New Exact Solutions for the Modified KdV-KP Equation

Using the Extended F-Expansion Method

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Abstract

A several exact solutions for the modified KdV-KP equation are obtained by means of extended F-expansion method. Interesting Jacobi doubly periodic wave solutions is obtained from the F-expansion (EFE) method with symbolic computation. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions.

Keywords: Extended F-expansion method; Exact solutions; Korteweg De Vries (KdV) equation; Kadomtsov-Petviashivilli (KP) equation; modified KdV-KP equation.

1 Introduction

The well known Korteweg De Vries (KdV) equation firstly observed by John Scott Russell in experiments, and then Lord Rayleigh and Joseph Boussinesq studied it theoretically and, finally, Korteweg and De Vries put it in its form in 1895.

\[ u_t + 6uu_x + u_{xxx} = 0. \]  

(1)

The KdV equation extended to several physical problems such as long internal waves in a density-stratified ocean, acoustic waves on a crystal lattice and so on[1]-[7].

Kadomtsov and Petviashivilli obtained the Kadomtsov-Petviashivilli (KP) equation [8]-[12] as an improvement of the Korteweg-de Vries equation (KdV)

\[ (u_t + 6uu_x + u_{xxx})_x + u_{yy} = 0, \]  

(2)
In both the KdV and the KP equations, waves travel in the positive x-direction. The KP equation can be used as model water waves of long wavelength with weakly non-linear restoring forces and frequency dispersion, model waves in ferromagnetic media.

In this paper we study the modified KdV-KP equation

\[ (u_t - \frac{3}{2} u_x + u^2 u_x + u_{xxx})_x + u_{yy} = 0. \]  

The investigation of exact solutions is the key of understanding the nonlinear physical phenomena. It is known that many physical phenomena are often described by nonlinear evolution equations (NLEEs). Many methods for obtaining exact travelling solitary wave solutions to NLEEs have been proposed. Among these are the tanh methods, the exp-function method, the sub-ODE method, Jacobi elliptic function (JEF) expansion methods, Hirota’s bilinear methods, the solitary wave ansatz method, the inverse scattering transform and so on. Recently F-expansion method was proposed to obtain periodic wave solutions of NLEEs, which can be thought of as a concentration of JEF expansion since F here stands for every one of JEFs.

In this paper, we apply the extended F-expansion (EFE) method with symbolic computation to Eq. (3) for constructing their interesting Jacobi doubly periodic wave solutions. It is shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. In addition the algorithm that we use here also a computerized method, in which generating an algebraic system.

### 2 Extended F-expansion method

In this section, we introduce a simple description of the EFE method, for a given partial differential equation

\[ G(u, u_x, u_y, u_{xy}, u_{xxx}, \ldots) = 0. \]  

We like to know whether travelling waves (or stationary waves) are solutions of Eq. (4). The first step is to unite the independent variables \( x \) and \( t \) into one particular variable through the new variable

\[ \zeta = x + \alpha y + \nu t, \quad u(x, t) = U(\zeta), \]

where \( \nu \) is wave speed, and reduce Eq. (4) to an ordinary differential equation (ODE)

\[ G(U, U', U'', U''', \ldots) = 0. \]  

Our main goal is to derive exact or at least approximate solutions, if possible, for this ODE. For this purpose, let us simply \( U \) as the expansion in the form,
New exact solutions for the modified KdV-KP equation

\[ u(x,t) = U(\zeta) = \sum_{i=0}^{N} a_i F^i + \sum_{i=1}^{N} a_{-i} F^{-i}, \]  

(6)

where

\[ F = \sqrt{A + BF^2 + CF^4}, \]  

(7)

the highest degree of \( \frac{d^p U}{d\zeta^p} \) is taken as

\[ O(\frac{d^p U}{d\zeta^p}) = N + p, \quad p = 1,2,3,\ldots, \]  

(8)

\[ O(U^q \frac{d^p U}{d\zeta^p}) = (q+1)N + p, \quad q = 0,1,2,\ldots, p = 1,2,3,\ldots. \]  

(9)

Where \( A, B \) and \( C \) are constants, and \( N \) in Eq. (5) is a positive integer that can be determined by balancing the nonlinear term(s) and the highest order derivatives. Normally \( N \) is a positive integer, so that an analytic solution in closed form may be obtained. Substituting Eqs. (4)- (7) into Eq. (5) , comparing the coefficients of each power of \( F(\zeta) \) in both sides, getting an over-determined system of nonlinear equations with respect to \( \alpha, \nu, a_0, a_1, \cdots \). Solving the over-determined system of nonlinear equations with Mathematica. The relations between values of \( A, B, C \) and corresponding JEF solution \( F(\zeta) \) of Eq. (6) are given in Table 1. Substitute the values of \( A, B, C \) and the corresponding JEF solution \( F(\zeta) \) chosen from table 1 into the general form of solution, then an ideal periodic wave solution expressed by JEF can be obtained.

