Delay Dependent Robust Stability of T-S Fuzzy Systems with Additive Time Varying Delays

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Abstract

This paper presents delay dependent stability conditions of T-S fuzzy systems with additive time varying delays. The approach is based on constructing a new Lyapunov-Krasovskii functional, and Finsler’s lemma. The perturbations considered are norm bounded and the results are expressed in terms of LMIs. Numerical examples are provided to show the effectiveness of the present technique, compared to some recent results.

**Keywords:** Additive time varying delays, T-S fuzzy system, Linear matrix inequality (LMI), Robust stability

1. Introduction

During the past two decades the stability analysis for Takagi-Sugeno (T-S) fuzzy systems [23] has been studied extensively. Lots of stability criteria of T-S fuzzy systems have been expressed in linear matrix inequality (LMI) via different approaches [6, 24, 26]. These fuzzy systems are described by a family of fuzzy IF–THEN rules. However, all the aforementioned methods are proposed for time-delay free T–S fuzzy systems. In practice time delay often appears in many practical systems such as chemical processes, metallurgical process, long transmission lines in pneumatic, mechanics, and communications networks, etc. [1,9]. Since time delay, is usually a source of instability and degradation of
systems performance, the analysis and synthesis issues of fuzzy systems with time
delay has received more attention in recent years [13, 5, 16, 19].
Some approaches developed for general delay systems have been applied to deal
with fuzzy systems with time delays, e.g., the Lyapunov–Krasovskii functional
approach [2, 3], Li et al. [13]. Applying the model transformation also called
Moon’s inequality [18] for bounding cross terms, Guan and Chen [4] have studied
the delay-dependent robust stability and guaranteed cost control of the time-delay
fuzzy systems. Recently a free weighting matrix approach has been employed in
[5, 16, 11, 15]. In Liu et al. [17], the problem of stability for uncertain T-S fuzzy
systems with time varying delay has been studied by employing a further
improved free weighting matrix method. The free weighting matrix approach has
been shown to be less conservative than the previous approaches.
In the literature the fuzzy systems with time-varying delay have been modelled as
a system with a single delay term in the state. Recently in [8, 12, 22], it was noted
that in networked controlled system, successive delays with different properties
are introduced in the transmission of signals between different points through
different segments of networks. Thus it is appropriate to consider different time-
delays $\tau_1(t)$ and $\tau_2(t)$ in the same state where, $\tau_1(t)$ is the time-delay induced
from sensor to controller and $\tau_2(t)$ is the delay induced from controller to the
actuator.
In this work, motivated by the above idea, we derive a new and improved delay-
dependent condition for asymptotic stability of T-S fuzzy system with two
additive delay components. The condition is extended to cover systems with norm
bounded uncertainties. The sufficient conditions for asymptotic stability and
robust stability analysis are derived by using Lyapunov-Krasovskii functional
method and making use of improved technique and Finsler’s lemma. By solving a
set of LMIs, the upper bounds of the time delays can be obtained. We provide two
illustrative examples to show that the new stability conditions proposed in this
paper are less conservative.

**Lemma 1** [7]: Consider a vector $\chi \in \mathbb{R}^n$, a symmetric matrix $Q \in \mathbb{R}^{n \times n}$ and
matrix $B \in \mathbb{R}^{m \times n}$, such that $\text{rank } (B) < n$. The following statements are equivalent:

i. $\chi^T Q \chi < 0$, $\forall \chi$ such that $B\chi = 0$, $\chi \neq 0$
ii. $B^T Q B < 0$
iii. $\exists \mu \in \mathbb{R} : Q - \mu B^T B < 0$
iv. $\exists F \in \mathbb{R}^{m \times n} : Q + F B + B^T F^T < 0$

Where $B^\perp$ denotes a basis for the null-space of $B$

**Lemma 2** [14]: for any constant matrix $M = M^T \in \mathbb{R}^{n \times n}$, $M > 0$, scalar
$\gamma \geq \eta(t) > 0$, vector function $\omega : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations in the
following are well defined, then:
\[ \eta(t) \int_0^{\eta(t)} \omega^T(\beta)M\omega(\beta)d\beta \geq \left[ \int_0^{\eta(t)} \omega(\beta)d\beta \right]^T M \left[ \int_0^{\eta(t)} \omega(\beta)d\beta \right] \]

