal flat plate embedded in a porous medium. Gorla et al. [4] changed the arbitrary wall temperature by a uniform wall temperature at the solid-liquid interface to analyze the velocity and temperature fields for aiding flow conditions. Cheng and Lin [5] examined the melting effect on mixed convective heat transfer from a porous vertical plate in liquid-saturated porous medium. They [6] have also examined transient mass transfer in mixed convective heat flow with melting effect from a vertical plate in a liquid-saturated porous medium. Bakier et al. [7] studied hydromagnetic heat transfer by mixed convection from a vertical plate in a liquid-saturated porous medium in the presence of melting effect. Chamkha et al. [8] considered the effects of melting and heat generation or absorption on steady mixed convection from a radiative vertical wall embedded in a non-Newtonian power-law fluid-saturated porous medium.

On the other hand, the study of magnetohydrodynamic (MHD) flow of an electrically conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of MHD flow and heat transfer in any medium due to the effect of a magnetic field on the boundary-layer flow control and on the performance of many systems using electrically conducting fluids. This type of flow has attracted the interest of many researchers due to its application in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. The effect of the magnetic field on the flow over unsteady stretching surface with or without heat and mass transfer was studied by a number of research workers. Chamkha [9] studied the unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching surface immersed in a porous medium. Xu and Liao [10] investigated the unsteady MHD flows of a non-Newtonian fluid over an impulsively stretching flat sheet. EL-Kabeir et al. [11] analyzed the unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium. EL-Kabeir et al. [12] discussed the unsteady MHD three-dimensional by natural convection from an inclined stretching surface.
Melting effect on unsteady heat and mass transfer


The aim of this paper is to study the simultaneous heat and mass transfer by unsteady electrically conducting fluid over impulsively stretched vertical surface in an unbounded quiescent fluid with assisting external laminar flow in the presence of a transverse magnetic field and melting effects. The effects of the governing parameters on the velocity, temperature and concentration profiles as well as the variation of the rate of heat and mass transfer for the whole transient from initial state to final steady-state are presented graphically and analyzed.

2. Problem Formulation

Consider unsteady, laminar, heat and mass transfer by mixed convection, boundary layer flow of an electrically-conducting fluid over a heated vertical linearly stretched sheet with assisting external laminar flow in the magnetic field and melting effects. A uniform magnetic field is applied in the transverse direction y normal to the plate. It is assumed that the wall is impulsively stretched with a velocity $U_e(x) = ax$ which is proportional to the distance along the sheet surface. It is assumed that the temperature of the melting surface is $T_m$, while the temperature at the free stream is $T_\infty$ such that $T_\infty > T_m$. At the same the plate surface is maintained at a constant concentration $C_w$, and the ambient concentration far away from the surface of the sheet $C_e$ is assumed to be uniform such that $C_e > C_w$. Initially $(t < 0)$ the ambient fluid-saturated porous medium is quiescent and its Temperature $T_e$ and concentration $C_e$, respectively. At $t = 0$ the fluid is impulsively started in motion with the velocity $U_e(x)$ and both of the temperature and concentration at the sheet are suddenly changed to a constant values $T_e > T_m$ and $C_e > C_w$, respectively. Further, The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. In addition, the Hall effect and the electric field are assumed negligible. The small magnetic Reynolds number assumption uncouples the Navier-Stokes equations from Maxwell’s equations. All physical properties are assumed constant except the density in the buoyancy force term. By invoking all of the boundary layer, Boussineq approximation, the governing equations for this investigation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$  

(1)
where \( t, x \) and \( y \) represent time, tangential distance, and transverse or normal distance, respectively. \( u, v, T \) and \( C \) are the fluid tangential velocity, normal velocity, and temperature, and concentration, respectively. \( g, \rho, \nu, \alpha \) and \( D \) are the acceleration due to gravity, fluid density, kinematic viscosity, thermal diffusivity and mass diffusivity, respectively. \( \sigma, B, \beta_T \) and \( \beta_c \) are the fluid electrical conductivity, magnetic induction, thermal expansion coefficient and concentration expansion coefficient, respectively.

