Peristaltic Pumping of a Jeffrey Fluid
under the Effect of Magnetic Field
in an Inclined Channel

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Abstract
The peristaltic pumping of a Jeffrey fluid in an inclined channel is studied under long wavelength and low Reynolds number assumptions. The magnetic field of uniform strength is applied in the transverse direction to the flow. The flow is investigated in a wave frame of reference moving with the velocity of the wave. The expressions for axial velocity, axial pressure gradient and frictional force have been obtained. The effect of various parameters on the flow characteristics are discussed with the help of graphs.

Keywords: Peristaltic pumping; Jeffrey fluid; inclined channel; magnetic field.
1. Introduction

Peristalsis is a mechanism of pumping of viscous fluids in ducts against an adverse pressure gradient by means of a series of moving contractile rings on the wall. It is an inherent property of many of the smooth muscle tubes such as the gastrointestinal tract, bile duct, ureter and similar ducts.

The study of peristaltic motion has gained considerable interest because of its extensive applications in urine transport from the kidney to bladder, transport of the spermatozoa in the ducts efferentes of the male reproductive tract, movement of the ovum in the fallopian tube, vasomotion of the small blood vessels, movement of the chyme in gastrointestinal tract and so forth. Peristaltic pumping is found in many applications such as, for the transport of slurries, sensitive or corrosive fluids, sanitary fluid and noxious fluids in the nuclear industry.

In physiological peristalsis the pumping fluid cannot always be treated as a Newtonian fluid. Several authors considered the fluid to behave like a Newtonian fluid for the physiological peristalsis including the flow of blood in arterioles. But such a model cannot be suitable for blood flow unless the non-Newtonian nature of the fluid is included in it. Raju and Devanathan [4] reported that a theoretical investigation for blood flow by considering blood as a non-Newtonian power – law fluid. A similar solution for viscoelastic liquids was subsequently presented by Bohme and Friedrich [2]. Srivastava and Srivastava [5] considered Casson fluid model which is applicable to blood flow. They also included a peripheral layer of a Newtonian fluid in the analysis.

MHD flows have numerous applications in bioengineering and medical devices. Specially, magnetic wound or cancer treatment causing hyperthermia, bleeding reduction during surgeries, and targeted transport of drugs using magnetic particles as drug carriers are few examples. In living creature, blood is a bio magnetic fluid because of complex interaction of the intercellular protein, cell membrane and the hemoglobin.

The magneto hydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids (e.g the blood flow in arteries). Sud et al. [6] studied the effect of magnetic field on blood flow. They observed that a suitable magnetic field accelerates speed of blood.

Agrawal and Anwaruddin [1] studied the effect of magnetic field on the peristaltic flow of blood using long wavelength approximation method and observed for the flow of blood in arteries with arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as blood pump in carrying out cardiac operations.

Mekheimer and Al-Arabi [3] studied the non linear peristaltic transport of MHD flow through a porous medium. The magnetohydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids (e.g the blood flow in arteries).
The Jeffrey model is relatively simpler linear model using time derivatives instead of convected derivatives for example the Oldroyd – B model does, it represents a rheology different from the Newtonian.

In view of this peristaltic pumping of an electrically conducting Jeffrey fluid in an inclined channel is investigated under long wavelength and low Reynolds number assumptions. The axial velocity distribution, the volume flow rate, the axial pressure rise and the frictional force are calculated. The effect of various parameters on the pumping characteristics is discussed through graphs.

2. Mathematical formulation of the problem

Consider the peristaltic pumping of an incompressible and electrically conducting Jeffrey fluid in a channel of half – width ‘a’ and inclined at an angle \( \theta \) to the horizontal. A longitudinal train of progressive sinusoidal waves takes place on the upper and lower walls of the channel. We assume that a uniform magnetic field strength ‘\( B_0 \)’ is applied along the direction of the Y-axis and the induced magnetic field is assumed to be negligible. For simplicity we restrict our discussion to the half width of the channel as shown in the Figure 1. The wall deformation is given by

\[
H(x, t) = a + b \cos \left( \frac{2\pi}{\lambda} (X - ct) \right)
\]

where \( b \) is the amplitude, \( \lambda \) is the wavelength and \( c \) is the wave speed.