**Lemma 3** [20]: Let \( Q = Q^T, \ H, \ E, \) and \( F(t) \) satisfying \( F^T(t)F(t) \leq I \) are appropriately dimensioned matrices, the following inequality:

\[
Q + HF(t)E + E^TF^T(t)H^T < 0
\]

Is true, if and only if the following inequality holds for any matrix \( Y > 0, \)

\[
Q + HY^{-1}H^T + E^TYE < 0.
\]

### 2. System description

Consider a T–S fuzzy time-varying delay system, which can be described by a T–S fuzzy model, composed of a set of fuzzy implications, and each implication is expressed by a linear system model. The \( i \)th rule of the T–S fuzzy model is described by following IF – THEN form:

**Plant Rule i:**

\[
\text{IF } z_1(t) \text{ is } W^i_1 \text{ and } \ldots \text{ and } z_g(t) \text{ is } W^i_g \text{ THEN }
\]

\[
\begin{align*}
\dot{x}(t) &= (A_{o_i} + \Delta A_{o_i}(t))x(t) + (A_{d_i} + \Delta A_{d_i}(t))x(t-h_1(t) - h_2(t)) \\
x(t) &= \phi(t), t \in [-\bar{h},0], i = 1,2,\ldots, r
\end{align*}
\]

(1)

where \( z_1(t), z_2(t), \ldots, z_g(t) \) are the premise variables, and \( W^i_j, j = 1,2,\ldots,g \) are fuzzy sets, \( x(t) \in \mathbb{R}^n \) is the state variable, \( r \) is the number of if-then rules, \( \phi(t) \) is a vector-valued initial condition, \( h_1(t) \) and \( h_2(t) \) is the time-varying delays satisfying

\[
0 \leq h_1(t) \leq \bar{h}_1, \quad 0 \leq h_2(t) \leq \bar{h}_2, \quad \hat{h}_1(t) \leq d_1, \quad \hat{h}_2(t) \leq d_2, \quad \bar{h} = \bar{h}_1 + \bar{h}_2 \quad \text{and} \quad d = d_1 + d_2
\]

(2)

The parametric uncertainties \( \Delta A_{o_i}(t) \) and \( \Delta A_{d_i}(t) \) are time-varying matrices with appropriate dimensions, which can be described as:

\[
\begin{bmatrix}
\Delta A_{o_i}(t) \\
\Delta A_{d_i}(t)
\end{bmatrix} = D_i F_i(t) \begin{bmatrix}
E_{o_i} \\
E_{d_i}
\end{bmatrix}, i = 1,2,\ldots,r
\]

(3)

Where \( D_i, E_{o_i}, E_{d_i} \) are known constant real matrices with appropriate dimensions and \( F_i(t) \) are unknown real time-varying matrices with Lebesgue measurable elements bounded by:

\[
F_i^T(t)F_i(t) \leq I, \quad i = 1,2,\ldots,r
\]

(4)

By using the center-average defuzzifier, product inference and singleton fuzzifier, the global dynamics of T-Z fuzzy system (1) can be expressed as
\[ \dot{x}(t) = \sum_{i=1}^{r} \mu_i(z(t))[(A_{0i} + \Delta A_{0i}(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - h_1(t) - h_2(t))] \quad (5) \]

Where,

\[ \mu_i(z(t)) = \omega_i(z(t)) / \sum_{i=1}^{r} \omega_i(z(t)), \quad \omega_i(z(t)) = \prod_{j=1}^{g} W_{ij}(z_j(t)) \]

And \( W_{ij}(z_j(t)) \) is the membership value of \( z_j(t) \) in \( W_j \), some basic properties of \( \mu_i(z(t)) \) are \( \mu_i(z(t)) \geq 0, \sum_{i=1}^{r} \mu_i(z(t)) = 1 \).