Table 1: Relation between values of \((A,B,C)\) and corresponding \(F(\zeta)\)

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(F(\zeta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(-1-m^2)</td>
<td>(m^2)</td>
<td>(\text{sn}(\zeta)) or (\text{cd}(\zeta) = \frac{\text{cn}(\zeta)}{\text{dn}(\zeta)})</td>
</tr>
<tr>
<td>(1-m^2)</td>
<td>(2m^2-1)</td>
<td>(-m^2)</td>
<td>(\text{cn}(\zeta))</td>
</tr>
<tr>
<td>(m^2-1)</td>
<td>(2-m^2)</td>
<td>(-1)</td>
<td>(\text{dn}(\zeta))</td>
</tr>
<tr>
<td>(m^2)</td>
<td>(-1-m^2)</td>
<td>(1)</td>
<td>(\text{ns}(\zeta) \frac{1}{\text{sn}(\zeta)}) or (\text{dc}(\zeta) = \frac{\text{dn}(\zeta)}{\text{cn}(\zeta)})</td>
</tr>
<tr>
<td>(-m^2)</td>
<td>(2m^2-1)</td>
<td>(1-m^2)</td>
<td>(\text{nc}(\zeta) = \frac{1}{\text{cn}(\zeta)})</td>
</tr>
</tbody>
</table>
-1 & 2-m² & m²-1 & \text{nd}(\zeta) = \frac{1}{dn(\zeta)} \\
1 & 2-m² & 1-m² & \text{sc}(\zeta) = \frac{sn(\zeta)}{cn(\zeta)} \\
1 & 2m²-1 & -m²(-1-m²) & \text{sd}(\zeta) = \frac{sn(\zeta)}{dn(\zeta)} \\
1-m² & 2-m² & 1 & \text{cs}(\zeta) = \frac{cn(\zeta)}{sn(\zeta)} \\
-m²(1-m²) & 2m²-1 & 1 & \text{ds}(\zeta) = \frac{dn(\zeta)}{sn(\zeta)} \\
\frac{1}{4} & 1-2m² & \frac{1}{2} & \text{ns}(\zeta) + \text{cs}(\zeta) \\
\frac{1-m²}{4} & \frac{1+m²}{2} & \frac{1-m²}{2} & \text{nc}(\zeta) + \text{sc}(\zeta) \\
\frac{1}{4} & \frac{m²-2}{2} & \frac{m²}{4} & \text{ns}(\zeta) + \text{ds}(\zeta) \\
\frac{m²}{4} & \frac{m²-2}{2} & \frac{m²}{4} & \text{sn}(\zeta) + \text{ics}(\zeta)

Where \( sn(\zeta), \ cn(\zeta) \) and \( dn(\zeta) \) are the JE sine function, JE cosine function and the JEF of the third kind, respectively. And

\[
\begin{align*}
  cn²(\zeta) = 1 - sn²(\zeta), & \quad dn²(\zeta) = 1 - m² sn²(\zeta), \\
\end{align*}
\]

with the modulus \( m \) (0 < m < 1).

When \( m \to 1 \), the Jacobi functions degenerate to the hyperbolic functions, i.e.,

\[
\begin{align*}
  sn\zeta & \to \tanh\zeta, & \quad cn\zeta & \to \sech\zeta, & \quad dn\zeta & \to \sech\zeta, \\
\end{align*}
\]

when \( m \to 0 \), the Jacobi functions degenerate to the triangular functions, i.e.,

\[
\begin{align*}
  sn\zeta & \to \sin\zeta, & \quad cn\zeta & \to \cos\zeta & \quad \text{and} & \quad dn \to 1.
\end{align*}
\]

3 Modifed KdV-KP equation

In this section, we will apply the extended method to study Modified KdV-KP equation (3)
New exact solutions for the modified KdV-KP equation

\[
(u_t - \frac{3}{2}u_x + u^2u_x + u_{xxx})_t + u_{yy} = 0, \tag{11}
\]

if we use \( \zeta = x + \alpha y + \nu t, \quad u(x, t) = U(\zeta) \) carries Eq. (11) into an ODE

\[
(U^{\nu} + 6U^2U^{\nu} + (\nu - \frac{3}{2})U^{\nu}) + \alpha^2 U^{\nu} = 0, \tag{12}
\]

if we integrate Eq. (12) twice we find:

\[
U^{\nu} + 2U^3 + (\alpha^2 + \nu - \frac{3}{2})U = 0. \tag{13}
\]

