Effect of Rotation, Gravity Field and Initial Stress on Generalized Magneto-Thermoelastic Rayleigh Waves in a Granular Medium

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Abstract. In the present paper, the effect of rotation, magnetic field, initial stress and gravity field on Rayleigh waves velocity in an elastic half-space of granular medium is investigated. The solution of the problem is obtained by using Lame's potential techniques. The frequency equation of Rayleigh waves in the form of a determinant containing a term involving the coefficient of friction of a granular medium is obtained. The numerical calculations are carried out for the frequency equation of Rayleigh waves velocity. The results are displayed graphically to illustrate the effect of rotation, relaxation times, magnetic and gravity fields and initial stress on Rayleigh wave velocity are very pronounced. Relevant results of previous investigations are deduced as special cases from this study.

Keywords: Rayleigh waves, rotation, magneto-thermoelastic, relaxation times, granular medium, initial stress and gravity field

1. Introduction

The granular medium under consideration is a discontinuous one and is composed of numerous large or small grains. Unlike a continuous body each element or grain cannot

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only translate but also rotate about its center of gravity. This motion is the characteristic of
the medium and has an important effect upon the equations of motion to produce internal
friction. It was assumed that the medium contains so many grains that it will never be
separated from each other during the deformation and that each grain has perfect
thermo-elasticity. The dynamical problem of magneto-thermoelasticity has received much
attention in the literature during the past decade. In recent years the theory of
magneto-thermoelasticity which deals with the interactions among strain, temperature and
electromagnetic fields has drawn the attention of many researchers because of its extensive
uses in diverse fields, such as geophysics for understanding the effect of Earth's magnetic
field on seismic waves, damping of acoustic waves in magnetic field, emissions of
electromagnetic radiations from nuclear devices, development of highly sensitive
superconduction magnetometer, electrical power engineering, optics etc. The dynamical
problem in granular medium of generalized magneto-thermoelastic waves has been studied
in recent times, necessitated by its possible applications in Soil mechanics, earthquakes
Science, Geophysics, mining Engineering and Plasma physics, etc. In the motion of a
deformable body, quantities at each point of it can't be recognized with out some
uncertainty, and must be regarded as average values over a small region about the point.
Dynamical problems of thermoelastic solids is investigated by El-Naggar and Abd-Alla
[1]. Some problems taking into account the influence of granular are discussed by Alexei
and Thomas [2]. Solutions of some problems in thermoelasticity under assumption special
cases are investigated by L. You [3] and El-Naggar [4]. The dynamical problem of a
generalized thermoelastic granular infinite cylinder under initial stress has been illustrated
by El-Naggar [5]. Variational theory for linear magneto-electro-elasticity is discussed by
He [6]. Rayleigh wave propagation of thermoelasticity or generalized thermoelasticity is
pointed out by Dawan and Chakraporty [7]. The generalized magneto-thermoelastic
Rayleigh waves in a granular medium under influence of gravity field and initial stress is
discussed by Abd-Alla et al. [8] and El-Naggar et al. [9]. Rayleigh waves in
magneto-thermo-microelastic half-space under initial stress is illustrated by El-Naggar
[10]. Generation of waves in an infinite micropolar elastic solid Body under initial stress is
propagation of waves in granular medium. Recently, Rayleigh waves in a thermoelastic
granular medium under initial stress has been explained by Ahmed [13]. Problem of
Rayleigh waves propagation in an orthotropic thermoelastic medium under gravity and
initial stress is discussed by Abd-Alla and Ahmed [14].

The present paper focuses on the study of Rayleigh waves with models fitting with
the earth. The granular medium under consideration is a discontinuous one and is
composed of large or small grains. The effects of rotation, magnetic and gravity fields and
initial stress on the propagation of Rayleigh waves in a granular medium under incremental
thermal stresses are investigated. The medium under consideration is granular half-space
overlying by a different granular layer and initial stress present in this medium have
considerable effect in the propagation of Rayleigh waves, Ahmed[13]. The wave velocity
equation has been derived in the form of twelfth-order determinant. The roots of this
equation are in general complex, the real part measures the Rayleigh wave velocity and the
imaginary part of an appropriate root measures the attenuation of the waves. When there is no coupling between the temperature and the strain field in the absence of the initial stress, the derived frequency equation reduces to an equation in the form of ninth-order determinant. Also, the classical frequency when both media are elastic and the other effects are absent is obtained.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( \theta )</td>
<td>is the ratio of the coefficients of heat transfer,</td>
</tr>
<tr>
<td>( t )</td>
<td>is the time,</td>
</tr>
<tr>
<td>( \vec{H}_0 )</td>
<td>is the constant primary magnetic field vector,</td>
</tr>
<tr>
<td>( E )</td>
<td>is the electric intensity,</td>
</tr>
<tr>
<td>( s )</td>
<td>is the specific heat per unit mass,</td>
</tr>
<tr>
<td>( \rho )</td>
<td>is the density of the material,</td>
</tr>
<tr>
<td>( P )</td>
<td>is the initial stress,</td>
</tr>
<tr>
<td>( F )</td>
<td>is the coefficient of friction,</td>
</tr>
<tr>
<td>( \tau_1, \tau_2 )</td>
<td>are the mechanical relaxation times,</td>
</tr>
<tr>
<td>( g )</td>
<td>is the earth gravity,</td>
</tr>
<tr>
<td>( \vec{u} )</td>
<td>is the component of displacement vector,</td>
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<tr>
<td>( T_i )</td>
<td>is the initial temperature,</td>
</tr>
<tr>
<td>( \vec{j} )</td>
<td>is the electric current density,</td>
</tr>
<tr>
<td>( \alpha_{ij} )</td>
<td>is the rotation vector,</td>
</tr>
<tr>
<td>( \mu_e )</td>
<td>is the magnetic permeability,</td>
</tr>
<tr>
<td>( k )</td>
<td>is the wave number,</td>
</tr>
<tr>
<td>( c )</td>
<td>is speed of Rayleigh waves,</td>
</tr>
<tr>
<td>( \lambda, \mu )</td>
<td>are the same constants,</td>
</tr>
<tr>
<td>( M )</td>
<td>is the third elastic constant,</td>
</tr>
<tr>
<td>( \eta )</td>
<td>is thermal conductivity,</td>
</tr>
<tr>
<td>( K )</td>
<td>is the thickness.</td>
</tr>
<tr>
<td>( \vec{h} )</td>
<td>is the perturbed magnetic field over the constant primary magnetic field,</td>
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2. **Formulation of the problem**

Let us consider a system of orthogonal Cartesian axes oxyz, the interface and the free surface of the granular layer resting on the granular half-space of different material being the planes \( z = K \) and \( z = 0 \), respectively. The origin O is any point on the free surface, \( z - axis \) is positive along the direction towards the exterior of the half-space, and the \( x - axis \) is positive along the direction of Rayleigh waves propagation. The both media are under initial compression stress \( P \) along \( x \)-direction and primary magnetic field \( \vec{H} \) parallel to \( y \)-axis, as well as gravity field and incremental thermal stresses as shown in (Fig. 1).