3. Main results

In this section, we will obtain the stability criteria for T-S fuzzy time-varying delay systems with two additive time varying delay based on a new Lyapunov-Krasovskii functional approach. First the following nominal system of system (5) will be considered:

\[
\begin{cases}
\dot{x}(t) = A_0x(t) + A_dx(t - h_1(t) - h_2(t)) \\
x(t) = \phi(t), \, t \in [-\bar{h},0]
\end{cases}
\]

Where \( A_0 = \sum_{i=1}^{r} \mu_i(z(t))A_{0i} \) and \( A_d = \sum_{i=1}^{r} \mu_i(z(t))A_{di} \)

**Theorem 1:** The system described by (6) and satisfying conditions (2) is asymptotically stable if there exist symmetric positive definite matrices \( P, Q_1, Q_2, R_1, R_2 \) and any appropriately dimensioned matrices, \( F_0, F_1, F_2 \), such that \( R_1-R_2>0 \) and the following LMIs are feasible for \( i = 1,2,\ldots,r \)

\[
\begin{bmatrix}
\Phi_{11} & \frac{1}{h_1}Q_1 & F_0A_{di} + A_{di}^TF_1^T & P - F_0 + A_{di}^TF_2^T \\
\ast & \Phi_{22} & \frac{1}{h_2}Q_2 & 0 \\
\ast & \ast & \Phi_{33} & -F_1 + A_{di}^TF_2^T \\
\ast & \ast & \ast & \Phi_{44}
\end{bmatrix} < 0
\]

Where,

\[ \Phi_{11} = -\frac{1}{h_1}Q_1 + R_1 + F_0A_{di} + A_{di}^TF_0^T \]
Proof: Define the following Lyapunov–Krasovskii functional

\[ V(x_i) = V_1(x_i) + V_2(x_i) + V_3(x_i) \] (8)

where,

\[ V_1(x_i) = x^T(t)Px(t) \] (9)

\[ V_2(x_i) = \int_0^t \int_{s=-h_i}^{s=t} \dot{x}^T(s)Q_1\dot{x}(s)dsdt + \int_{s=-h_i}^{s=t} \int_{s=-h_2}^{s=s} \dot{x}^T(s)Q_2\dot{x}(s)dsdt \] (10)

\[ V_3(x_i) = \int_{t-h_i(t)}^{t} x^T(s)R_1x(s)ds + \int_{t-h_i(t)-h_2(t)}^{t} x^T(s)R_2x(s)ds \] (11)

Computing the time derivative of (9)-(11) one obtains,

\[ \dot{V}_1(x_i) = 2\dot{x}^T(t)Px(t) \] (12)

\[ \dot{V}_2(x_i) = \int_0^t \left( \dot{x}^T(t)Q_1\dot{x}(t) - \dot{x}^T(t+h_i(t))Q_1\dot{x}(t+h_i(t)) \right)dt + \int_{s=-h_i}^{s=t} \int_{s=-h_2}^{s=s} \dot{x}^T(s)Q_2\dot{x}(s)dsdt \]

\[ = \bar{h}_1\dot{x}^T(t)Q_1\dot{x}(t) - \int_{t-h_i(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds + \bar{h}_2\dot{x}^T(t)Q_2\dot{x}(t) - \int_{t-h_i(t)-h_2(t)}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds \] (13)

For any symmetric positive definite matrices \( Q_1 \) and \( Q_2 \) the following inequalities always hold, see [22].

\[ - \int_{t-h_i(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \leq - \int_{t-h_i(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds 
\]

\[ - \int_{t-h_2(t)}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds \leq - \int_{t-h_2(t)}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds 
\]

Where \( h(t) = h_1(t) + h_2(t) \)

Applying the above inequalities to the integral terms in (13) one obtains,

\[ \dot{V}_2(x_i) \leq \bar{h}_1\dot{x}^T(t)Q_1\dot{x}(t) - \int_{t-h_i(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds + \bar{h}_2\dot{x}^T(t)Q_2\dot{x}(t) - \int_{t-h_i(t)-h_2(t)}^{t} \dot{x}^T(s)Q_2\dot{x}(s)ds \] (14)

