Abstract

The Goal Programming techniques handles decision situation involving a single goal or multiple goals. These goals may be treated as conditions which must be met as closely as possible. The goals may be specified values or a range of such values, situation of the later category arise in goal interval programming the expression “as closely as possible” may later to metric or non-metric properties. This technique is particularly useful in a problem situation having conflicting goals or objectives if such goals are written as constraints in the problem, the result would be no feasible solution. Consequently, some of the goals may have to be incorporated in the objective function. Goal Programming is important to a decision maker who is a satisfies later than an optimized, in this paper a numerical example on course affiliation will be presented to demonstrate the application of the Goal Programming model.

Keywords: Academic resource allocation: Goal Programming

INTRODUCTION

We know all the mathematical programming models has involved optimizing an objective relating to single entity e.g., profit, total distance traveled
subject to stated constraints. The reasons as to why these models are of such importance are that they do indeed replicate the contexts within which may real world decision are made.

In many decision making problems some goals are so important that analysis these are achieved the decision maker cannot consider that achievement of the other Goals. This way of using the goals is known as primitive ordering. The detailed analyses of the problems are given by Arenas (2002), Gonzalez, Romero (2001), Hodgkin (2003) and Jones (2003).

**GENERAL FORMULATION**

Consider the general linear programming formulation i.e.,

\[
\text{Optimize : } \sum_{j=1}^{n} c_j x_j \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (1.1)
\]

such that a \[ \sum_{j=1}^{n} a_{ij} x_j \leq or = or \geq b_i, i = 1 \ldots m \ldots \] (1.2)

\[ x_j \geq 0, j = 1 \ldots n \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ (1.3) \]

Up to this, the \( b_i \) values in the constraints (1.2) have been viewed as the available amounts of source, resources in the case of ‘less than or equals’ constraints and as requirements to be equaled or exceeded in the case of ‘greater than or equals’ constraints, We shall now view these values as goals to be achieved as closely as possible. Specifically, a set of values is required for the \( x_j \) such that

\[
\sum_{j=1}^{n} a_{ij} x_j \rightarrow b_i; i = 1 \ldots m \ldots \] (1.4)

Where ‘\( \rightarrow \)’ is interpreted as meaning ‘approaches as closely as possible to’. In practice, for a given solution, the values of \( x_j \) will probably be such as to cause the left hand side of 1.4 to fall short of or exceed \( b_j \) at least for certain of the constraint expressions. We can thus rewrite 1.4 as:

\[
\sum_{j=1}^{n} a_{ij} x_j + D_{ij}^{-} - D_{ij}^{+} = b_i; i = 1 \ldots m \ldots \] (1.5)

where \( D_{ij}^{-} \) gives the amount by which the solution values of \( x_j \) cause the quantity \( \sum_{j=1}^{n} a_{ij} x_j \) fall short of \( b_i \) and \( D_{ij}^{+} \) gives the amount by which the solution values of \( x_j \) cause the quantity \( \sum_{j=1}^{n} a_{ij} x_j \) to exceed \( b_i \). Quite obviously, for a particular constraint expression and set of solution values \( x_{ij} j = 1, \ldots, n \):
Goal programming approach

\[ D^-_i \text{ and/or } D^+_i = 0 \ldots \ldots \ldots (1.6) \]

If the values of \( x_i \) are such as to cause an expression of the form (1.5) to hold exactly. The larger the values of \( D^-_i \) or \( D^+_i \)
\[ D^-_i = 0 ; \quad D^+_i = 0 \ldots \ldots \ldots (1.7) \]
the greater the amount which the goal expressed as \( b_i \) is under fulfilled or over fulfilled.

The conditions (1.5) together with (1.3) and
\[ D^-_i \geq 0 ; \quad D^+_i \geq 0, \quad i = 1, \ldots, m \ldots (1.8) \]
(which cause (1.6) to hold automatically) are those appropriate to a goal programming formulation derived linear programming.