![Fig. 1. Displaying of the problem.](image-url)
The state of deformation in the granular medium is described by the displacement vector $\vec{u}(u, 0, w)$ of the centre of gravity of a grain and the rotation vector $\vec{\xi}(\xi, \eta, \zeta)$ of the grain about its centre of gravity. The stress-temperature equation is given by
\[ \kappa \nabla^2 T = \rho S \frac{\partial}{\partial t} \left[ 1 + \tau_2 \frac{\partial}{\partial t} \right] T + \gamma T_o \frac{\partial}{\partial t} \left[ 1 + \tau_2 \frac{\partial}{\partial t} \right] \nabla \cdot \vec{u}. \] (1)

The vector equation of motion is
\[ \tau_{ij,i} + F_j = \rho \left[ \ddot{u} + \left( \vec{\Omega} \times \vec{\Omega} \times \ddot{u} \right) \right] \] (2)
where $\vec{\Omega} \times \vec{\Omega} \times \ddot{u}$ is the centripetal acceleration due to the time varying motion only and $2\vec{\Omega} \times \ddot{u}$ is the Coriolis acceleration. Here $\ddot{u}$ is the dynamic displacement vector measured from steady state deformed position and supposed to be small, $\vec{F}$ is the Lorentz's body forces vector, then equation (2) is
\[ \begin{aligned}
\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{31}}{\partial z} + \frac{\rho}{2} \frac{\partial \omega}{\partial x} + F_x &= \rho \left( \frac{\partial^2 u}{\partial t^2} + 2 \vec{\Omega} \frac{\partial w}{\partial t} - \Omega^2 u \right), \\
\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{32}}{\partial z} + F_y &= 0, \\
\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} + \frac{\rho}{2} \frac{\partial \omega}{\partial x} + F_z &= \rho \left( \frac{\partial^2 w}{\partial t^2} - 2 \vec{\Omega} \frac{\partial u}{\partial t} - \Omega^2 w \right)
\end{aligned} \] (3)
where, $F = (-\mu_e H_o^2 \nabla^2 \Phi, 0, \mu_e H_o^2 \nabla^2 \Phi)$ and
\[ \begin{aligned}
\tau_{32} - \tau_{23} + \frac{\partial M_{11}}{\partial x} + \frac{\partial M_{12}}{\partial z} &= 0, \\
\tau_{31} - \tau_{13} + \frac{\partial M_{12}}{\partial x} + \frac{\partial M_{13}}{\partial z} &= 0, \\
\tau_{12} - \tau_{21} + \frac{\partial M_{13}}{\partial x} + \frac{\partial M_{13}}{\partial z} &= 0.
\end{aligned} \] (4)

The stress-strain-temperature relations have the forms
\[ \begin{aligned}
\tau_{11} &= (\lambda + 2\mu + P) \frac{\partial u}{\partial x} + (\lambda + P) \frac{\partial \omega}{\partial z} - \gamma (1 + \tau_1 \frac{\partial}{\partial t}) T, \\
\tau_{33} &= \lambda \frac{\partial u}{\partial x} + (\lambda + 2\mu) \frac{\partial \omega}{\partial z} - \gamma (1 + \tau_1 \frac{\partial}{\partial t}) T, \\
\tau_{13} &= \mu (\frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial z}) + F \frac{\partial \eta}{\partial t}, \\
\tau_{12} &= -F \frac{\partial \xi}{\partial t}, \\
\tau_{32} &= -F \frac{\partial \xi}{\partial t}
\end{aligned} \] (5)
and
\[ \begin{aligned}
M_{11} &= M \frac{\partial \xi}{\partial x}, \\
M_{13} &= M \frac{\partial \xi}{\partial z}, \\
M_{33} &= M \frac{\partial \zeta}{\partial z}, \\
M_{21} &= M \frac{\partial \zeta}{\partial x}, \\
M_{22} &= M \frac{\partial \zeta}{\partial x}.
\end{aligned} \] (6)

From equations (5) and (6), equations (3) and (4) tend to
\[ (\lambda + 2\mu + P + \mu_e H_o^2) \frac{\partial^2 u}{\partial x^2} + (\lambda + \mu + P + \mu_e H_o^2) \frac{\partial^2 \omega}{\partial z^2} + \left(\mu + \frac{p}{2}\right) \frac{\partial^2 u}{\partial x^2} + \left(\mu + \frac{p}{2}\right) \frac{\partial^2 \omega}{\partial z^2} + (\lambda + 2\mu + P) \frac{\partial \omega}{\partial z} - \gamma (1 + \tau_1 \frac{\partial}{\partial t}) T - F \frac{\partial \eta}{\partial t} - \rho g \frac{\partial \omega}{\partial x} = \rho \left( \frac{\partial^2 u}{\partial t^2} + 2 \vec{\Omega} \frac{\partial w}{\partial t} - \Omega^2 u \right), \] (7)
\[ -\gamma \frac{\partial}{\partial x} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T - F \frac{\partial \eta}{\partial t} - \rho g \frac{\partial \omega}{\partial x} = \rho \left( \frac{\partial^2 u}{\partial t^2} + 2 \vec{\Omega} \frac{\partial w}{\partial t} - \Omega^2 u \right), \] (8)
Effect of the rotation

\( \lambda + \mu + \rho \left( \frac{\partial^2 w}{\partial x^2} + (\mu - \frac{\rho}{2}) \frac{\partial^2 w}{\partial y^2} \right) \frac{\partial^2 u}{\partial x \partial y} + (\mu - \frac{\rho}{2}) \frac{\partial^2 w}{\partial x^2} + (\lambda + 2\mu + \mu e H_0^2) \frac{\partial^2 w}{\partial y^2} \)

\(-\gamma \frac{\partial}{\partial z} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \frac{\partial}{\partial z} - \rho \left( \frac{\partial^2 \eta}{\partial x^2} + 2\Omega \frac{\partial u}{\partial t} - \Omega^2 \right) \),

\( \nabla^2 \xi - s_2 \frac{\partial^2 \xi}{\partial t^2} = 0, \)

\( \nabla^2 (\eta + \omega_2) - s_2 \frac{\partial \eta}{\partial t} = 0, \)

\( \nabla^2 \xi - s_2 \frac{\partial^2 \xi}{\partial t^2} = 0. \)

3. Solution of the problem

We assume that the displacements \( \vec{u} \) are derivable from the displacement potentials \( \Phi \) and \( \Psi \) by the relation

\( \vec{u} = \nabla \Phi + \nabla \times \vec{\Psi}, \quad \vec{\Psi} = (0, \Psi, 0) \)

which reduces to

\( u = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \quad w = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}. \)

Substituting from equations (14) into equations (7), (9) and (11), the wave equations tend to

\[ \alpha^2 \nabla^2 \Phi - \frac{\gamma}{\rho} (1 + \tau_2 \frac{\partial}{\partial t}) \nabla T - \frac{\partial \Psi}{\partial x} - \frac{\partial^2 \Phi}{\partial z^2} - 2\Omega \frac{\partial \Psi}{\partial t} + \Omega^2 \Phi = 0, \]

\[ \beta^2 \nabla^2 \Psi - s_1 \frac{\partial \eta}{\partial t} + g \frac{\partial \Phi}{\partial x} - \frac{\partial^2 \Psi}{\partial z^2} - 2\Omega \frac{\partial \Phi}{\partial t} + \Omega^2 \Psi = 0, \]

\[ \nabla^2 \eta - s_2 \frac{\partial \eta}{\partial t} + \nabla^4 \Psi = 0 \]

where,

\[ s_1 = \frac{F}{\rho}, \quad s_2 = \frac{F}{M}, \quad \alpha^2 = \frac{\lambda + 2\mu + \mu e H_0^2}{\rho}, \quad \beta^2 = \frac{2\mu - p}{2\rho}. \]

Using equations (14), the heat conduction equation (1) becomes

\[ \kappa \nabla^2 T = \rho s \frac{\partial}{\partial t} \left[ 1 + \tau_2 \frac{\partial}{\partial t} \right] T + \gamma T_0 \frac{\partial}{\partial t} \left[ 1 + \tau_2 \frac{\partial}{\partial t} \right] \nabla^2 \Phi. \]

From equations (15) and (19), we get

\[ \nabla^2 - \frac{1}{\rho s} \frac{\partial}{\partial t} \left( 1 + \tau_2 \frac{\partial}{\partial t} \right) \left[ \alpha^2 \nabla^2 \Phi - \frac{\partial^2 \Phi}{\partial z^2} - g \frac{\partial \Phi}{\partial x} - 2\Omega \frac{\partial \Phi}{\partial t} + \Omega^2 \Phi \right] - \epsilon \frac{\partial}{\partial t} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) \nabla^2 \Phi = 0 \]

where,

\[ \dot{u} = \frac{\kappa}{\rho s} \epsilon, \quad \epsilon = \frac{\tau_0 y^2}{\rho k}. \]