The constitutive equations for an incompressible Jeffrey fluid are

\[
\bar{T} = -\bar{P}I + \bar{S}
\]

\[
\bar{S} = \frac{\mu}{1 + \lambda_1} \left( \frac{\partial \gamma}{\partial t} + \lambda_2 \frac{\partial^2 \gamma}{\partial t^2} \right)
\]

where \( \bar{T} \) and \( \bar{S} \) are Cauchy stress tensor and extra stress tensor, \( \bar{P} \) is the pressure, \( I \) is the identity tensor, \( \lambda_1 \) is the ratio of the relaxation to retardation times, \( \lambda_2 \) is the retardation time and \( \gamma \) is the shear rate.

Under the assumptions that the channel length is an integral multiple of the wavelength \( \lambda \) and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame \((x, y)\) moving with velocity \( c \) away from the fixed(laboratory) frame \((X, Y)\). The transformation between these frames is given by

\[
x = X - ct , \ y = Y , \ u(x,y) = U(X-ct , Y) \text{ and } v(x,y) = V(X-ct , Y)
\]

where \( U \) and \( V \) are velocity components in the laboratory frame and \( u \) and \( v \) are velocity components in the wave frame. In the many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the wavelength is infinite. So the flow is of Poiseuille type at each local cross - section.

Using the non – dimensional quantities
\[ x = \frac{2\pi x}{\lambda}, \quad y = \frac{y}{d}, \quad u = \frac{u}{c}, \quad v = \frac{v}{c^2}, \quad \delta = \frac{2\pi x}{\lambda}, \quad \rho = \frac{2\pi d^2 p}{\mu c \lambda}, \quad \lambda = \frac{2\pi t}{\lambda_1}, \quad h = \frac{H}{a}, \quad \phi = \frac{b}{a} \]

in the equations governing the motion (dropping bars) are
\[
\frac{\partial}{\partial y} \left( \frac{1}{1+\lambda_1} \frac{\partial u}{\partial y} \right) - M^2 (u + 1) + GS in\theta = \frac{\partial p}{\partial x} \tag{4}
\]
\[
\frac{\partial p}{\partial y} = 0 \tag{5}
\]

where \( G = \frac{\rho g d^2}{\mu c} \) is gravitational parameter, \( M^2 = \frac{\sigma e B^2_a}{\mu} \) is the magnetic parameter.

The non-dimensional boundary conditions are given by
\[
\frac{\partial u}{\partial y} = 0 \quad \text{at} \quad y = 0 \tag{6}
\]
\[
u = -1 \quad \text{at} \quad y = h \tag{7}
\]

3. Solution of the problem

Solving the equation (4) with the boundary conditions (6) and (7), we get the velocity as
\[
u = \frac{1}{M^2} \left\{ \frac{\partial p}{\partial x} - G \sin \theta \cosh (M \sqrt{1 + \lambda_1}) y \right\} - (M^2 - G \sin \theta + \frac{\partial p}{\partial x}) \cosh (M \sqrt{1 + \lambda_1}) h \tag{8}
\]

The volume flux ‘q’ through each cross section in the wave frame is given by
\[
q = \int_0^h ud\gamma
\]
\[
= \frac{1}{M^2} \left\{ \frac{\partial p}{\partial x} - G \sin \theta \tanh (M \sqrt{1 + \lambda_1}) h \right\} - (M^2 - G \sin \theta + \frac{\partial p}{\partial x}) h \tag{9}
\]

The instantaneous volumetric flow rate \( Q(x,t) \) in the laboratory frame between the central line and the wall is
\[
Q(x,t) = \int_0^h ud\gamma = q + 1 \tag{10}
\]

From the equation (9)(4.20), we have
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\[
\frac{\partial p}{\partial x} = \left\{ \frac{M \sqrt{1+\lambda_i}(GM^2 + (M^2 - G\sin \theta)h + G\sin \theta \tanh(M \sqrt{1+\lambda_i}h))}{\tanh(M \sqrt{1+\lambda_i}h) - hM \sqrt{1+\lambda_i}} \right\}
\]

(A11)

Averaging the equation (10) over one period yields the time mean flow (time averaged flow rate) \( \bar{Q} \) as

\[
\bar{Q} = \frac{1}{T} \int_0^T Q dt = q + 1
\]

(A12)

The Pumping characteristics

Integrating the equation (11) with respect to \( x \) over one wavelength, we get the pressure rise of the peristaltic wave as

\[
\Delta p = \int_0^1 \frac{M \sqrt{1+\lambda_i}((Q-1)M^2 + (M^2 - G\sin \theta)h + G\sin \theta \tanh(M \sqrt{1+\lambda_i}h))}{\tanh(M \sqrt{1+\lambda_i}h) - hM \sqrt{1+\lambda_i}} dx
\]

