Comparing Neural Networks: Hopfield Network and RBF Network

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Abstract

The two well-known neural network, Hopfield networks and Radial Basis Function networks, have different structures and characteristics. Hopfield neural network and RBF neural network are two of the most commonly-used types of feedback networks and feedforward networks respectively. This study gives an overview for Hopfield neural network and RBF neural network in architectures, the learning processing, and their applications, as well as the comparison between these two networks from several different aspects.

Keywords: Artificial Neural Networks, RBF Network, Hopfield Network

1 Introduction

Artificial neural networks or ANNs (is also called connectionist systems) have received particular attention because of their ability to analyze complex non-linear data sets (e.g.[21], [14], [3]). Neural Networks (NNs) are artificial intelligence structures which include some mathematical and graphical models. These models are used to show the structure of artificial neural network. All neural networks perform essentially the same function: vector mapping. Also, all neural network applications are special cases of vector mapping. It consists of a large number of simple processing elements called neurons, cells, unites or nodes. These neurons are connected with each other by direct communication links, each with an associated weight. There are two main topologies in neural network according to their connectivity. The first one is feed forward network - such as RBF network - if the neurons belonging to the same layer receive inputs from neurons of the previous layer and send their values only to neurons of the next layer, and the second one is feedback network - such as Hopfield network - if the neurons belonging to the same layer send their output to neurons of the
next and previous layers. Different types of ANNs have been designed for specific problems such as finding solutions to constrained optimization problems, data compression, forecasting and risk assessment, control, content addressable memories, pattern recognition, and diagnostics. The training phase requires the availability of a relatively large number of input-output vectors for the network.

The training involves adjusting the weights on the interconnections in the network until the error which is the difference between the actual output and the target output is small. Unsupervised learning mechanisms differ from supervised learning, where the differentiable characteristic of supervised ANNs lies in the inclusion of "a teacher" in their learning process, while unsupervised networks do not have "a teacher" in the training data set. Supervised learning is based on direct comparison between the actual output and the target output, in other words it is formulated as the minimization of an error function such as the fitness function which is Mean absolute percentage error between the actual output and the target output summed over all available data. Unsupervised learning is solely based on the correlations among input data. No information on correct output is available for learning. Essence of a learning algorithm is the learning rule, i.e., a weight-updating rule which determines how connection weights are changed for example Hebbian rule. After the network is trained, it can be used to solve the problem at hand by merely presenting the inputs to the network. The corresponding value of the output is found virtually instantaneously as the inputs are propagated through the network. This paper is structured as follows. In Section 2, we will review architectures, rule learning process, and applications for RBF network. In Section 3, we will also review the properties of Hopfield network and their applications. In section 4, We will summarize the differences between Hopfield network and RBF neural network. In section 5, we give some concluding remarks of this work.

2 RBF Neural Network

The Radial Basis Function neural network (RBFNN) have been subjected to extensive research over recent years and have successfully been employed to various problem domains (e.g. [9], [8]). The idea of a RBFNN is to allocate each RBF neuron to respond to each of sub-spaces of a pattern class, formed by the clusters of training samples. As a result of that, learning at the hidden layer, is commonly configured as the problem of finding these clusters and their parameters by certain means of functional optimization. The name RBFNN comes from the fact that the basis functions in the hidden layer neurons are radially symmetric.
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The Radial Basis Function (RBF) network typically has three layers: an input layer, a hidden layer with a non-linear RBF activation functions and a linear output layer is a special class of multilayer feed-forward network. The hidden layer neurons receive the input information, followed by certain decomposition, extraction, and transformation steps to generate the output information. A layer is a vector of units. RBF networks are very popular for function approximation, curve fitting, time series prediction, and control and classification problems. The radial basis function network is different from other neural networks, possessing several distinctive features. Because of their universal approximation, more compact topology and faster learning speed, RBF networks have attracted much attention and they have been widely applied in many science and engineering fields.

RBF network sets the constant of the weight between input layer and hidden layer and updates the weight between hidden layer and output layer. The hidden-to-output weights are usually 0 or 1; for each hidden unit, a weight of 1 is used for the connection to the output which is true for it, and all other connections are given weights of 0. The input layer is made up of source neurons that connect the network to its environment. The neurons in the hidden layer are associated with centers that determine the behavior structure of network. The output layer provides the response of the network to the activation pattern of the input layer that serves as a summation unit. The radial basis function $\phi_i$ computed by the ith hidden neurons is maximum when the input vector is near the center (the mean) of that neuron. There are many types in radial basis function such as