From equations (16) and (17), by eliminated \( \eta \), we get

\[ (\nabla^2 - s_2 \frac{\partial}{\partial t}) \left( \beta^2 \nabla^2 \Psi - \frac{\partial^2 \Psi}{\partial z^2} + \Omega^2 \Psi + g \frac{\partial \Phi}{\partial x} - 2\Omega \frac{\partial \Phi}{\partial t} \right) + s_1 \nabla^4 \frac{\partial \Psi}{\partial t} = 0. \]

For a plane harmonic wave propagation in the x-direction, we assume

\[ \Phi = \Phi_1 \exp(ik(x - ct)), \]

(23)
\[ \Psi = \Psi_1 \exp(ik(x - ct)), \quad 2018 \]

(24)

\[ (\xi, \eta, \zeta) = (\xi_1, \eta_1, \zeta_1) \exp(ik(x - ct)). \]

(25)

Substituting from equation (25) into equations (8), (10) and (12), one may obtain

\[ \begin{align*}
D\xi_1 - ik\zeta_1 &= 0, \\
D^2\xi_1 + q^2\xi_1 &= 0, \\
D^2\zeta_1 + q^2\zeta_1 &= 0,
\end{align*} \]

(26a), (26b), (26c)

where,

\[ q^2 = ikcs_2 - k^2, \quad D \equiv \frac{d}{dx}. \]

Solutions of equations (26b) and (26c) are

\[ \begin{align*}
\xi_1 &= A_1 e^{iqx} + A_2 e^{-iqx}, \\
\zeta_1 &= B_1 e^{iqx} + B_2 e^{-iqx},
\end{align*} \]

(27)

where, \( A_1, A_2, B_1 \) and \( B_2 \) are arbitrary constants, from equations (26a) and (27) we obtain

\[ q(A_1 e^{iqx} - A_2 e^{-iqx}) - k(B_1 e^{iqx} + B_2 e^{-iqx}) = 0, \]

(28)

then

\[ A_j = \frac{(-1)^{j-1}k}{q} B_j, \quad j = 1, 2. \]

(29)

Substituting from equations (23) and (24) into equations (20) and (22), we obtain

\[ \begin{align*}
&\alpha^2 D^4 + (k^2(c^2 - 2\alpha^2) + \frac{ikc}{\hat{u}}(\alpha^4 a^2 \Gamma_2 + \hat{u}e\Gamma_1\Gamma_3) + \Omega^2)D^2 + k^4(\alpha^2 - c^2) \\
&+ \frac{ikc \Gamma_2(c^2 - \alpha^2)}{\hat{u}} - i\epsilon c k^2 \Gamma_1 \Gamma_3 + \frac{ikc \Gamma_2}{\hat{u}} \Omega^2] \Phi_1 - ikg[(D^2 - k^2)\left(1 - \frac{2\Omega c}{g}\right) \\
&+ \frac{ikc \Gamma_2}{\hat{u}}] \Psi_1 = 0, \\
&(\beta^2 - ic\Gamma_1)D^4 + [k^2(c^2 - 2\beta^2) + i\epsilon c \beta^2 c_2 - k^2 s_2)]D^2 + k^4(\beta^2 - c^2) \\
&+ i\epsilon c (c_2(c^2 - \beta^2) - k^2 s_2)] \Psi_1 + ikg[D^2 - k^2 + ic\Gamma_2 - \frac{2\Omega c}{g}] \Phi_1 = 0
\end{align*} \]

(30)

(31)

where,

\[ \Gamma_1 = 1 - ic\tau_1, \quad \Gamma_2 = 1 - ic\tau_2, \quad \Gamma_3 = 1 - ic\tau_2 \delta. \]

The solutions of equations (30) and (31) take the forms

\[ \Phi_1 = \sum_{j=1}^{4} C_je^{ikN_j x} + D_j e^{-ik\xi x}, \quad \Psi_1 = \sum_{j=1}^{4} E_je^{ik\xi_j x} + F_j e^{-ik\xi_j x} \]

(32)

where, the constants \( E_j \) and \( F_j \) are related with the constants \( C_j \) and \( D_j \) as the form

\[ \begin{align*}
&E_j = m_j C_j, \quad F_j = m_j D_j, \quad j = 1, 2, 3, 4, \\
m_j = \frac{-i}{g(-k(N_j^2 + 1) + \frac{ick}{\hat{u}})} \left[ \alpha^2 k^2 N_j^4 - (k^2(c^2 - 2\alpha^2) \\
+ \frac{ikc}{\hat{u}} (\alpha^2 \Gamma_2 + \epsilon\hat{u}\Gamma_1\Gamma_3) + \Omega^2)N_j^2 + (\alpha^2 - c^2)(k^2 - \frac{ic\Gamma_2}{\hat{u}}) \\
- i\epsilon c \alpha^2 \Gamma_1 \Gamma_3 + \frac{i\epsilon c \Gamma_2}{\hat{u}} \Omega^2] \right]
\end{align*} \]

(33)

where, \( N_1, N_2, N_3 \) and \( N_4 \) are taken to be the complex roots of equation

\[ \begin{align*}
N^8 + t_1 N^6 + t_2 N^4 + t_3 N^2 + t_4 &= 0,
\end{align*} \]

(34)
Effect of the rotation

From equations (16), (24), (25), (32a) and (32b), we obtain

$$t_1 = -2k^2 + c^2k^2 + \frac{ic(k^2r_2 + 3 + k^2r_3)}{\omega^2} + \frac{\Omega^2}{\omega^2} + \frac{1}{\beta^2 - ikcs_1}(k^2(c^2 - 2\beta^2))$$  \hspace{1cm} (35a)

$$t_2 = k^4 - c^2k^4 - \frac{ic(k^2r_1 + 3)}{\omega^2} + \frac{ik^2r_2\Omega^2}{\beta^2 - ikcs_1} + \frac{k^4(\beta^2 - c^2)}{\beta^2 - ikcs_1} + \frac{ik^3c(s_2(c^2 - \beta^2) - k^2s_1)}{\beta^2 - ikcs_1}$$

$$+ \frac{1}{\alpha^2(\beta^2 - ikcs_1)}[k^2(c^2 - 2\beta^2) + ic(k^2s_2 - k^2s_1)[k^2(c^2 - 2\alpha^2)]$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3)}{\omega^2} - \frac{\Omega^2}{\omega^2} - \frac{k^2g}(1 - \frac{2\alpha c}{g})(ic(k^2 + 2\alpha^2))$$

$$t_3 = \frac{1}{\alpha^2(\beta^2 - ikcs_1)}[(k^2(c^2 - 2\beta^2) + ic(k^2s_2 - k^2s_1))(k^4(\alpha^2 - c^2))$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3 + ic(k^2r_2\Omega^2)}{\omega^2} - g^2k^2(1 - \frac{2\alpha c}{g})(ic(k^2s_2 + 2\alpha c))$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3 + ic(k^2r_2\Omega^2)}{\omega^2}$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3 + ic(k^2r_2\Omega^2)}{\omega^2}$$

$$t_4 = \frac{1}{\alpha^2(\beta^2 - ikcs_1)}[(k^4(\beta^2 - c^2) + ic(k^2s_2(c^2 - \beta^2) - k^2s_1))(k^4(\alpha^2 - c^2))$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3 + ic(k^2r_2\Omega^2)}{\omega^2} - g^2k^2(k + 2\alpha c)(ic(k^2s_2 + 2\alpha c))$$

$$+ \frac{ic(k^2r_2 + 3 + k^2r_3 + ic(k^2r_2\Omega^2)}{\omega^2}$$

From equations (16), (24), (25), (32a) and (32b), we obtain

$$\eta_1 = \sum_{j=1}^{4} \left[ \frac{1}{\alpha^4} \left( k(\beta^2(N_j^2 + 1) - c^2) - \frac{\Omega^2}{\omega^2} \right) m_j - ig - 2\Omega \right]$$

$$C_j e^{ikN_j z} + D_j e^{-ikN_j z}$$

(36)

using equations (43a) and (43b), we obtain

$$T = \frac{\rho}{\gamma^4} \sum_{j=1}^{4} \left[ -\alpha^2 k^2(N_j^2 + 1) + k^2c^2 - ik(g - 2\Omega)c m_j + \Omega^2 \right]$$

$$C_j e^{ikN_j z} + D_j e^{-ikN_j z} e^{ik(x - ct)}$$

(37)