By using lemma 2 we obtain:
\[ V_2(x(t)) \leq \frac{1}{\lambda_1} [x(t) - x(t-h_1(t))]^T Q_1 [x(t) - x(t-h_1(t))] + \frac{1}{\lambda_2} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ \leq \frac{1}{\lambda_1} [x(t) - x(t-h_1(t))]^T Q_1 [x(t) - x(t-h_1(t))] + \frac{1}{\lambda_2} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ \leq \frac{1}{\lambda_1} [x(t) - x(t-h_1(t))]^T Q_1 [x(t) - x(t-h_1(t))] + \frac{1}{\lambda_2} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

\[ + \frac{1}{\lambda_1} [x(t-h_1(t)) - x(t-h(t))]^T Q_2 [x(t-h_1(t)) - x(t-h(t))] \]

Where we assume that \( R_1 > R_2 \). Let

\[ \xi(t) = \begin{pmatrix} x(t) & x(t-h_1(t)) & x(t-h_2(t)) & \dot{x}(t) \end{pmatrix}^T \]

Taking account of (12), (15) and (16), and letting

\[ \Psi = \begin{pmatrix} \frac{1}{\lambda_1} Q_1 + R_1 & \frac{1}{\lambda_1} Q_1 & 0 & P \\ \frac{1}{\lambda_1} Q_1 & \frac{1}{\lambda_1} Q_1 & \frac{1}{\lambda_1} Q_1 & 0 \\ 0 & 0 & \frac{1}{\lambda_1} Q_2 & 0 \\ \frac{1}{\lambda_2} Q_2 & 0 & \frac{1}{\lambda_2} Q_2 & \frac{1}{\lambda_2} Q_2 \end{pmatrix} \]

We obtain:

\[ \dot{V}(x(t)) \leq \xi(t)^\top \Psi \xi(t) \] (18)

Now let \( \tilde{B} = [A_0 \ 0 \ A_d \ -1] \) and \( F = [F_0^T \ F_1^T \ F_2^T]^T \). Then, since

\[ \sum_{i=1}^{r} \mu_i(z(t)) = 1 \]

we can verify that \( \tilde{B} \xi = 0, \ \forall \xi \neq 0 \). Since condition (7) holds, it follows that the matrices \( \Psi + \tilde{B} \tilde{F} + \tilde{B}^\top \tilde{F}^\top < 0 \) and therefore by lemma 1 we have \( \xi(t)^\top \Psi \xi(t) < 0 \) which implies that \( \dot{V}(x(t)) < 0 \). This completes the proof. \( \square \)
Theorem 2: The uncertain system (5) satisfying conditions (2) is robustly stable if there exist symmetric positive definite matrices \( P, Q_1, Q_2, R_1, R_2, Y \) and any appropriately dimensioned matrices, \( F_0, F_1, F_2 \), such that \( R_1 - R_2 > 0 \) and the following LMIs are feasible for \( i = 1, \ldots, r \):

\[
\begin{bmatrix}
\Phi_{11} + E_{0i}^T Y E_{0i} & \frac{1}{\ell_1} Q_1 & F_0 A_{di} + A_{0i}^T F_1 & E_{0i}^T Y E_{di} & P - F_0 + A_{0i}^T F_2 & F_0 D_i \\
* & \Phi_{22} & \frac{1}{\ell_2} Q_2 & 0 & 0 \\
* & * & \Phi_{33} + E_{di}^T Y E_{di} & -F_1 + A_{di}^T F_2 & F_1 D_i \\
* & * & * & \Phi_{44} & F_2 D_i \\
* & * & * & * & -Y
\end{bmatrix} < 0 \quad (19)
\]

Where \( \Phi_{11}, \Phi_{22}, \Phi_{33} \) and \( \Phi_{44} \) are defined in (7).

Proof: Replacing \( A_{0i} \) and \( A_{di} \) by \( A_{0i} + D_i F_i(t) E_{0i} \) and \( A_{di} + D_i F_i(t) E_{di} \) in (7), respectively, the corresponding formula of (7) for system (5) can be rewritten as follows:

\[
\Phi_i + H F_i(t) E + E^T F_i^T(t) H^T < 0 \quad (20)
\]

Where \( H^T = \begin{bmatrix} D_i^T F_i^T & 0 & D_i^T F_i^T & D_i^T F_i^T \end{bmatrix} \) and \( E = \begin{bmatrix} E_{0i} & 0 & E_{di} & 0 \end{bmatrix} \). According to Lemma 3, (20) is true if there exist \( Y > 0 \), such that the following inequality holds:

\[
\Phi_i + H Y^{-1} H^T + E^T Y E < 0 \quad (21)
\]

By Schur complement, (21) is equivalent to (19). This completes the proof.