(A13)

The pressure rise required to produce zero average flow rate is denoted by \( \Delta p_0 \).

\[
\Delta p_0 = \int_0^1 \frac{M \sqrt{1+\lambda_i}(-M^2 + (M^2 - G\sin \theta)h + G\sin \theta \tanh(M \sqrt{1+\lambda_i}h))}{\tanh(M \sqrt{1+\lambda_i}h) - hM \sqrt{1+\lambda_i}} dx
\]

(A14)

The dimensionless frictional force at the wall across one wave length in the inclined channel is given by

\[
F = \int_0^1 (h - \frac{dp}{dx}) dx
\]

(A15)

4. Discussion of the results

From equation (11), we have calculated the pressure difference \( \Delta p \) as a function of time averaged flow rate \( \bar{Q} \) to study the effect of various parameters on the pumping characteristics through graphs (Figures 2 - 9).

From figure 3, we can observe the effect of magnetic parameter on the pumping phenomenon. It is observed that an increase in the magnetic parameter ‘M’ results in an increase in the peristaltic pumping rate and also an increase in the pressure rise.

Figure 4 is drawn to study the effect of the amplitude ratio on the pressure rise with time averaged flow rate for fixed values of remaining parameters. We observe that the larger the amplitude ratio, the greater the pressure rise against which the pump works.

Figure 5 shows the variation of pressure rise \( \Delta p \) with \( \bar{Q} \) for different values of angle of inclination \( \theta \) (\( 0 \leq \theta \leq \pi / 2 \)) of \( \lambda_i=2 \), \( \phi =0.6 \) and \( M =1 \). We observe that the pumping, free pumping and co – pumping increase as \( \theta \) increases in \( [0, \pi / 2] \). For a given \( \Delta p \), the flux \( \bar{Q} \) increases with increasing \( \theta \) in \( [0, \pi / 2] \). And also it is observed that for an inclined channel, the peristaltic wave passing over the
channel wall pumps against more pressure rise compared to the horizontal channel ($\theta = 0$).

From equation (15), we have calculated frictional force $F$ as a function of $Q$ for different values of the Jeffrey parameter ($\lambda_1$) and the angle of inclination ($\theta$) and is shown in figures 6 and 7. It is observed that the frictional force $F$ has the opposite behavior compared to the pressure rise ($\Delta p$).

From equation (8), we have calculated the axial velocity $u$ as a function of $y$ for different values of magnetic parameter $M$ and is shown in figures 8 and 9. From figure 8 it is observed the maximum velocity decrease when $M$ increases for $\frac{\partial p}{\partial x} = 0$.

But from figure 9 it is observed that the maximum velocity increase as $M$ increases when $\frac{\partial p}{\partial x} < 0$. This behavior is mainly because of peristalsis. And also it is observed that the velocity profiles are nearly parabolic in nature.

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Figure 2: The variation of $\Delta p$ with $\bar{Q}$ for different values of $\lambda_1$ with $\phi=0.7$, $\theta = \pi/6$ and $M = 0.5$.

Figure 3: The variation of $\Delta p$ with $\bar{Q}$ for different values of $M$ with $\phi=0.6$, $\lambda_1=2$, $\theta = \pi/3$. 
Figure 4: The variation of $\Delta p$ with $Q$ for different values of $\phi$ with $\lambda_1 = 1$, $\theta = \pi / 3$

Figure 5: The variation of $\Delta p$ with $Q$ for different values of $\theta$ with $\lambda_1 = 2$, $\phi = 0.6$ and $M = 1$. 
Figure 6: The variation of F with $\bar{Q}$ for different values of $\lambda_1= 2$ with $\phi =0.6 , \lambda_1= 0$,

$\theta = \pi / 6$ and M =1.
Figure 7: The variation of $F$ with $\bar{Q}$ for different values of $\theta$ with $\phi = 0.6$, $\lambda_{1} = 2$ and $M = 1$.

Figure 8: The variation of axial velocity $u$ with $y$ at $x = 0.1$ for different values of magnetic parameter $M$ with $\phi = 0.6$, $\lambda_{1} = 1$, $\theta = \pi / 4$ and for $\frac{\partial p}{\partial x} = 0$. 
Figure 9: The variation of axial velocity $u$ with $y$ at $x = 0.1$ for different values of magnetic parameter $M$ with $\phi = 0.6$, $\lambda_1 = 0.8$, $\theta = \pi / 6$ and for $\frac{\partial p}{\partial x} = -0.5$.

References


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