Balancing the term \( U^{\nu} \) with the term \( U^3 \) we obtain \( N = 1 \) then

\[
U(\zeta) = a_0 + a_1 F + a_{-1} F^{-1}, \quad F = \sqrt{A + BF^2 + CF^4}. \tag{14}
\]

Substituting Eq. (14) into Eq. (13) and comparing the coefficients of each power of \( F \) in both sides, to get an over-determined system of nonlinear equations with respect to \( \nu, \ a_i, \ i = 0, 1, -1 \). Solving the over-determined system of nonlinear equations with Mathematica, we obtain three groups of constants:

1.

\[
a_0 = 0, \quad a_1 = \pm i\sqrt{C}, \quad a_{-1} = \pm i\sqrt{A} \quad \text{and} \quad \nu = \frac{3 - 2B \pm 12\sqrt{AC} - 2\alpha^2}{2}. \tag{15}
\]

2.

\[
a_0 = a_1 = 0, \quad a_{-1} = \pm i\sqrt{A} \quad \text{and} \quad \nu = \frac{3 - 2B - 2\alpha^2}{2}. \tag{16}
\]

3.

\[
a_0 = a_{-1} = 0, \quad a_1 = \pm i\sqrt{C} \quad \text{and} \quad \nu = \frac{3 - 2B - 2\alpha^2}{2}. \tag{17}
\]

Now, the solutions of Eq. (11) can be written as follows:

\[
u_1 = \pm i\sin(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1 + m^2) \pm 12m}{2} t), \tag{18}
\]

\[
u_2 = \pm i\cos(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1 + m^2) \pm 12m}{2} t), \tag{19}
\]
\[ u_\gamma = \pm 0.5i (ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2) \pm 3}{2} t)) + cs(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2) \pm 3}{2} t) \]

\[ \pm 0.5i (ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2) \pm 3}{2} t))^{-1} + cs(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2) \pm 3}{2} t)^{-1} \]
\[ u_6 = \pm \sqrt{0.5m^2 - 0.5} \]
\[ \times (nc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2) \pm 12\sqrt{(0.5 - 0.5m^2)(0.25 - 0.25m^2)}}{2} t)
+ sc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2) \pm 12\sqrt{(0.5 - 0.5m^2)(0.25 - 0.25m^2)}}{2} t)) \]
\[ \pm \sqrt{0.25m^2 - 0.25}\left(nc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2) \pm 12\sqrt{(0.5 - 0.5m^2)(0.25 - 0.25m^2)}}{2} t)
+ sc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2) \pm 12\sqrt{(0.5 - 0.5m^2)(0.25 - 0.25m^2)}}{2} t))^{-1}, \]
\[ (25) \]
\[ u_9 = \pm 0.5i m(ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m}{2} t)
+ ds(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m}{2} t)) \]
\[ \pm 0.5i \left(ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m}{2} t)
+ ds(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m}{2} t))^{-1}, \]
\[ (26) \]
\[ u_{10} = \pm 0.5i m(sn(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m^2}{2} t)
+ ics(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m^2}{2} t)) \]
\[ \pm 0.5i m(sn(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m^2}{2} t)
+ ics(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1) \pm 3m^2}{2} t))^{-1}, \]
\[ (27) \]
\[ u_{11} = \pm ins(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1 + m^2)}{2} t), \]
\[ (28) \]
\[ u_{12} = \pm idc(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1 + m^2)}{2} t), \]
\[ (29) \]
\[ u_{13} = \pm \sqrt{m^2 - 1} nc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2m^2 - 1)}{2} t), \]
\[ (30) \]
\[ u_{14} = \pm \sqrt{1-m^2} nd(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2-m^2)}{2} t), \] (31)
\[ u_{15} = \pm i\sqrt{1-m^2} sc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2-m^2)}{2} t), \] (32)
\[ u_{16} = \pm im\sqrt{1+m^2} sd(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2m^2 - 1)}{2} t), \] (33)
\[ u_{17} = \pm 0.5i(ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5-m^2)}{2} t) \] 
\[ + cs(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5-m^2)}{2} t))^{-1}, \] (34)
\[ u_{18} = \pm \sqrt{0.25m^2 - 0.25} (nc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5+0.5m^2)}{2} t) \] 
\[ + sc(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5+0.5m^2)}{2} t))^{-1}, \] (35)
\[ u_{19} = \pm 0.5i(ns(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t) + ds(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t))^{-1}, \] (36)
\[ u_{20} = \pm 0.5im(sn(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t) + ics(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t))^{-1}, \] (37)
\[ u_{21} = \pm imsn(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1+m^2)}{2} t), \] (38)
\[ u_{22} = \pm imcd(x + \alpha y - \frac{-3 + 2\alpha^2 - 2(1+m^2)}{2} t), \] (39)
\[ u_{23} = \pm mcn(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2m^2 - 1)}{2} t), \] (40)
\[ u_{24} = \mp dn(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2-m^2)}{2} t), \] (41)
\[ u_{25} = \pm ics(x + \alpha y - \frac{-3 + 2\alpha^2 + 2(2-m^2)}{2} t), \] (42)
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\[ u_{26} = \pm ids(x + ay - \frac{-3 + 2\alpha^2 + 2(2m^2 - 1)}{2} t), \]