Remark 1. To the best of our knowledge, all the results studying T-S fuzzy systems with time delay consider systems with single delay term as:

\[
\begin{align*}
\text{IF } z_i(t) & \text{ is } W_i^1 \text{ and } \ldots \text{ and } z_\ell(t) \text{ is } W_\ell^1 \text{ THEN } \\
\dot{x}(t) & = (A_{0i} + \Delta A_{0i}(t)) x(t) + (A_{di} + \Delta A_{di}(t)) x(t - h(t)) \\
x(t) & = \phi(t), t \in [0,0], i = 1,2,\ldots,r
\end{align*}
\]

Where \( 0 \le h(t) \le \bar{h} \) and \( 0 < \dot{h}(t) \le d \), and there is no results dealing with additive time varying delay.

Remark 2. For time delay systems with single delay term, free weighting matrices approach has been used in [5, 16, 15, 17, 25] and less conservative results have been established compared with Moon’s inequality approach employed in [4, 10]. In [19] stability criteria for T-s Fuzzy systems with delay...
have been developed by employing neither free weighting matrices nor model
transformation and derived less conservative results than those in the above
references. In this paper, we use a new Lyapunov Krasovskii functional and our
method is based on Finsler’s Lemma. Comparing theorem 1 with corollary 2 of
[19], concerning the stability of system (6) with $h_1(t) + h_2(t) = h(t)$ and
$0 \leq h(t) \leq \bar{h}$, the numbers of variables required in theorem 1 and corollary 2 are
$3n^2 + \frac{5n(n+1)}{2}$ and $6n^2 + 2n$ respectively. It can be seen that theorem 1
requires less number of variable that is $\frac{n(n-1)}{2}$. Consequently, with our results
the computational demand on searching for the solution of stability conditions can
be alleviated. This advantage can be revealed especially for systems with large
dimension $n$. A second advantage of our approach is that we expect a reduced
conservatism. This is illustrated in the examples.

4. Numerical examples

In this section, we aim to demonstrate the effectiveness of the proposed approach
presented in this paper by theorem 1 and theorem 2.

Example 1: Consider a system with the following rules:

Rule 1: If $z_1(t)$ is $W_1$, then

$$\dot{x}(t) = A_{a1}x(t) + A_{d1}x(t - h_1(t) - h_2(t))$$

If $z_2(t)$ is $W_2$, then

$$\dot{x}(t) = A_{a2}x(t) + A_{d2}x(t - h_1(t) - h_2(t))$$

And the membership functions for rule 1 and rule 2 are

$$\mu_1(z_1(t)) = \frac{1}{1 + \exp(-2z_1(t))}, \mu_2(z_1(t)) = 1 - \mu_1(z_1(t))$$

Where,

$$A_{a1} = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, A_{a2} = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}$$

Applying Theorem 1, we fix different values of $\bar{h}_1$ and search for the corresponding
upper bounds $\bar{h}_2$ of $h_2(t)$. Hence we fix different values of $\bar{h}_2$ and search for the
corresponding upper bounds $\bar{h}_1$ of $h_1(t)$. In order to compare with the literature
results, since to our knowledge, there is no results dealing with fuzzy systems with additive delays, we let \( h_1(t) + h_2(t) = h(t) \) such that \( 0 \leq h(t) \leq \overline{h} \). Then we apply the literature conditions. From our results we compute \( \overline{h} \) as in (2). The results are summarized in table 1.