With the lower medium, we use the symbols with dashes, for $\xi_1$, $\zeta_1$, $\eta_1$, $T$, $\Phi_1$, $\Psi_1$ and $q$, for $z > K$,

$$\xi_1 = -\frac{k}{q} - B_2 e^{-iqa} z, \quad \zeta_1 = B_2 e^{-iqa} z,$$

$$\eta_1 = \sum_{j=1}^{4} \left[ \frac{1}{i\alpha^4} \left( k(\beta^2(N_j^2 + 1) - c^2) - \frac{\Omega^2}{\omega^2} \right) m_j - ig - 2\Omega \right]$$

$$\frac{d}{dz} \left[ \frac{\rho}{\gamma^4} \sum_{j=1}^{4} \left[ -\alpha^2 k^2(N_j^2 + 1) + k^2c^2 - ik(g - 2\Omega)c m_j + \Omega^2 \right] e^{-ik(x - ct)} \right]$$

(38)

$$\Phi_1 = \sum_{j=1}^{4} D_j e^{-i\alpha N_j z}, \quad \Psi_1 = \sum_{j=1}^{4} F_j e^{-i\alpha N_j z}.$$
4. Boundary conditions and frequency equation

In this section, we obtain the frequency equation for the boundary conditions which specify on the interface $z = K$, i.e.,

(i) $u = u\,'$,  (ii) $w = w\,'$,  (iii) $\xi = \xi\,'$,  (iv) $\eta = \eta\,'$,  (v) $\zeta = \zeta\,'$,  (vi) $M_{33} = M_{33}'$,  (vii) $M_{31} = M_{31}'$,  (viii) $M_{32} = M_{32}'$,  (ix) $\tau_{33} + \bar{\tau}_{33} = \tau_{33}' + \bar{\tau}_{33}'$,  

\[ (x) \quad \tau_{31} + \bar{\tau}_{31} = \tau_{31}' + \bar{\tau}_{31}', \quad (xi) \quad \tau_{32} + \bar{\tau}_{32} = \tau_{32}' + \bar{\tau}_{32}', \quad (xii) \quad T = T\,', \quad (xiii) \quad \frac{\partial T}{\partial z} + \theta T = \frac{\partial T}{\partial z} + \theta T\,'. \]

The boundary condition on the free surface $z = 0$ are

\[ \text{(xiv) } M_{33} = 0, \quad \text{(xv) } M_{31} = 0, \quad \text{(xvi) } M_{32} = 0, \quad \text{(xvii) } \tau_{33} + \bar{\tau}_{33} = 0, \quad \text{(xviii) } \tau_{31} + \bar{\tau}_{31} = 0, \quad \text{(xix) } \tau_{32} + \bar{\tau}_{32} = 0, \quad \text{(xx) } \frac{\partial T}{\partial z} + \theta T = 0, \]

from the conditions (iii), (v), (vi), (vii), we get

\[ B_1 e^{iqK} - B_2 e^{-iqK} = -B_2 e^{-iqK}, \quad B_1 e^{iqK} + B_2 e^{-iqK} = B_2 e^{-iqK}, \]

\[ M(B_1 e^{iqK} - B_2 e^{-iqK}) = -M B_2 e^{-iqK}, \quad M(B_1 e^{iqK} + B_2 e^{-iqK}) = -M B_2 e^{-iqK}. \]

hence

\[ B_1 = B_2 = B_2\,' = 0, \quad \xi = \xi\,' = \zeta\,' = \zeta\,' = 0. \]

The other significant boundary conditions are responsible for the following relations

(i) \[ \frac{4}{j_1} (1 - N_j m_j) C_j e^{ikN_j K} + (1 + N_j m_j) D_j e^{-ikN_j K} - (1 - N_j m_j) D_j \,' e^{-ikN_j K} = 0, \]

(ii) \[ \sum_{j=1}^{4} \left( N_j + m_j \right) C_j e^{ikN_j K} + (m_j - N_j) D_j e^{-ikN_j K} - (m_j - N_j) D_j \,' e^{-ikN_j K} = 0, \]

(iv) \[ \sum_{j=1}^{4} \left( \frac{1}{s_1} \right) (\beta^2 k (N_j^2 + 1) - kc^2 - \frac{\Omega^2}{c}) m_j \,' - ig - 2i\Omega c) (C_j e^{ikN_j K} + D_j e^{-ikN_j K}) \]

\[- \sum_{j=1}^{4} \left( \frac{1}{s_1} \right) (\beta^2 k (N_j^2 + 1) - kc^2 - \frac{\Omega^2}{c}) m_j \,' - ig - 2i\Omega c) D_j \,' e^{-ikN_j K} = 0, \]

(viii) \[ \sum_{j=1}^{4} MN_j \left( \frac{1}{s_1} \right) (\beta^2 k (N_j^2 + 1) - kc^2) m_j \,' - ig) - k^2 m_j \,' (N_j \,'^2 + 1) D_j \,' e^{-ikN_j K} = 0, \]
(ix) \[ \sum_{j=1}^{4} [-k^2(\lambda(1 - N_j m_j) + N_j(\lambda + 2\mu)(m_j + N_j))]C_j e^{ikN_j^K} - [-k^2(\lambda(1 + N_j m_j) - N_j(\lambda + 2\mu)(m_j - N_j))]D_j e^{-ikN_j^K} \\
+ [k^2(N_j^2 + 1)(\rho \gamma \alpha^2 - \mu e H_o^2) - k \rho \gamma (kc^2 - igm_j)](C_j e^{ikN_j^K} + D_j e^{-ikN_j^K}) \\
- [-k^2(\lambda(1 + N_j m_j) - N_j(\lambda + 2\mu)(m_j - N_j))]D_j e^{-ikN_j^K} \\
- [k^2(N_j^2 + 1)(\rho \gamma \alpha^2 - \mu e H_o^2) - k \rho \gamma (kc^2 - igm_j)]D_j e^{-ikN_j^K} = 0, \]