\[ u_{27} = \pm 0.5i(ns(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2)}{2} t) + cs(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5 - m^2)}{2} t)), \]

\[ u_{28} = \pm \sqrt{0.5m^2 - 0.5}(nc(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2)}{2} t) + se(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5 + 0.5m^2)}{2} t)), \]

\[ u_{29} = \pm 0.5im(ns(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t) + ds(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t)), \]

\[ u_{30} = \pm 0.5im(sn(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t) + is(x + ay - \frac{-3 + 2\alpha^2 + 2(0.5m^2 - 1)}{2} t)). \]
Fig. 1 The modulus of solitary wave solution $u_t$ (Eq. 18) where $\alpha = m = 0.5$. 
New exact solutions for the modified KdV-KP equation

Fig. 2 The modulus of solitary wave solution $u_2$ (Eq. 19) where $\alpha = m = 0.5$. 
Fig. 3 The modulus of solitary wave solution $u_3$ (Eq. 20) where $\alpha = m = 0.5$. 
New exact solutions for the modified KdV-KP equation

The modulus of solitary wave solution \( u_4 \) (Eq. 21) where \( \alpha = \beta = \epsilon = \gamma = \lambda = m = \delta = 0.5 \).

The modulus of solutions \( u_1, u_2, u_3 \) and \( u_4 \) are displayed in figures 1, 2, 3 and 4 respectively, with values of parameters listed in their captions.

### 3.1 Soliton solutions

Some solitary wave solutions can be obtained, if the modulus \( m \) approaches to 1 in Eqs. (18)-(47)

\[
\begin{align*}
 u_{31} & = \pm i \tanh (x + \alpha y - \frac{2\alpha^2 - 7}{2} t), \\
 u_{32} & = \pm \text{sech} (x + \alpha y - \frac{2\alpha^2 - 1}{2} t), \\
 u_{33} & = \pm i \text{csch} (x + \alpha y - \frac{2\alpha^2 - 1}{2} t), \\
 u_{34} & = \pm i \coth (x + \alpha y - \frac{2\alpha^2 - 7}{2} t), \\
 u_{35} & = \pm i \sqrt{2} \sinh (x + \alpha y - \frac{2\alpha^2 - 1}{2} t), \\
 u_{36} & = \pm i \tanh (x + \alpha y - \frac{2\alpha^2 - 7 \pm 12}{2} t) \pm i \coth (x + \alpha y - \frac{2\alpha^2 - 7 \pm 12}{2} t), \\
 u_{37} & = \pm i \text{csch} (x + \alpha y - \frac{2\alpha^2 - 1 \pm 12\sqrt{2}}{2} t) \pm i \sqrt{2} \sinh (x + \alpha y - \frac{2\alpha^2 - 1 \pm 12\sqrt{2}}{2} t),
\end{align*}
\]
\[ u_{38} = \pm 0.5i (\coth(x + \alpha y - (\alpha^2 - 2)t) + \csch(x + \alpha y - (\alpha^2 - 2)t))^{-1}, \quad \text{(55)} \]
\[ u_{39} = \pm 0.5i (\tanh(x + \alpha y - (\alpha^2 - 2)t) + i\csch(x + \alpha y - (\alpha^2 - 2)t))^{-1}, \quad \text{(56)} \]
\[ u_{40} = \pm 0.5i (\coth(x + \alpha y - (\alpha^2 - 2)t) + \csch(x + \alpha y - (\alpha^2 - 2)t)), \quad \text{(57)} \]
\[ u_{41} = \pm 0.5i (\tanh(x + \alpha y - (\alpha^2 - 2)t) + i\csch(x + \alpha y - (\alpha^2 - 2)t)), \quad \text{(58)} \]
\[ u_{42} = \pm 0.5i (\coth(x + \alpha y - \frac{2\alpha^2 - 4\pm 3}{2} t) + \csch(x + \alpha y - \frac{2\alpha^2 - 4\pm 3}{2} t))^{-1}, \quad \text{(59)} \]
\[ u_{43} = \pm 0.5i (\tanh(x + \alpha y - \frac{2\alpha^2 - 4\pm 3}{2} t) + i\csch(x + \alpha y - \frac{2\alpha^2 - 4\pm 3}{2} t))^{-1}, \quad \text{(60)} \]