The corresponding initial and boundary conditions for this problem can be written as:

\[
\begin{align*}
    u &= U_e = \alpha x, \quad T = T_m, \quad C = C_w, \quad k \frac{\partial T}{\partial y} = \rho(\lambda' + C_s(T_m - T_s)) \psi \quad \text{at} \quad y = 0, \\
    u &\to 0, \quad T \to T_m, \quad C \to C_w \quad \text{as} \quad y \to \infty,
\end{align*}
\]  

where \( a \) is a constant and \( C_s \) and \( \lambda \) are the heat capacity of the solid surface, the latent heat of the fluid, respectively. \( T_s, T_m \) and \( C_w \) are the surface temperature, melting temperature and the wall concentration, respectively. \( k, T_m \) and \( C_w \) are the thermal conductivity, ambient temperature and ambient concentration, respectively. Equation (5) states that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the solid surface temperature \( T_s \) to its melting temperature \( T_m \) (see Epstein and Cho [15]). The detailed derivation of Eq. (5) can be found in the paper by Roberts [16].

It is convenient to non-dimensionalize and transform Eqs. (1) through (4) by using \( \psi = (av)^{1/2} \xi^{1/2} \eta (\xi, \eta), \eta = (a/\nu)^{1/2} \xi^{-1/2} y, \xi = 1 - e^{-t'}, t' = at \)

\[
\begin{align*}
    \theta(\xi, \eta) &= \frac{T - T_m}{T_s - T_m}, \quad \phi(\xi, \eta) = \frac{C - C_w}{C_s - C_w}, \quad u = \frac{\partial \psi}{\partial \xi}, \quad \nu = -\frac{\partial \psi}{\partial \eta}.
\end{align*}
\]  

Substituting Eqs. (6) into Eqs. (1) through (4) yields:

\[
\begin{align*}
    f'''' + \frac{1}{2} \eta(1-\xi)f''' + \xi(f'' - f^{-1}f'' - Hdf' + \lambda(\theta + N \phi)) &= \xi(1-\xi) \frac{\partial f'}{\partial \xi}, \\
    \frac{1}{Pr} \theta'' + \frac{1}{2} \eta(1-\xi)\theta' + \xi \theta' &= \xi(1-\xi) \frac{\partial \theta}{\partial \xi}, \\
    \frac{1}{Sc} \phi'' + \frac{1}{2} \eta(1-\xi)\phi' + \xi \phi' &= \xi(1-\xi) \frac{\partial \phi}{\partial \xi},
\end{align*}
\]
where Eq. (1) is identically satisfied. In Eqs. (7)-(9), a prime indicates differentiation with respect to $\eta$ and the parameters

$$
Ha = \frac{\sigma B^2}{\rho a}, \quad M = \frac{C_p(T_\infty - T_m)}{[\lambda' + C_s(T_m - T_s)]}, \quad N = \frac{\beta_c(C_\infty - C_w)}{\beta_l(T_\infty - T_m)},
$$

$$
\lambda = (g \beta(T_\infty - T_m)x^3 / \nu^2)/(U_x / \nu)^2 = \text{Gr}_x / \text{Re}_x^2, \quad \text{Pr} = \frac{\nu}{\alpha}, \quad \text{Sc} = \frac{\nu}{D},
$$

are the Hartmann number, melting parameter, concentration to thermal buoyancy ratio, Prandtl number, mixed convection parameter, Prandtl number and Schmidt number, respectively. It can be noticed that $\lambda > 0$ corresponds to the assisting (aiding) flow case (included in the present results), $\lambda < 0$ to the opposing flow case, where $\text{Gr}_x$ is the local Grashof number and $\text{Re}_x$ is the local Reynolds number.