Gaussian function

$$\phi(x) = e^{-\beta x^2}, \text{ for some } \beta > 0,$$

Multiquadric function

$$\phi(x) = \sqrt{x^2 + \beta^2}, \text{ for some } \beta > 0,$$

Polyharmonic Spline

$$\phi(x) = x^k, k = 1, 3, 5, \ldots$$

An approximation RBF network by a combination of Gaussian functions[5] be on the form

$$\phi_i(x) = e^{-\frac{\|x-c_i\|^2}{\sigma_i^2}} \quad (1)$$
Figure 1: Structure of RBF network

where \( i = 1, \ldots, l \) and \((l)\) the number of hidden neurons, \( c_i \) is the center of \( ith \) RBF hidden unit, which is a vector whose dimension is equal to the number of inputs to the neuron \( i \), \( \sigma_i \) is the width of the receptive field in the input space from neuron \( i \). This implies that \( \phi_i \) has an appreciable value only when the distance \( \| x - c_i \| \) is smaller than the width \( \sigma_i \). \( \| \cdot \| \) indicates the Euclidean norm on the input space as follows:

\[
\| x - c_i \|_2 = \sqrt{\sum_{m=1}^{M} (x_m - c_{m}^i)^2} \quad (2)
\]

Determination of the number of neurons in the hidden layer is very important because it affects the network complexity and the generalizing capability of the network. The position of the centers in the hidden layer also affects the network performance considerably, so determination of the optimal locations of centers is an important task. The training procedure of RBF networks also includes the optimization of centers of each neuron. Afterwards, the weights between the hidden layer and the output layer must be selected appropriately. Finally, the bias values which are added with each output are determined in
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Figure 2: The processing for RBFNN

the RBF network training procedure. In RBF networks each RBF neuron in the model [6] inserts into the state space an open basin of attraction around its center, thus introducing a stable fixed point.

An admissible RBF $\phi$ possesses the following attributes: [1]]

$\phi$ is continuous and bounded.

$\phi$ Attains its unique maximum at the center (zero distance).

$\phi$ Having considerable value in the close neighborhood of center.

$\phi$ Having negligible value in far distances (where are close to other centers).

if $\lim ||x - c|| \to \infty$ then $\phi(x) = 0$

Differentiability.

where $\phi_i(x) = e^{-\frac{||x-c_i||^2}{\sigma_i^2}}$.

The output[5] $f : \mathbb{R}^n \to \mathbb{R}$ of the network is thus

$$f_j(x) = \sum_{i=1}^{N} W_{ij}\phi_i(x) \quad (3)$$
where $W_{ij}$ is the connection weight of the $i$th RBF unit to the $j$th output node. $x$ is the input vector.

There is another type [24] of activity functions for output sigmoid function:

$$z_j(x) = \frac{1}{1 + e^{-f_j(x)}}$$  

(4)

where

$$f_j(x) = \sum_{i=1}^{N} W_{ij}\phi_i(x)$$

and $z_j(x)$ is the output.

Two types of learning with RBFNN are supervised and unsupervised learning (hybrid learning). An important issue concerning supervised learning - such as Back-Propagation Learning Algorithm - is the problem of error convergence. The aim is to determine a set of weights which minimizes the error. When the training phase is complete the weights are fixed. Unsupervised clustering algorithm used to estimate kernel positions (the centers) and kernel widths includes K-means [5] and Kohonen learning algorithms. The role of clustering in the design of RBFNN is to set up an initial distribution of receptive fields (hidden neurons) across the input space of the input variables. There are new learning algorithms such as Particle Swarm Optimization (PSO) [22], Genetic Algorithm (GA) [20], Kalman Filtering (KF) Algorithm, and Artificial Bee Colony (ABC) algorithm [7].

There are a lot of applications for this network through the modeling such as managements of water distribution system (WDS) [10] where is needed to use water quality models. The modeling of the Automatic Depth Control Electrohydraulic System (ADCES) of a certain mine-sweeping weapon using RBFNN [2].

### 3 Hopfield Neural Network

The Hopfield neural network which popularized by John Hopfield in 1982, is one of the ANNs based on energy minimization. It has been reported that minimizing the constructed energy function leads to getting trapped in an early local minima in the energy landscape. Its architecture consists of a two-dimensional connected neural network in which the linking strengths between
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neurons-which are binary threshold neurons- are determined based on the constraints and solution criteria of the optimization problem to be solved. Each unit is connected to all other units except itself, as a result, avoids a permanent feedback of its own state value. Figure 3.3. shows the architecture of Hopfield network. A solution is active after the network is relaxed and reaches a stable state. The energy function in higher order is thus[15]:

\[
E = \ldots - \frac{1}{3} \sum_{i} \sum_{j} \sum_{k} J_{i,j,k}^{(3)} S_i S_j S_k - \frac{1}{2} \sum_{i} \sum_{j} J_{i,j}^{(2)} S_i S_j - \sum_{i} J_i S_i \tag{5}
\]