<table>
<thead>
<tr>
<th>Method</th>
<th>Upper bound ( \overline{h} )</th>
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<tbody>
<tr>
<td>Wang et al. [21]</td>
<td>1.597</td>
</tr>
<tr>
<td>Tian and Peng [25] Corollary 1</td>
<td>1.597</td>
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<tr>
<td>Chen et al. [5]</td>
<td>1.597</td>
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<tr>
<td>Fan et al. [17]</td>
<td>1.597</td>
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<tr>
<td>This paper</td>
<td>upper bound ( \overline{h}_1 ) for given ( \overline{h}_1 ) upper bound ( \overline{h}_2 ) for given ( \overline{h}_2 )</td>
</tr>
<tr>
<td>Theorem 1 of this paper</td>
<td>( \overline{h}_1 = 1 ) ( \overline{h}_1 = 1.2 ) ( \overline{h}_1 = 1.5 ) ( \overline{h}_2 = 0.2 ) ( \overline{h}_2 = 0.3 ) ( \overline{h}_2 = 0.5 )</td>
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<td>1.323</td>
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We can see that the maximum allowable upper bound, \( \overline{h} = \overline{h}_1 + \overline{h}_2 \) obtained by theorem 1 is greater than the bound \( \overline{h} \) obtained by the results of [21, 25, 5, 17]. Our approach leads to less restrictive condition although computing \( \overline{h} = \overline{h}_1 + \overline{h}_2 \) may be conservative, in fact the delays \( h_1(t) \) and \( h_2(t) \) may have sharply different properties and when \( h_1(t) + h_2(t) \) reaches its maximum, we do not necessarily have both \( h_1(t) \) and \( h_2(t) \) reach their maximum at the same time.

**Example 2:** Consider the following uncertain fuzzy system with two additive time varying delay:

\[
\dot{x}(t) = \sum_{i=1}^{2} \mu_i(z_i(t)) \left[ (A_{0i} + \Delta A_{0i}(t))x(t) + (A_{di} + \Delta A_{di}(t))x(t - h_1(t) - h_2(t)) \right]
\]

where,

\[
A_{0i} = \begin{bmatrix} -2 & 1 \\ 0.1 & -1 \end{bmatrix}, \quad A_{di} = \begin{bmatrix} -1 & 0.5 \\ -1 & -1 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -1.6 & 0 \\ 0 & -1 \end{bmatrix}
\]

\[
E_{0i} = \begin{bmatrix} 1.6 & 0 \\ 0 & 0.05 \end{bmatrix}, \quad E_{di} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad E_{02} = \begin{bmatrix} 1.6 & 0.5 \\ 0 & -0.05 \end{bmatrix}, \quad E_{d2} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & -0.3 \end{bmatrix}
\]

\[
D_i = \begin{bmatrix} 0.05 & 0 \\ 0 & -0.05 \end{bmatrix}
\]
Employing Theorem 2 of this paper, we calculate the upper bound $\mu_1$ of $h_1(t)$ or $\mu_2$ of $h_2(t)$, when the other is known. The upper bound of $\mu$ is obtained by summing the two delay bounds $\mu_1$ and $\mu_2$. In order to compare with the literature results, we consider the above system as an uncertain fuzzy systems with a single delay term $h(t)$, i.e., $h_1(t) + h_2(t) = h(t)$ satisfying $0 \leq h(t) \leq \mu$. Then we apply the literature conditions. The results are presented in Table 2.

Table 2: Upper bound $\mu$ for invariant delay

<table>
<thead>
<tr>
<th>Method</th>
<th>Upper bound $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chen et al. [5]</td>
<td>1.431</td>
</tr>
<tr>
<td>Fan et al. [17]</td>
<td>1.439</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$h_1$ for given $h_2$</th>
<th>$h_2$ for given $h_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1 = 1$</td>
<td>$\mu_1 = 1.1$</td>
</tr>
<tr>
<td>$\mu_1 = 1.3$</td>
<td>$\mu_2 = 0.6$</td>
</tr>
<tr>
<td>$\mu_2 = 0.7$</td>
<td>$\mu_2 = 0.8$</td>
</tr>
</tbody>
</table>

The results guarantee the stability of uncertain fuzzy system for invariant delays. It is shown that the upper bound $\mu$ obtained by theorem 2 is better then those obtained by single delay approach in [5, 17].

5. Conclusion

We have established delay dependent conditions for asymptotic stability of T-S fuzzy systems with two additive time varying delays. The results are extended to cover the class of uncertain T-S fuzzy systems. The perturbations considered are assumed to be norm-bounded. The LMIs proposed have been obtained by utilizing a new Lyapunov Krasovskii functional and Finsler’s lemma. The less conservativeness of the results is shown by two numerical examples in which we obtained a large delay upper bound as shown in table 1 and table 2.

References

Delay dependent robust stability


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