The transformed initial and boundary conditions become:

$$
f'(\xi, 0) = 0, \quad \theta(\xi, 0) = 0, \quad \phi(\xi, 0) = 0, \quad \text{Pr} \xi f'(\xi, 0) + M \theta'(\xi, 0) = 0
$$

$$
f'(\xi, \infty) = 0, \quad \theta(\xi, \infty) = \phi(\xi, \infty) = 1,
$$

Of special significance for this type of flow and heat transfer situation are the local skin-friction coefficient, local Nusselt number and the local Sherwood number. These physical parameters can be defined in dimensionless form as:

$$
C_{f_x} = -\frac{\mu(\partial u / \partial y)_{y=0}}{\rho U_e^2} = -\text{Re}_x^{-1/2} \xi^{-1/2} f''(\xi, 0),
$$

$$
\text{Nu}_x = -\frac{x(\partial T / \partial y)_{y=0}}{(T_\infty - T_m)} = -\text{Re}_x^{-1/2} \xi^{-1/2} \theta''(\xi, 0),
$$

$$
\text{Sh}_x = -\frac{x(\partial C / \partial y)_{y=0}}{(C_\infty - C_w)} = -\text{Re}_x^{-1/2} \xi^{-1/2} \phi''(\xi, 0),
$$

3. Numerical Method

The initial-value problem represented by Eqs. (7) through (10) is nonlinear and possesses no analytical solution. Therefore, a numerical solution is sought for this problem. The standard implicit, iterative, finite-difference method discussed by Blottner [17] has proven to be adequate and accurate for this type of problems and therefore, it is chosen for the solution of Eqs. (7)-(9) subject to Eqs. (10). The computational domain is divided into 196 by 196 nodes in the $\xi$ and $\eta$ directions, respectively. Since the changes in the dependent variables are large in the immediate vicinity of the plate while these changes decrease greatly as the distance above the plate increases, variable step sizes in the $\eta$ direction are used. For the same reason, variable step sizes in the $\xi$ direction are also employed. The initial step sizes employed were $\Delta \eta_1 = 0.001$ and $\Delta \xi_1 = 0.001$ and the growth
factors were $K_\eta = 1.03$ and $K_\xi = 1.03$ such that $\Delta \eta_n = K_\eta \Delta \eta_{n-1}$ and $\Delta \xi_m = K_\xi \Delta \xi_{m-1}$. The convergence criterion used was based on the relative difference between the current and the previous iterations which was set to $10^{-3}$ in the present work. For more details on the numerical procedure, the reader is advised to read the paper by Blottner [17].

4. Results and Discussion

Figures 1 through 12 represent typical numerical results based on the solution of Eqs. (7)-(10). These results are obtained to illustrate the influences of the Hartmann number, melting parameter, and the mixed convection parameter on the profiles of the fluid tangential velocity and temperature and the concentration as well as the transient developments of the local skin-friction coefficient $C_{fl}$, local Nusselt number $Nu_x$, and the local Sherwood number $Sh_x$. It should be mentioned that in all the results, the conditions are intended for an electrically-conducting fluid such as liquid metal gallium ($Pr=0.007$) polluted by water vapor ($Sc=0.6$) and The value of the corresponding buoyancy force parameter (ratio of the buoyancy force due to mass diffusion to the buoyancy force due to the thermal diffusion) $N$ takes the value 1.0 for low concentration.

**Figure 1:** Effect of $Ha$ on the (a) velocity, (b) temperature, (c) concentration profiles

**Figure 2:** Effect of $Ha$ on the local skin-friction coefficient

**Figure 3:** Effect of $Ha$ on the local Nusselt number

**Figure 4:** Effect of $Ha$ on the local Sherwood number
Figures 1(a)-(c) show typical unsteady-state fluid tangential velocity $f'$, temperature $\theta$ and concentration $\phi$ for various values of the magnetic Hartmann number $Ha$, respectively. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of the flow. This has a tendency to reduce the fluid tangential velocity and its temperature and consequently, the concentration. This is indicative from the decreases in $f'$, $\theta$, and $\phi$ as $Ha$ increases shown in Figures 1(a)-(c), respectively.