\[
h_i = \ldots + \sum_{i} \sum_{j} \sum_{k} J_{j,k}^{(3)} S_j S_k + \sum_{i} \sum_{j} J_{i,j}^{(2)} S_j + \sum_{i} J_i \tag{6}
\]

where the system in Hopfield model consists of N formal neurons each of which is described by an Ising variable \( S_i \) (is called the state of neuron \( i \)), \( S_i \in \{1, -1\} \) and \( S_i = \text{sign}(h_i) \). The updating rule maintains \( S_i(t + 1) = \text{sign}(h_i(t)) \). \( J_{i,j}^{(2)} \) is the synaptic strength from neuron \( i \) to neuron \( j \), where the synaptic strength are symmetric with zero diagonal[15]
\[ J^{(n)}_{ij\ldots k} = J^{(n)}_{ji\ldots k} \text{ and } J^{(n)}_{ii\ldots i} = J^{(n)}_{jj\ldots j} = 0, \]

and thus necessary conditions for the convergence of an asynchronous totally connected network to a stable state, which means that only one neuron at a time updates itself in the output layer. However, if the Hopfield networks units are updating synchronous, that is, all neurons update themselves at the same time, it may not converge to a stable state.

Hopfield network must have a learning rule which sets the connection weights between all pairs of neurons such as Hebbian learning method, Storkey learning [18], [17] and Wan Abdullah method. In addition, every learning algorithm for perceptrons can be turned into a learning method for Hopfield network. Hebbian learning method used to calculate the changes in connection weights depending on the activities of the neurons as follows:

\[ \Delta J^{(n)}_{i,j\ldots,m} = \lambda S_i S_j \ldots S_m \] (7)

where \( \lambda \) is a learning rate.

\[ J^{(n)}_{i,j\ldots,m} = \lambda^{(n)} \sum_{k=1}^{M} S_i S_j \ldots S_m \] (8)

where \( \lambda^{(n)} = \frac{1}{(n-1)!} \)

In which time and learning thresholds can be taken into account.

In Hopfield neural network the basin of attraction [23] is the set of states in the system within which almost all states flow to one attractor, and it’s usually measured by Hamming distance. Attractor represent the stored patterns. The basin of attraction of a fixed point(a stable state)is defined to be the set of states that are attracted to that fixed point. Different learning rules produce different basins of attraction[18]. With the knowledge that, Capacity and attraction basins are related. For any starting point the state slides down hill until it comes to rest(stable state)at one of these minima(attractors). The basins of attractions correspond to the valley around each minimum.

There is two phases in the operation of the Hopfield network; Phase Storage and phase Retrieval. In the storage Phase, we suppose that we wish to store a vectors fundamental memories, and then retrieval phase; initial state is given to the network, neuron states are updated asynchronously, and the network finally produces a time invariant state vector whose individual elements satisfy the condition for stability.
One of the important factors which influence the quality of a solution obtained by Hopfield network is the difference in the frequency that a neuron receives information from other neurons. The function of the relaxation rate, $R$ [16] is to adjust the speed of the relaxation so that solutions with better quality can be obtained. When network relaxed too fast, there will be fewer opportunities for exchange of the information between neurons, and therefore a solution formed under this condition has poor quality. There is many applications for Hopfield network such as solving travelling salesman problem, for storing multiple sequences of varying duration, Optimal Edge Selection, and many applications.

4 Comparison between Hopfield network and RBF network

The fundamental difference between Hopfield network and RBF network is that Hopfield network is single layer feedback network proposed as an associative memory system (content addressable memory CAM). It is also called Associative Memories or Hopfield Memories, because it has been able to store the correct pattern. In other word, its asynchronous of a bidirectional associative memory without hidden neurons. In addition, Hopfield network is tool for solving an optimization problem. RBF network - which have widely been used for function approximation - is 3-layers feedforward network with three types of parameters that need to be chosen: the center vectors, the output weights and the $RBF$ width parameters. Importance of these parameters comes from the fact that RBF network is a local network which means that the input vectors in the close neighborhood of centers produce activation and the other inputs have negligible values. In addition, $RBF$ is symmetry around the center and maximum near this center, thus the center and the width are necessary conditions for the convergence connected network to a stable state. On the contrary, Hopfield network is global network which means all input vectors produce activation. Energy function (Lyapunov function) used in their input activation, which becomes exactly minimum, when creating a pattern. The weight $W$ in Hopfield network is a symmetric matrix with zero diagonal elements which are necessary conditions for the convergence of an asynchronous totally connected network. The activation function (energy function) is the weighted sum of the inputs, where the activation is passed through a threshold function to obtain the output with taking into account that Lyapunov function can take negative values; that is, however, just a matter of scaling.