(x) \[ \sum_{j=1}^{4} [\mu k^2(m_j(N_j^2 - 1)) + \frac{iFk}{s_1}(\beta^2 k(N_j^2 + 1) - kc^2)m_j - ig)] \\
(C_j e^{ikN_j^K} + D_j e^{-ikN_j^K}) + 2\mu k^2 N_j (-C_j e^{ikN_j^K} + D_j e^{-ikN_j^K}) \\
- [\mu k^2(N_j^2 - 1) + 2N_j] + \frac{iFk}{s_1}(\beta^2 k(N_j^2 + 1) - ig)]D_j e^{-ikN_j^K} = 0, \]
\[-\rho \frac{\partial}{\partial \gamma} \left[-\alpha^2 k^2 (N_j - 1) + k^2 c^2 - ik(g - 2\Omega c) m_j + \Omega^2 \right] D_j e^{-ikN_j k} = 0,\]

\[\text{(xii) } \sum_{j=1}^{4} \frac{\rho}{\gamma} \left[-\alpha^2 k^2 (N_j^2 + 1) + k^2 c^2 - ik(g - 2\Omega c) m_j + \Omega^2 \right] [(\theta + ikN_j) C_j e^{ikN_j k} \right. \]
\[\left. + (\theta - ikN_j) D_j e^{-ikN_j k} \right] \times (\theta - ikN_j) D_j e^{-ikN_j k} = 0,\]

\[\text{(xvi) } \sum_{j=1}^{4} \left( \frac{1}{s_1 C} (\beta^2 k(N_j^2 + 1) - kc^2) m_j - ig(M N_j(C_j - D_j)) + k^2 m_j(N_j^2 + 1) \right) \]
\[\quad \times (C_j + D_j) = 0,\]

\[\text{(xvii) } \sum_{j=1}^{4} -\lambda k^2 [(1 - N_j m_j) C_j + (1 + N_j m_j) D_j] + (\lambda + 2\mu)[\alpha^2 k^2 (m_j + N_j) C_j \]
\[+ (m_j - N_j) D_j] + [k^2 (N_j^2 + 1)(\rho_1^2 \alpha^2 + \mu_1 H_j^2) - k\rho (kc^2 - igm_j))(C_j + D_j) \]
\[ \quad = 0,\]

\[\text{(xviii) } \sum_{j=1}^{4} -2k^2 N_j(C_j - D_j) + k^2 m_j(N_j^2 - 1) + \frac{iFk}{s_1} [(\beta^2 k(N_j^2 + 1) - kc^2) m_j \]
\[\quad - ig](C_j + D_j) = 0,\]

\[\text{(xx) } \sum_{j=1}^{4} [-\alpha^2 k^2 (N_j^2 + 1) + k^2 c^2 - ik(g - 2\Omega c) m_j + \Omega^2 \right] [(\theta + ikN_j) C_j \]
\[\left. + (\theta - ikN_j) D_j \right] = 0\]

By eliminating \(C_j, D_j\) and \(D_j\) from the relevant results, we will get a determinant of a twelfth-order form which determines the wave velocity and attenuation coefficient.

### 5- Special cases and discussion

#### 5.1 In presence of rotation and absence of the gravity field

In this case, we put \((g = 0, \Omega = 0)\) equations (30) and (31) take the forms

\[\alpha^2 D^4 + (k^2(c^2 - 2\alpha^2) + \frac{iKC}{\alpha} (\alpha^2 \Gamma_2 + \hat{u} \hat{u} \Gamma_1 \Gamma_3) + \Omega^2) D^2 \]
\[+ k^4 (\alpha^2 - c^2) + \frac{iKC}{\alpha} \left[\Gamma_2 (c^2 - \alpha^2 + k^2) - \hat{u} \hat{u} \Gamma_1 \Gamma_3 \right] \Phi_1 + 2ic\Omega \Psi_1 = 0,\]

\[\text{(c) } (\beta^2 - ikcs_1) D^4 + 2[k^2(c^2 - 2\beta^2) + ikc(s_2 \beta^2 - s_1 k^2)] D^2 \]
\[+ 2k^4(2\beta^2 - c^2) + 2ics_2 k^3(c^2 - \beta^2) - ics_1 k^5] \Psi_1 - 2ic\Omega \Phi_1 = 0\]
which take the solutions
\[ \Phi_1 = \sum_{j=1}^{4} L_j e^{ikN_j z} + Q_j e^{-ikN_j z}, \quad \Psi_1 = \sum_{j=1}^{4} n_j e^{ikN_j z} + p_j e^{-ikN_j z}, \]  
where,
\[ n_j = L_j L_j, \quad p_j = L_j Q_j \]
where,
\[ I_j = \frac{-2i c k \Omega}{[\alpha^2 k^2 N_j^4 - (k^2 (c^2 - 2 \alpha^2) + \frac{IC}{\alpha^2 (\alpha^2 + 1 + \epsilon \alpha^2 + 2 \Omega^2))} N_j^4 + \frac{1}{[\alpha^2 - c^2] (k^2 - i c k t) \epsilon \alpha^2 + 1 + \epsilon \alpha^2 + 2 \Omega^2]} \]
\[ R^8 + \Lambda_1 R^6 + \Lambda_2 R^4 + \Lambda_3 R^2 + \Lambda_4 = 0, \]
\[ \Lambda_1 = \frac{-2i c (c^2 - 2 \beta^2)}{[\beta^2 - i k e \Sigma_1]} - \frac{k}{k^2 \alpha^2} \]
\[ \Lambda_2 = \frac{1 - \alpha^2}{[\beta^2 - i k e \Sigma_2]} \left[ \sum \left( k^2 (c^2 - 2 \beta^2) + i c k (\beta^2 s_2 - k^2 s_1) \right) \right] \]
\[ \Lambda_3 = \frac{2 \alpha^2}{[\beta^2 - i k e \Sigma_2]} \left[ \left( c^2 - 2 \beta^2 + \frac{C}{k^2} \right) - \frac{\epsilon \alpha^2}{k^4} \right] \]
\[ \Lambda_4 = \frac{2 \alpha^2}{[\beta^2 - i k e \Sigma_2]} \left[ \left( c^2 - 2 \beta^2 + \frac{C}{k^2} \right) - \frac{\epsilon \alpha^2}{k^4} \right] \]

Also
\[ \eta_1 = \sum_{j=1}^{4} \left[ \frac{1}{s_1 c} \left( k (\beta^2 (R^2) + 1) - c^2 - \frac{C}{k} \right) I_j - 2 i \Omega \epsilon \right] (L_j e^{ikR_j z} + Q_j e^{-ikR_j z}) \]
\[ T = \rho \sum_{j=1}^{4} \left[ -\alpha^2 k^2 (R_j + 1) + k^2 c^2 + i k (2 \Omega \epsilon) I_j + \Omega^2 \right] (L_j e^{ikR_j z} + Q_j e^{-ikR_j z}) \]