Figures 2-4 illustrate the transient development of the local skin-friction coefficient $C_{fx}$, local Nusselt number $Nu_x$ and the local Sherwood number $Sh_x$ for different values of $Ha$, respectively. As mentioned before, increases in $Ha$ cause respective decreases in $f'$, $\theta$, and $\phi$. This results in increasing the slope of the tangential velocity and decreasing the slopes of the temperature and concentration. This has the direct effect of increasing $C_{fx}$ and decreasing both $Nu_x$ and $Sh_x$ due to increases in $Ha$ as depicted in Figures 2-4, respectively. In addition, it is observed that, in general, both of local Nusselt number and local Sherwood number increase as the dimensionless time $\xi$ increases.

Figures 5(a)-(c) present the influence of the melting parameter $M$ on the unsteady-state tangential velocity, temperature and concentration profiles, respectively. It is obvious that increasing the melting parameter $M$ causes higher acceleration to the fluid flow which, in turn, increases its motion and causes decreases in the temperature and concentration profiles. This is accompanied by respective increases in the boundary-layer thicknesses of velocity, temperature and concentration.
Figures 6-8 depict the influence of the melting parameter $M$ on the time histories of the local skin-friction coefficient, local Nusselt number and the local Sherwood number, ($C_{fx}$, $Nu_x$ and $Sh_x$), respectively. It is clear that increasing the value of the melting parameter $M$ yields decreases in all of the local skin-friction coefficient, local Nusselt number and the local Sherwood number because the melting phenomenon is analogous to the blowing effect at vertical surface (Epstein and Cho [15] and Roberts [16]). According to the principle of boundary condition (5), a phenomenon can be expected that the thermal flux of heat conduction to the melting surface is reduced with increasing convection strength due to heat is removed by stronger convection.

Figures 9(a)-(c) depict the influence of increasing the mixed convection parameter $\lambda$ on the behavior of the unsteady profiles of velocity, temperature and concentration, respectively. It can be seen that increasing the value of the mixed
convection parameter $\lambda$ has a tendency to accelerate the flow. This, in turn, produces increases in all of the velocity, temperature and concentration profiles. These behaviors are clearly shown in Figures 9(a)-(c). Moreover, As $\lambda$ increases, hydrodynamic, thermal and solutal boundary layers increase.

Figures 10 and 11 present the effects of the mixed convection parameter $\lambda$ on the time histories of the local skin-friction coefficient $C_{fx}$, local Nusselt number $Nu_x$ and the local Sherwood number $Sh_x$, respectively. As indicated before, increasing the mixed convection parameter $\lambda$ causes increases in the velocity, temperature and concentration profiles causing the negative wall slope of velocity, temperature and concentration profiles to increase. Moreover, for $\lambda>0$ (assisting flow) which is considered in this problem, there is a favorable pressure gradient due to the buoyancy forces which result in flow acceleration and
consequently, there is a larger skin-friction coefficient than in the non-buoyant case ($\lambda\equiv0$). This yields enhancement in the local skin-friction coefficient, local Nusselt and Sherwood numbers.

5. Conclusion

The problem of unsteady, laminar, coupled heat and mass transfer by MHD mixed convective boundary-layer flow of electrically conducting fluid over impulsively stretched vertical surface in an unbounded quiescent fluid with aiding external flow in the presence of a transverse magnetic field and melting effects was formulated. The obtained non-similar differential equations were solved numerically by an efficient implicit finite-difference method. It was found that the local skin-friction coefficient increased as either of the Hartmann number or mixed convection parameter increased, while it decreased as the melting parameter was increased. In addition, the local Nusselt number was increased as either of the melting parameter, Hartmann number or the strength of the magnetic field was increased. Furthermore, the local Sherwood number was increased as mixed convection parameter, while the opposite behavior was obtained as the strength of the magnetic field and the melting parameter were increased. At present our results may not have any direct industrial application. However, they could be useful if the quantitative design procedures for industrial processing operations using the results of fluid-mechanical, rheological and molecular researches are developed.

References


Received: May, 2012