The basin of attraction of a stable state is defined to be the set of states that are attracted to that stable state. In Hopfield network Attractor repre-
sent the stored patterns, note that energy function cannot be further reduced. In relation to the RBF network, each RBF neuron in the model inserts into the state space an open basin of attraction around its center, thus introducing a fixed point. Note that, the hidden layer of a RBF network is non linear and the output layer is linear and the activation function in a hidden layer unit computes the Euclidean norm between the input vector and the center of the unit. The Hopfield network is structured such that each unit connects to very other unit, with the exception that no unit has any connection to itself, each neuron has two states similar to those of McCulloch and Pitts. While each RBF network layer connects to other layer by their units, each RBF neuron in hidden layer is allocated to each of subspaces of a pattern class, formed by the clusters of training samples. Associative memory is a major application of the Hopfield network. In addition, pattern recognition, solving liner programming problems and solving combinatorial optimization problems(COPs). There are manifold examples to illustrate the properties of Hopfield networks such as N-Queens Problem[13], Traveling Salesman Problem and Race Traffic problem. whereas applications that are used RBF network effectively include: Function approximation, classification, modeling of dynamic systems and time series. In addition to Combined Economic and Emission Dispatch and many other topics using RBF neural network. Of applications that benefited from all attributes of RBF network and Hopfield network; neuro-vision system for 3D reconstruction[4], where the system uses RBF network in camera calibration task and Hopfield network in stereo matching which are the process tasks of this system. Nearest Neighbor Classifier(NNC)system[6]uses Hopfield network but using RBF network within hybrid network is more better. The XOR problem can be solved by using Hopfield network[1], [13]and RBF network[11].

RBF neural networks are often trained by hybrid learning algorithms, in which centers and widths of the hidden units are first obtained by unsupervised learning,after which the output weights are obtained by supervised learning. Accordingly, hybrid training is not applied to Hopfield networks which use unsupervised learning to find patterns in the input space in order to train it. The Hopfield network training algorithm has two basic phases: storage and retrieval. In the first phase, the network is required to store a set of states, or fundamental memories. In the second phase, an unknown corrupted or incomplete version of the fundamental memory is presented to the network. Note that, the Hopfield network output is calculated and feedback to adjust the input, then the network repeats this process until the output becomes constant. We summarized some of the differences between these two types of neural networks in table.
5 Conclusion

Hopfield neural network can perform some of the functions of memory recall in a manner analogous to the way the brain functions. In addition, pattern recognition, solving linear programming problems and solving combinatorial optimization problems (COPs). One of the advantages of Hopfield network: simple technical implementation using electronic or optical device. For RBF network, there are indeed topics where it can succeed such as financial markets. There are several applications benefit from the characteristics of this network, which includes: Function approximation, classification, modeling of dynamic systems and time series. Note that there are systems harness the properties of both networks, such as neuro-vision system for 3D reconstruction. In addition to there are problems where can be solved by both these networks such as XOR problem.

References


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<th><strong>Hopfield network</strong></th>
<th><strong>RBF network</strong></th>
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<td>John Hopfield proposed this feedback network (recurrent network) as an associative memory system.</td>
<td>Moody and Darken proposed this feed-forward network, which employs local receptive fields to perform function mapping.</td>
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<td>It is called Associative Memories or Hopfield Memories, because it has been able to store the correct pattern. On other word, its asynchronous of a bidirectional associative memory without hidden neurons. In addition, it is tool for solving an optimization problem, pattern recognition and solving liner programming problems.</td>
<td>It consisting of a single hidden layer, which have widely been used for function approximation. In addition, classification, modeling of dynamic systems and time series. The idea of a RBF network is to allocate each RBF neuron to respond to each of sub-spaces of a pattern class, formed by the clusters of training samples.</td>
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<td><strong>Global network</strong> which means all input vectors produce activation. When the network is trained, the memories correspond to minima of Energy function. The weight W is a symmetric matrix with zero diagonal elements which is necessary conditions for the convergence of an asynchronous totally connected network to a stable state. In a stable state the energy cannot be further reduced.</td>
<td><strong>Local network</strong> which means that the input vectors in the close neighborhood of centers produce activation and the other inputs have negligible values. whereas the output of hidden layer is nonlinear and the output of output layer is linear. In addition, RBF is symmetry around the center and maximum near this center, thus the center and the width are necessary conditions for the convergence connected network to a stable state.</td>
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<td>Hopfield networks using unsupervised learning to find patterns in the input space in order to train it. The Hopfield network training algorithm has two basic phases: storage and retrieval. The activation is the weighted sum of the inputs, where the activation is passed through a threshold function.</td>
<td>RBF neural networks are often trained by hybrid learning algorithms, in which centers and widths of the hidden units are first obtained by unsupervised learning, after which the output weights are obtained by using supervised learning by minimizing the error.</td>
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