With the lower medium, we use the symbols with dashes, for \( \xi_1, \zeta_1, \eta_1, T, \Phi_1, \Psi_1 \) and \( q \), for \( z > K \),
\[ \xi_1 = -\frac{k}{q} B_2 e^{-iq z}, \quad \zeta_1 = B_2 e^{-iq z}, \]
\[ \eta_1 = \sum_{j=1}^{4} \left[ \frac{1}{s_1 c} \left( k (\beta^2 (R_j + 1) - c^2) - \frac{C}{k} \right) I_j - 2 i \Omega \epsilon \right] Q_j e^{-ikR_j z}, \]
\[ T = \rho \sum_{j=1}^{4} \left[ -\alpha^2 k^2 (R_j + 1) + k^2 c^2 + 2 i \Omega \epsilon k l_j + \Omega^2 \right] Q_j e^{-ik(R_j z + \epsilon + c t)} \]
\[ \Phi_1 = \sum_{j=1}^{4} Q_j e^{-ikR_j z}, \quad \Psi_1 = \sum_{j=1}^{4} p_j e^{-ikR_j z} \]
$$d_{1j} = (1 - R_j l_j) L_j e^{i k R_j K} + (1 + R_j l_j) Q_j e^{-i k R_j K}, \quad d_{1j} = (1 - R_j l_j) Q_j e^{-i k R_j K},$$
$$d_{2j} = (R_j + l_j) L_j e^{i k R_j K} + (l_j - R_j) Q_j e^{-i k R_j K}, \quad d_{2j} = (l_j - R_j) Q_j e^{-i k R_j K},$$
$$d_{3j} = \frac{1}{s_1 c} (-i k \beta^2 (R_j^2 + 1) + i k c^2 - \frac{\Omega^2}{i k}) l_j + (-2 \Omega c)(L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{3j} = \frac{1}{s_1 c} (-i k \beta^2 (R_j^2 + 1) + i k c^2 - \frac{\Omega^2}{i k}) l_j + (-2 \Omega c) Q_j e^{-i k R_j K},$$
$$d_{4j} = M V_j \left( \frac{1}{s_1 c} (-i k \beta^2 (R_j^2 + 1) + i k c^2 - \frac{\Omega^2}{i k}) l_j + (-2 \Omega c) \right) (L_j e^{i k R_j K} - Q_j e^{-i k R_j K}) + k^2 L_j (R_j^2 + 1) (L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{4j} = \left( \frac{1}{s_1 c} (-i k \beta^2 (R_j^2 + 1) + i k c^2 - \frac{\Omega^2}{i k}) l_j + (-2 \Omega c) \right) Q_j e^{-i k R_j K} + k^2 L_j (R_j^2 + 1) (L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{5j} = (-k^2 [\lambda (1 - R_j l_j) + R_j (\lambda + 2 \mu)(l_j - R_j)]) L_j e^{i k R_j K} + [-k^2 (\lambda (1 + R_j l_j) - R_j (\lambda + 2 \mu)(l_j - R_j)]) Q_j e^{-i k R_j K} + [k^2 (R_j^2 + 1) (p \rho \alpha^2 - \mu_e H_0^2) - k \rho_y (k c^2)] (L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{5j} = [-k^2 (\lambda (1 + R_j m_j) - R_j (\lambda + 2 \mu)(l_j - R_j))] Q_j e^{-i k R_j K} - [k^2 (R_j^2 + 1) (p \rho \alpha^2 - \mu_e H_0^2) - k \rho_y (k c^2)] Q_j e^{-i k R_j K},$$
$$d_{6j} = [\mu k^2 (l_j (R_j^2 - 1) + 2 R_j)] + \frac{i F_k}{s_1} ((\beta^2 k (R_j^2 + 1) - k c^2) l_j) \times (L_j e^{i k R_j K} + Q_j e^{-i k R_j K}) + 2 \mu k^2 R_j (-L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{6j} = [\mu k^2 (l_j (R_j^2 - 1) + 2 R_j)] + \frac{i F_k}{s_1} ((\beta^2 k (R_j^2 + 1) - k c^2) l_j) Q_j e^{-i k R_j K},$$
$$d_{7j} = \frac{\rho}{\gamma} [\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 - i k (2 \Omega c) l_j + \Omega^2] (L_j e^{i k R_j K} + Q_j e^{-i k R_j K}),$$
$$d_{7j} = \frac{\rho}{\gamma} [\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 - i k (2 \Omega c) l_j + \Omega^2] Q_j e^{-i k R_j K},$$
$$d_{8j} = \frac{\rho}{\gamma} [\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 - i k (2 \Omega c) l_j + \Omega^2] \theta e^{-i k R_j K},$$
$$d_{8j} = \frac{\rho}{\gamma} [\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 - i k (2 \Omega c) l_j + \Omega^2] (\theta - i k R_j) Q_j e^{-i k R_j K},$$
$$d_{8j} = \frac{\rho}{\gamma} [\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 - i k (2 \Omega c) l_j + \Omega^2] (\theta - i k R_j) Q_j e^{-i k R_j K},$$
Effect of the rotation

\[ d_{ij} = \frac{1}{s_1 c} \left( -ik\beta^2 (R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik} I_j + (-2\Omega c) \right) \]

\[ (MR_j(L_j - Q_j)) + k^2 I_j(R_j^2 + 1)(L_j + Q_j) \]

\[ d_{10j} = -\lambda k^2[(1 - Rjm_j)C_j + (1 + Rjm_j)D_j] + (\lambda + 2\mu)(-k^2R_j[(L_j + R_j)L_j + (I_j - R_j)D_j]) - [k^2(R_j^2 + 1)(\rho\gamma a^2 + \mu_eH_j^2) - k\rho y(kc^2)](L_j + Q_j), \]

\[ d_{11j} = -2k^2 R_j(L_j - Q_j) + k^2 I_j(R_j^2 - 1) + \frac{iFk}{s_1}[(\beta^2k(R_j^2 + 1) - kc^2)I_j \]

\[ d_{12j} = [-\alpha^2 k^2(R_j^2 + 1) + k^2(c^2 - ik(-2\Omega c)L_j + \Omega^2)[(\theta + ikR_j)L_j + (\theta - ikR_j)Q_j], \]

\[ d_{0j} = d_{10j} = d_{11j} = d_{12j} = 0, \quad j = 3, 4, 5, 6. \] (54)

Equation (54) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient due to the friction of the granular nature of the medium. Analytically, one may observed that the Rayleigh wave velocity and attenuation coefficient depend on the rotation, magnetic field, initial stress, gravity field, granular rotation and thermal relaxation times. It’s shown from equation (34) that due to effect of the thermal field and gravity field change from fourth-order to eight-order which involves four positive roots. The transcendental equation (54) in the determinant form, represents the required wave velocity equation of wave propagated in generalized magneto-thermoelastic granular body under the influence of rotation, gravity field and initial stress.

5.2 In presence of rotation and there is uncoupling between the temperature and strain field

In this case \( \Omega \neq 0, \quad g = 0, \quad P = 0, \quad H_o = 0 \) and \( \theta = 0 \), we obtain

\[ \lim_{\epsilon \to 0} \Lambda_1 = \frac{-2c^2(2b^2 - K_s)}{(b^2 - i k c s_1)} - \frac{ic\beta^2 s_2 - k^2 s_1}{k^2 a^2} - \frac{ic\alpha^2}{k^2} \]

\[ \Gamma_2 \] (55)

\[ \lim_{\epsilon \to 0} \Lambda_2 = 2 \left[ \frac{(c^2 - 2a^2) + i c a^2}{k^2} \right] \Gamma_2 + \frac{(c^2 - 2\beta^2) + i c k(\beta^2 s_2 - k^2 s_1)}{k^2 a^2} \]

\[ \left[ 1 - \frac{c^2}{a^2} + \frac{ic}{\alpha^2} \left( \frac{c^2 - \alpha^2}{k^2} \right) \right] \Gamma_2 + \frac{2(b^2 - c^2)}{(b^2 - i k c s_1)} + \frac{2ic s_2 [c^2 - b^2 - i c k^2 s_1]}{k^2 (b^2 - i k c s_1)} \]

\[ \lim_{\epsilon \to 0} \Lambda_3 = \frac{1}{a^2(b^2 - i k c s_1)k^2} \left[ 2a^2(c^2 - a^2) + \frac{ic}{\alpha^2} \left( \frac{c^2 - a^2}{k^2} \right) \Gamma_2 \left( c^2 - a^2 + \frac{\Omega^2}{k^2} \right) \right] \]

\[ \times \left[ k^2(c^2 - 2\beta^2) + i c k(\beta^2 s_2 - k^2 s_1) \right] \] (56)

\[ \lim_{\epsilon \to 0} \Lambda_4 = \frac{1}{a^2(b^2 - i k c s_1)k^2} \left[ \frac{(a^2 - c^2)}{k^4} + \frac{ic}{\alpha^2} \left( \frac{c^2 - a^2}{k^2} \right) \Gamma_2 \left( c^2 - a^2 + \frac{\Omega^2}{k^2} \right) \right] \]

\[ \times [2k^4(2\beta^2 - c^2) + 2ic k^3 s_2(c^2 - \beta^2) - i c k^5 s_1] \] (58)