We now consider the formulation of an appropriative. Given that it is required that the goal (1.5) be as closely as possible, it is required that the values \( D^-_i \) of be as small as possible. Thus one appropriate object is:
\[
\text{Minimize: } \sum_{i=1}^{n} (D^-_i + D^+_i) \ldots \ldots \ldots (1.9)
\]

In many goal programming applications, it can be argued the fulfillment of some goals is more important than of in order to take account of this, the terms in 1.9 are weighted to yield:
\[
\text{Minimize: } \sum_{i=1}^{n} w_i (D^-_i + D^+_i) \ldots \ldots \ldots (1.10)
\]

Where \( w_i \) represents the relative importance of satisfy the \( i^{\text{th}} \) goal. Specifically, if it is considered of immense importance that the \( i^{\text{th}} \) constraint be satisfied, \( w_i \) will assigned a large value which, given the nature of the minimization and in a similar manner to be incorporation of in a linear programming minimization objective will cause \( D^-_i \), \( D^+_i \) to tend to zero. The objective (1.10) presupposes that the weight given to the \( i^{\text{th}} \) goal being under fulfilled \( (D^-_i > 0) \) or fulfilled and \( (D^+_i > 0) \) is the same. If, the problem that this is not so, separate weights, \( w^-_i \) and \( w^+_i \) rarely can be assigned to \( D^-_i \) and \( D^+_i \) to yield the revised objective:
\[
\text{Minimize: } \sum_{i=1}^{n} w^-_i D^-_i + \sum_{i=1}^{n} w^+_i D^+_i \ldots \ldots \ldots (1.11)
\]

Thus, for example if the \( i^{\text{th}} \) constraint is such that very important that the values of \( x_j \) be such as \( b_i \) achieved (i.e., \( D^-_i = 0 \)) but it is of little concerned that is exceeded, then while \( w^-_i \) would be chosen to be a relatively large number while \( w^+_i \) would be chosen to be relatively small. In the case of certain goals their nature is such as to disallow either underestimation or overestimation which will cause the term \( D^-_i \) or \( D^+_i \) as appropriate to be dropped from (1.5) and the objective. Consider, for example, the task of subdividing an area of land between \( n \) users. If \( x_j \) is the proportion of land allocated to the \( j^{\text{th}} \) use, \( j = 1 \ldots n \) and it is a goal that all of the land be allocated.
Then: \[ \sum_{j=1}^{n} x_j \rightarrow 1 \cdots \cdots \cdots \cdots \cdots \cdots \cdots (1.12) \]

Quite obviously, the summation in (1.12) cannot exceed one i.e., over use of the land cannot occur. Thus introducing \( D^-_i \) only (1.12) becomes:

\[ \sum_{j=1}^{n} x_j + D^-_i = 1 \cdots \cdots \cdots (1.13) \]

and only \( D^-_i \) appears in the objective. In order to indicate one further development of the basic goal programming model, we shall reconsider the objective (1.10); the development could be discussed equally well within the context of (1.11) or (1.12) and where certain of the terms \( D^-_i, D^+_i, i = 1, \ldots, m \) are deleted. Suppose that in (1.10), a set of weighting values \( w_i = p_i, i = 1, \ldots, m \) is chosen such that:

\[ p_i \gg p((i+1)); i = 1, \ldots, (m-1) \cdots \cdots (1.14) \]

where the inequality ‘\( \gg \)’ means ‘is very much greater than’ the revised objective is now:

\[ \text{Minimize: } \sum_{i=1}^{m} p_i(D^-_i + D^+_i) \cdots \cdots (1.15) \]

Where the coefficient if the first term is very large, the coefficient of the next term is significantly smaller and so on. Provided the coefficient \( p_i \) are different enough from each other (for a summary of how this can be achieved see for example Field (1973) and, once more, following the logic underlying the big-M method, the optimization will proceed by satisfying the first goal (i.e., causing \( D^-_i, D^+_i \), to tend to zero) to the greatest extent possible first and then, and only then, by satisfying the second goal to the greatest extent possible and so on. A goal programming problem where the weights satisfy the condition (1.14) i.e., with an objective of the form (1.15) is referred to as a pre-emptive goal programming problem; a problem with an objective of the form (1.10) comprises a weighted goal programming problem. It should be noted that the pre-emptive and weighted formulation can be combined. Specifically, suppose that in a given goal programming problem comprising \( m \) goals, the first subgroup of \( a \) goals whose weights relative to each other are \( w_1, w_2, \ldots, w_a \) respectively are of top priority and must be satisfied in so far as is possible before all others, that the next subgroup of \( b \) goals of weight \( w_{a+1}, \ldots, w_{a+b} \) are of second priority and so on, then assigning pre-emptive weights \( p_1, p_2, \ldots \) to the sub groups, the appropriate objective is:

\[ \text{Minimize: } P_1 W_1 (D^-_i + D^+_i) + P_1 W_a (D^-_a + D^+_a) + P_2 W_{(a+1)} (D^-_{(a+1)}) + \cdots + P_2 W_{(a+b)} (D^-_{(a+b)} + D^+_{(a+b)}) \cdots \cdots (1.16) \]

**CASE STUDY**

A Simplified numerical example will be presented to demonstrate the
Goal programming approach

application of the general model. Let us assume that the Principal, SJCS BHOPAL provided the following priority structure for academic goals and information on constants:

Problem with given data:
The required information is given in the following table-1

<table>
<thead>
<tr>
<th>Table-1</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Teaching Loads, Average Salaries, Desired Proportions of Total Staff</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Teaching Load</th>
<th>Required Proportion</th>
<th>Salary Rs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Middle</td>
<td>Secondary</td>
<td>Maximum</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$x_2$</td>
<td>6</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>$x_3$</td>
<td>12</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>$x_4$</td>
<td>9</td>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>$x_5$</td>
<td>9</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>$x_6$</td>
<td>6</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$x_7$</td>
<td>3</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$x_8$</td>
<td>0</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$x_9$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$y_1$</td>
<td>6</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$y_2$</td>
<td>6</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$y_3$</td>
<td>3</td>
<td>3</td>
<td>-</td>
</tr>
<tr>
<td>$y_4$</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$y_5$</td>
<td>0</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

The constraints in the model needed for affiliation were given the higher priority by the Principal of the School. These goals will be considered first in the programming model, followed by the lower priority goals.

where

$x_1 =$ Number of primary teaching assistants
$x_2 =$ Number of middle teaching assistants
$x_3 =$ Number of Secondary teaching assistants
$x_4 =$ Number of Primary teachers without terminal degrees
$x_5 =$ Number of Middle division teachers without terminal degrees
$x_6 =$ Number of Secondary division teachers without terminal degrees
$x_7 =$ Number of Expert teachers without terminal degrees
$x_8 =$ Number of Senior teachers without terminal degrees
$x_9 =$ Number of supporting staff
$y_1 =$ Number of Teaching assistant with terminal degree
$y_2 =$ Number of Primary teachers with terminal degree
$y_3 =$ Number of Secondary teachers with terminal degree
$y_4 =$ Number of Expert teachers with terminal degree
$y_5 =$ Number of Senior teachers with terminal degree

*Terminal degree represents Ph.D., M.Ed and B.Ed
Priority Structures:
P_7 = Maintain the necessary requirements for affiliation by Board
P_6 = Assure adequate salary increases for the academic staff, teacher assistants and general staff.
P_5 = Assure adequate number of teachers by meeting desired teacher/students ratio and by having instruction available for the needed students credit hours.
The upper division teacher/students requirements are considered to be twice as important as the primary teacher requirement
P_4 = Attain a desire distribution of the academic staff with respect to rank.
P_3 = Maintain desired teacher/staff ratio
P_2 = Maintain desired teacher/teacher assistant ratio
P_1 = Minimize Cost

Constraints for Affiliation:

It is required that 80 percent of the academic staff be full-time Teachers according to Board. Since in our model x_3 to x_6 and x_8, y_1 to y_3 and y_5 are considered full-time, we may write:

\[ \sum_{i=3}^{6} x_i + x_8 + \sum_{i=1}^{3} y_i + y_5 - 0.80(\sum_{i=1}^{8} x_i + \sum_{i=1}^{5} y_i) + d_i^- + d_i^+ = 0 \ldots (1.17) \]