### 3.2 Triangular periodic solutions

Some trigonometric function solutions can be obtained, if the modulus \( m \) approaches to zero in Eqs. (18)-(46)

\[ u_{44} = \pm \cot(x + \alpha y - \frac{2\alpha^2 - 1}{2} t), \quad \text{(61)} \]
\[ u_{45} = \pm \sin(x + \alpha y - \frac{2\alpha^2 - 5}{2} t), \quad \text{(62)} \]
\[ u_{46} = \pm \tan(x + \alpha y - \frac{2\alpha^2 + 1}{2} t), \quad \text{(63)} \]
\[ u_{47} = \pm \csc(x + \alpha y - \frac{2\alpha^2 - 5}{2} t), \quad \text{(64)} \]
\[ u_{48} = \pm \sec(x + \alpha y - \frac{2\alpha^2 - 5}{2} t), \quad \text{(65)} \]
\[ u_{49} = \pm \cot(x + \alpha y - \frac{2\alpha^2 + 1\pm 12}{2} t) \pm \tan(x + \alpha y - \frac{2\alpha^2 + 1\pm 12}{2} t), \quad \text{(66)} \]
\[ u_{50} = \pm 0.5i (\csc(x + \alpha y - (\alpha^2 - 1)t) + \cot(x + \alpha y - (\alpha^2 - 1)t))^{-1}, \quad \text{(67)} \]
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$$u_{s_1} = \pm \frac{i}{2} (\sec(x + \alpha y - (\alpha^2 - 1)t) + \tan(x + \alpha y - (\alpha^2 - 1)t))^{-1},$$

(68)

$$u_{s_2} = \pm 0.5i (\csc(x + \alpha y - (\alpha^2 - 1)t) + \cot(x + \alpha y - (\alpha^2 - 1)t)),$$

(69)

$$u_{s_3} = \pm i \sqrt{0.5} (\sec(x + \alpha y - (\alpha^2 - 1)t) + \tan(x + \alpha y - (\alpha^2 - 1)t)),$$

(70)

$$u_{s_4} = \pm 0.5i (\csc(x + \alpha y - \frac{2\alpha^2 - 2\pm 3}{2}t) + \cot(x + \alpha y - \frac{2\alpha^2 - 2\pm 3}{2}t)),$$

(71)

$$\pm 0.5i (\csc(x + \alpha y - \frac{2\alpha^2 - 2\pm 3}{2}t) + \cot(x + \alpha y - \frac{2\alpha^2 - 2\pm 3}{2}t))^{-1},$$

$$u_{s_5} = \pm i \sqrt{0.5} (\sec(x + \alpha y - (\alpha^2 - 1\pm \frac{3}{\sqrt{2}})t) + \tan(x + \alpha y - (\alpha^2 - 1\pm \frac{3}{\sqrt{2}})t)),$$

(72)

$$\pm \frac{i}{2} (\sec(x + \alpha y - (\alpha^2 - 1\pm \frac{3}{\sqrt{2}})t) + \tan(x + \alpha y - (\alpha^2 - 1\pm \frac{3}{\sqrt{2}})t))^{-1},$$

4 Conclusion

we have been able to obtain a unified way with the aid of symbolic computation system-mathematica, a series of solutions including single and the combined Jacobi elliptic function. Also, we have shown that soliton solutions and triangular periodic solutions can be established as the limits of Jacobi doubly periodic wave solutions. When $m \to 1$, the Jacobi functions degenerate to the hyperbolic functions and given the solutions by the extended hyperbolic functions methods. When $m \to 0$, the Jacobi functions degenerate to the triangular functions and given the solutions by extended triangular functions methods.

References


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