After tanking \( \gamma \to 0 \), boundary conditions become
\[ d_{1j} = (1 - R_j l_j) L_j e^{ikR_j l_j} + (1 + R_j l_j) Q_j e^{-ikR_j l_j}, \quad d_{1j}^j = (1 - R_j l_j) Q_j e^{-ikR_j l_j} \]

\[ d_{2j} = (R_j + l_j) L_j e^{ikR_j l_j} + (l_j - R_j) Q_j e^{-ikR_j l_j}, \quad d_{2j}^j = (l_j - R_j) Q_j e^{-ikR_j l_j} \]

\[ d_{3j} = \left[ \frac{1}{s_1} (-ik\beta^2 (R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}) l_j - 2\Omega c \right] (L_j e^{ikR_j l_j} + Q_j e^{-ikR_j l_j}), \quad d_{3j}^j = (l_j - R_j) Q_j e^{-ikR_j l_j} \]

\[ d_{4j} = MR_j \left( \frac{1}{s_1} (-ik\beta^2 (R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}) l_j - 2\Omega c \right) (L_j e^{ikR_j l_j} - Q_j e^{-ikR_j l_j}) \]

\[ + k I_j^2 (R_j^2 + 1) (L_j e^{ikR_j l_j} + Q_j e^{-ikR_j l_j}), \quad d_{4j}^j = (l_j - R_j) Q_j e^{-ikR_j l_j} \]

\[ d_{5j} = (-k^2 \lambda (1 - R_j l_j) + R_j (\lambda + 2\mu) (l_j + R_j)] L_j e^{ikR_j l_j} + [-k^2 (1 + R_j)] L_j e^{ikR_j l_j} \]

\[ d_{5j} = [-k^2 (\lambda - 2\mu) (l_j + R_j)] Q_j e^{-ikR_j l_j} \]

\[ d_{6j} = [\mu k^2 (l_j (R_j^2 - 1)) + \left( \frac{iF_k}{s_1} \right) (\beta^2 k (R_j^2 + 1) - kc^2) l_j] \times (L_j e^{ikR_j l_j} + Q_j e^{-ikR_j l_j}) \]

\[ + Q_j e^{-ikR_j l_j} + 2\mu k^2 R_j (-L_j e^{ikR_j l_j} + Q_j e^{-ikR_j l_j}), \quad d_{6j}^j = [\mu k^2 (l_j (R_j^2 - 1)) + 2R_j] \]

\[ + \left( \frac{iF_k}{s_1} \right) (\beta^2 k (R_j^2 + 1) - kc^2) l_j] Q_j e^{-ikR_j l_j} \]

\[ d_{7j} = \left( \frac{1}{s_1} (-ik\beta^2 (R_j^2 + 1) + ikc^2 - \frac{\Omega^2}{ik}) l_j + (-2\Omega c) (MR_j (L_j - Q_j)) \right) \]

\[ + k^2 l_j (R_j^2 + 1) (L_j + Q_j) \]

\[ d_{8j} = -\lambda k^2 [(1 - R_j l_j) L_j + (1 + R_j l_j) Q_j] + (\lambda + 2\mu) [-k^2 R_j [(l_j + R_j) L_j \]

\[ + (l_j - R_j) Q_j]], \quad d_{8j} = -2k^2 R_j (L_j - Q_j) + k^2 l_j (R_j^2 - 1) + \left( \frac{iF_k}{s_1} \right) (\beta^2 k (R_j^2 + 1) - kc^2) l_j \]

\[ d_{9j} = -\alpha k^2 (R_j^2 + 1) + k^2 c^2 - i\lambda (2\Omega c) l_j + \Omega^2 ([i(kR_j) L_j + (ikR_j) Q_j], \quad d_{9j} = d_{8j} \]

\[ d_{10j} = d_{10j} = d_{10j} = 0 \quad j = 3, 4, 5, 6. \]

\[ \text{det} \ (d_{ij}) = 0, \quad (59) \]

Equation (59) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient. The frequency equation (54) which determines the wave velocity equation for the Rayleigh waves in a granular medium, when the rotation and initial stress are absent, we have

\[ \alpha^2 = \frac{\lambda + 2\mu}{\rho}, \quad \beta^2 = \frac{\mu}{\rho}, \quad (60) \]
Finally, if there is (i) no rotation, magnetic field, thermal relaxation times, gravity field, and granular rotation is vanishes and (ii) absence of the magnetic field, thermal relaxation times, gravity field, there is uncoupling between temperature and strain field, and the granular rotation is vanishes, the results obtained by Ahmed [15] are deduced as special case from this study with slight changes in symbols with additional to the graphs that not included in the last work.

For a computation work by half-interval method program, we use Sand Stone as a granular medium and Nepheline as a granular layer taking into consideration that the friction coefficient $F = 0.5$ and third elastic constants $M_1 = 0.4$, $M_2 = 0.6$.

### 5.3 In presence of rotation and there is no coupling between the temperature and strain field

In this case $\Omega \neq 0$, $g = 0$, $H_0 = 0$, $P = 0$, $\theta = 0$, $\tau_1 = \tau_2 = 0$, and $s_1 = s_2 = 0$ we obtain

\[
\lim_{\varepsilon \to 0} \Lambda_1 = -\frac{2(c^2 - 2\beta^2)}{\beta^2} - \frac{(c^2 - 2\alpha^2) + \Omega^2}{k^2\alpha^2} - \frac{ic}{\hat{u}k},
\]

\[
\lim_{\varepsilon \to 0} \Lambda_2 = 2 \left[ \frac{(c^2 - 2\alpha^2)}{k^4} + i\alpha c^2 \frac{k^2}{k^5} + \frac{\Omega^2}{k^4} \right] \times [k^2(c^2 - 2\beta^2)]
\]

\[
+ \left[ 1 - \frac{c^2}{\alpha^2} + \frac{ic}{\hat{u}k\alpha^2} \left( \left( c^2 - \alpha^2 + \frac{\Omega^2}{k^2} \right) \right) \right] + \frac{2(\beta^2 - c^2)}{\beta^2}
\]

\[
\lim_{\varepsilon \to 0} \Lambda_3 = \frac{1}{\alpha^2 \beta^2 k^2} \left[ \left( 2(\alpha^2 - c^2) + \frac{ic}{\hat{u}k} \left( c^2 - \alpha^2 + \frac{\Omega^2}{k^2} \right) \right) \right]
\]

\[
\times [k^2(c^2 - 2\beta^2)]
\]

\[
\lim_{\varepsilon \to 0} \Lambda_4 = \frac{1}{\alpha^2 \beta^2 k^2} \left[ \left( \frac{(c^2 - \alpha^2)}{k^4} + \frac{ic}{\hat{u}k^5} \left( c^2 - \alpha^2 + \frac{\Omega^2}{k^2} \right) \right) \right]
\]

\[
\times [2k^4(\beta^2 - c^2)] - \frac{2ic\Omega}{k^7}
\]

\[
d_{1j} = (1 - R_j l_j) L_j e^{ik_{R_j}k} + (1 + R_j l_j) Q_j e^{-ik_{R_j}k},
\]

\[
d_{1j} = (1 - R_j l_j) Q_j e^{-ik_{R_j}k},
\]

\[
d_{2j} = (R_j + l_j) L_j e^{ik_{R_j}k} + (l_j - R_j) Q_j e^{-ik_{R_j}k},
\]

\[
d_{2j} = (l_j - R_j) Q_j e^{-ik_{R_j}k},
\]

\[
d_{3j} = (-k^2(\lambda(1 - R_j l_j) + R_j(\lambda + 2\mu)(l_j + R_j))L_j e^{ik_{R_j}k} + [-k^2(\lambda(1 + R_j l_j) - R_j(\lambda + 2\mu)(l_j - R_j))Q_j e^{-ik_{R_j}k},
\]

\[
d_{3j} = [k^2(\lambda(1 + R_j l_j) + R_j(\lambda + 2\mu)(l_j - R_j))]Q_j e^{-ik_{R_j}k},
\]

\[
d_{4j} = -\lambda k^2 [(1 - R_j l_j) L_j + (1 + R_j l_j) Q_j] + \lambda(\lambda + 2\mu) - k^2 R_j (l_j + R_j) L_j,
\]

\[
d_{4j} = -\lambda k^2 [(1 - R_j l_j) L_j + (1 + R_j l_j) Q_j] + (\lambda + 2\mu)(-k^2 R_j (l_j + R_j) L_j,
\]

\[
d_{4j} = -\lambda k^2 [(1 - R_j l_j) L_j + (1 + R_j l_j) Q_j] + (\lambda + 2\mu)(-k^2 R_j (l_j + R_j) L_j,
\]
\[ +(i_j - R_j)Q_j])\]

\[ d_{5j} = -2k^2 R_j(L_i - Q_j) + k^2 l_j (R_j^2 - 1)(L_j + Q_j), \]

\[ d_{6j} = [-\alpha^2 k^2 (R_j^2 + 1) + k^2 c^2 + 2ik\Omega c l_j + \Omega^2][(ikR_j)L_j - (ikR_j)Q_j], \]

\[ d_{4j} = d_{5j} = d_{6j} = 0 , \quad j = 1, 2 \]