It is also required that at least 30 percent of the academic teaching staff at the middle level possess terminal coverage. This is expressed as:

\[ \sum_{i=1}^{3} y_i - 0.30(x_8 + \sum_{i=2}^{7} x_i + \sum_{i=1}^{3} y_i) + d_i^- + d_i^+ = 0 \ldots (1.18) \]

At least 65 percent of the academic staff teaching upper division studies are required to possess terminal coverage. This is expressed as:

\[ \sum_{i=3}^{6} y_i - 0.65(x_8 + \sum_{i=1}^{6} x_i) + d_i^- + d_i^+ = 0 \ldots (1.19) \]

Constraints for Academic Staff:

To determine the teacher required, it is necessary forecast the total number of student credit hours of instruction needed. In this example, the projected student enrolment is 2340, the average number of credit hours/student taken at the school is 6, and the desired class size is set at 30. Therefore, total students credit hours can be calculated by means of the following formula:

\[ 6x_1 + 12x_2 + 9x_3 + 9x_5 + 6x_6 + 3x_7 + 6y_1 + 6y_3 + 3y_5 + d_4^- + d_4^+ = 780 \ldots (1.20) \]

For the middle division student credit hours of instruction, we forecast 200 hours per session. The procedure is similar to the upper division forecast and constraint becomes:

\[ 3x_8 + 3y_1 + 3y_2 + 3y_3 + 3y_4 + 3y_5 + d_7^- + d_7^+ = 200 \ldots (1.21) \]

The next aspect to be considered in the determination of the required academic staff is the desired faculty/student ratio at both the graduate and
undergraduate level. The forecasted enrollments in the next year at undergraduate and graduate levels are 2340 and 200, respectively. The desired undergraduate faculty/student ratio is about 1/30 and the desired graduate teacher/student ratio at about 1/15. These constraints then become, for the undergraduate requirements:

\[ \sum_{i=1}^{7} x_i + \sum_{i=1}^{3} y_i + d^-_6 + d^+_6 = (0.03)(2,340) = 78 \ldots \ldots (1.22) \]

And for the graduate faculty:

\[ x_8 + \sum_{i=1}^{5} y_i + d^-_7 + d^+_7 = (0.06)(200) = 20 \ldots \ldots (1.23) \]

**Constraints for the Distribution of Academic Staff:**

It is necessary to impose some constraints on the distribution of the academic teachers according to the desired proportion of the total teachers for each type of discipline. The desired faculty/student ratio is based on the CBSE Affiliation regulations and the Principle’s academic policy.

\[
\begin{align*}
0.07T - x_2 + d^-_8 + d^+_8 &= 0 \\
0.07T - x_3 + d^-_9 + d^+_9 &= 0 \\
0.15T - x_4 + d^-_{10} + d^+_{10} &= 0 \\
0.05T - x_5 + d^-_{11} + d^+_{11} &= 0 \\
0.02T - x_6 + d^-_{12} + d^+_{12} &= 0 \\
0.01T - x_7 + d^-_{13} + d^+_{13} &= 0 \\
0.01T - x_8 + d^-_{14} + d^+_{14} &= 0 \\
0.211T - y_1 + d^-_{15} + d^+_{15} &= 0 \\
0.04T - y_2 + d^-_{16} + d^+_{16} &= 0 \\
0.023T - y_3 + d^-_{17} + d^+_{17} &= 0 \\
0.02T - y_4 + d^-_{18} + d^+_{18} &= 0 \\
0.02T - y_5 + d^-_{19} + d^+_{19} &= 0
\end{align*}
\]

Where \( T = \sum_{i=2}^{8} x_i + \sum_{i=1}^{5} y_i \)

**Number of Staff:**

In order to insure adequate staff for clerical and administrative work, the desired teacher /staff ratio is set at 6 to 1 by the principle. The constraint is then:

\[ T - 6x_9 + d^-_{20} + d^+_{20} = 0 \]
Number of Graduate Research Assistant:

We set the desired teacher /teacher assistant ratio at 4 to 1. Hence, the constraint is:

$$\sum_{i=3}^{8} x_i + \sum_{i=1}^{5} y_i - 4x_1 + d_{21}^- + d_{21}^+ \ldots \ldots (1.24)$$

Cost of Academic Staff, Graduate Assistant, and Staff:

The total salary increase constraint can be expressed as:

$$0.08 \left\{ 3000 \sum_{i=1}^{5} x_i \right\}$$

$$+0.10 \left\{ 8500x_1 + 14000x_2 + 6000x_3 + 19000x_4 + 3000x_5 + 35000x_6 + 15000y_1 + 17000y_2 + 17000y_3 + 3000y_4 + 35000y_5 \right\} + 0.08(6000x_6) + w + d_{23}^- + d_{23}^+ = 0 \ldots \ldots (1.25)$$

Where there is a 8 percent increase for lower division teacher and staff and a 10 percent increase for Upper division teacher The total payroll constraint for the entire school will be:

$$3,000x_1 + 3,000x_2 + 8,500x_3 + 14,000x_4 + 6,000x_5 + 19,000x_6 + 3,000x_7 + 35,000x_8 + 15,000y_1 + 17,000y_2 + 17,000y_3 + 3,000y_4 + 35,000y_5 + 6,000x_6 + w + d_{23}^- - d_{23}^+ = 0$$

Objective Function:

$$\text{Min.} Z = p_1 \sum_{i=1}^{5} d_i + p_2 d_{23}^+ + 2p_3 d_{23}^- + 2p_4 d_i^- + p_5 d_i^+ + p_6 \sum_{i=8}^{13} d_i^- + p_7 d_i^+ + p_8 d_{18}^+ + p_9 \sum_{i=14}^{17} d_i^- + p_9 d_{18}^+ + p_{10} d_{20}^+ + p_{11} d_{21}^- + p_{12} d_{23}^+$$

RESULT

This will be obtained by using QSB Computer Software may be interpreted as follows:

<table>
<thead>
<tr>
<th>Goal Attainment</th>
<th>Achieved</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth in strength of the school</td>
<td>Achieved</td>
</tr>
<tr>
<td>Salary increase</td>
<td>Achieved</td>
</tr>
<tr>
<td>Teacher/student ratios</td>
<td>Achieved</td>
</tr>
<tr>
<td>Teacher/staff ratios</td>
<td>Achieved</td>
</tr>
<tr>
<td>Teacher/distribution</td>
<td>Achieved</td>
</tr>
<tr>
<td>Teacher/Teacher assistant ratio</td>
<td>Achieved</td>
</tr>
<tr>
<td>Minimize cost</td>
<td>2674000</td>
</tr>
</tbody>
</table>
### Variable

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
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</tr>
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<tr>
<td>$x_3$</td>
<td>13</td>
</tr>
<tr>
<td>$x_4$</td>
<td>24</td>
</tr>
<tr>
<td>$x_5$</td>
<td>9</td>
</tr>
<tr>
<td>$x_6$</td>
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<tr>
<td>$x_7$</td>
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</tr>
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</tr>
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<td>5</td>
</tr>
<tr>
<td>$y_9$</td>
<td>40</td>
</tr>
</tbody>
</table>

$W = 2,06,000$

### CONCLUSION

Virtually all models developed for school management have focused upon the analysis of input (resource) requirements. They have generally neglected or often ignored the system outputs, unique institutional values, and bureaucratic decision structures. However, these are important environmental factors, which greatly influence the decision of the decision process. In this study the GP approach is utilized because it allows the optimization of goal attainments while permitting an explicit consideration of the existing decision environment.

Developing and solving the GP model points out where some goals cannot be achieved under the desired policy and hence, where tradeoff must occur due to limited resources. Furthermore, the model allows the administrator to review critically the priority structure in view of the solution derived by the model. Indeed, the most important Property of the GP model is its great flexibility which allows model simulation with numerous variations of constraints and priority structures of goals.

### REFERENCES


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