So, equation (59) tends to

\[ \text{det} \ (d_{ij}) = 0, \quad (61) \]

Equation (61) has complex roots, the real part gives Rayleigh wave velocity and the imaginary part gives the attenuation coefficient, where

\[ d_{11} = (1 - R_1 l_1)e^{ikR_1 K}, \quad d_{12} = (1 + R_1 l_1)e^{-ikR_1 K}, \quad d_{13} = (1 - R_2 l_2)e^{ikR_2 K}, \]

\[ d_{14} = (1 + R_2 l_2)e^{-ikR_2 K}, \quad d_{15} = (1 - R_1 l_1)e^{-ikR_1 K}, \]

\[ d_{16} = (1 - R_2 l_2)e^{-ikR_2 K}, \quad d_{21} = (1 - R_1 l_1)e^{ikR_1 K}, \quad d_{22} = (l_1 - l_1)e^{-ikR_1 K}, \quad d_{23} = (l_2 + l_2)e^{ikR_2 K}, \]

\[ d_{24} = (l_2 - R_2)e^{ikR_1 K}, \quad d_{25} = (l_1 - R_1)e^{-ikR_1 K}, \quad d_{26} = (l_2 - R_2)e^{-ikR_2 K}, \]

\[ d_{31} \{ -k^2[\lambda(l_1 - R_1 l_1) + R_1(\lambda + 2\mu)(l_1 + R_1)]e^{ikR_1 K}, \]

\[ d_{32} \{ -k^2[\lambda(l_1 + R_1 l_1) - R_1(\lambda + 2\mu)(l_1 - R_1)]e^{-ikR_1 K}, \]

\[ d_{33} \{ -k^2[\lambda(l_2 - R_2 l_2) + R_2(\lambda + 2\mu)(l_2 + R_2)]e^{ikR_2 K}, \]

\[ d_{34} \{ -k^2[\lambda(l_2 - R_2 l_2) + R_2(\lambda + 2\mu)(l_2 + R_2)]e^{-ikR_2 K}, \]

\[ d_{35} \{ 2\{ (1 + 1 + 1 + 1 + 2)(1 - 1 - 1) \} - 1, \]

\[ d_{36} \{ 2\{ (1 + 1 + 1 + 1 + 2)(1 - 1 - 1) \} - 2, \]

\[ d_{41} = -2 (1 - 1 - 1) + 2 \{ (1 + 2)(1 + 1)\}, \]

\[ d_{42} = -2 (1 + 1 + 1) - 2 \{ (1 + 2)(1 - 1)\}, \]

\[ d_{43} = -2 (1 + 1 + 1) - 2 \{ (1 + 2)(1 + 1)\}, \]

\[ d_{44} = -2 (1 + 2 + 2) + 2 \{ (1 + 2)(1 - 1)\}, \]

\[ d_{45} = -2 \{ (1 + 2 + 2)(2 - 1)\}, \]

\[ d_{46} = 2 \{ (1 + 2 + 2)(2 - 1)\}, \]

\[ d_{51} = -2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{52} = 2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{53} = 2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{54} = 2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{55} = 0, \quad d_{56} = 0, \]

\[ d_{61} = (1) \{ -2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{62} = (-2) \{ -2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{63} = (1) \{ -2 (2 + 2) + 2 \{ (2 - 1)\}, \]

\[ d_{64} = (-2) \{ -2 (2 + 2) + 2 \{ (2 - 1)\}. \]

\[ d_{65} = 0, \quad d_{66} = 0. \]
6. Numerical results and discussion

We wish to investigate the variation of Rayleigh wave velocity in a perfectly conducting granular medium under effect of rotation $\Omega$, magnetic field, initial stress and gravity field, for computational work. From Fig. 2, Show the effects of rotation $\Omega$ on non-dimensional frequency, which it increases with increasing of thickness and it decreases with increasing the rotation. Fig.3: Show the effects of magnetic field on non-dimensional frequency, which it increases with increasing of thickness and it increases with increasing the magnetic field. Fig.4: Show the effects of wave number on non-dimensional frequency, which it increases with increasing of thickness and it increases with increasing the relaxation time. Fig.5: Show the effects of relaxation time on Variation of attenuation coefficient with respect to initial stress with the various values of rotation $\Omega$, $1 = 10^{-5}$, $1 = 10^{-2}$, $1 = 10^{-3}$, $4 \times 10^{3}$ which it decreases with increasing of initial stress and it increases with increasing of rotation $\Omega$. Fig.6: Show the effects of rotation $\Omega$ and magnetic field on Variation of attenuation coefficient with respect to initial stress with the various values of magnetic field $1 = 10^{-5}$, $1 = 10^{-2}$, $1 = 10^{-3}$, which it decreases with increasing of initial stress and it decreases with increasing of magnetic field. Fig.7: Show the effects of rotation $\Omega$ and wave number on Variation of attenuation coefficient with respect to initial stress with the various values of wave number $1 = 10^{-2}$, $1 = 10^{-3}$, $4 \times 10^{3}$, which it decreases with increasing of initial stress and it decreases with increasing of wave number. Fig.8: Show the effects of rotation $\Omega$ and wave number on Variation of attenuation coefficient with respect to initial stress with the various values of wave number $1 = 10^{-2}$, $1 = 10^{-3}$, $4 \times 10^{3}$, which it decreases with increasing of initial stress and it decreases with increasing of wave number. Fig.9: Show the effects of relaxation time $\eta$ on Variation of Rayleigh wave velocity with respect to thickness with the various values of relaxation time $1 = 10^{-5}$, $1 = 10^{-3}$, $4 \times 10^{3}$, which it increases with increasing of thickness and it increases with decreasing of rotation $\Omega$. Finally, from the previous results obtained in general case and special cases, it is shown that the rotation, magnetic field, initial stress, gravity field, thermal relaxation times, and granular rotations have utilitarian aspects on Rayleigh wave velocity and attenuation coefficient.

7. Conclusion

The effects of rotation, relaxation times, magnetic and gravity fields and initial stress are very pronounced on Rayleigh wave velocity. When the medium is an orthotropic and the effect of rotation is neglected, the frequency equation reduces to previous work. Special cases are investigated. The results indicate that the effect of rotation is very sensitive.
Fig. 2: Show the effects of rotation \( \Omega \) on non-dimensional frequency.

Fig. 3: Show the effects of magnetic field \( H_0 \) on non-dimensional frequency.

Fig. 4: Show the effects of relaxation time on non-dimensional frequency.

Fig. 5: Show the effects of wave number on non-dimensional frequency.

Fig. 6: Variation of attenuation coefficient with respect to initial stress with the various values of rotation.

Fig. 7: Variation of attenuation coefficient with respect to initial stress with the various values of magnetic field.
Effect of the rotation

References


Received: 8 August 2010; Revised:14 October 2010; Accepted 12 November 2010