Effect of Rotation and Initial Stress on Generalized-Thermoelastic Problem in an Infinite Circular Cylinder

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Abstract

In this paper, we investigated the influence of rotating and initial stress on the propagation of Rayleigh waves in a homogeneous isotropic, generalized thermoelastic body, subject to the boundary conditions that the outer surface is traction free. In addition, it is subject to insulating thermal conduction. General solution is obtained by using Hankel transform and Lame' potentials. It has been found that frequency equation of waves contains a term involving the rotating and initial stress. Therefore the phase velocity of Rayleigh waves changes with respect to this rotating and initial stress. When the rotating and initial stress vanishes, the derived frequency equation reduces to that obtained in classical generalized thermo-elastic case which includes the relaxation time of heat conduction. The numerical results have been obtained and presented graphically in 2D-plots.

1-Introduction

There many previous study of generalized thermoelastic from the old. Lord and Shulman [1] have solved the dynamical theory of generalized thermoelasticity to take into account the time needed for acceleration of the heat flow. The generalized thermoelastic waves are discussed by Sharma and Singh [2] and Singh and Sharma [3]. Tanaka, et al. [4] studied the application of boundary element method to 3D problems of...
coupled thermoelasticity. Effect of initial stress on generalized thermoelastic problem in an infinite circular cylinder is discussed by El-Naggar and Abd-Alla [5].

The problem of rotating disks or cylinders has its application in high-speed cameras, steam and gas turbines, planetary landings and in many other domains. Various authors have formulated these generalized theories on different grounds. Lord and Shulman [1] have developed a theory on the basis of a modified heat conduction law which involves heat-flux rate, and Green and Lindsay [6] have developed a theory by including temperature-rate among the constitutive variables. Lebon [7] has formulated a theory by considering heat-flux as an independent variable. The effect of rotation and relaxation time in generalized thermoelasticity is discussed by Othman [8,9]. Eigenvalue approach to study the effect of rotation and relaxation time in generalized thermoelasticity have been studied by Sinha and Bera [10].

In this work, we discuss the frequency equation under effect the initial stress and rotation after determined the displacement components and stress components, also studied special cases from these equation at coupling or no coupling between the temperature and the strain fields for initial stress and rotation takes the different values and clarification that displayed by graphics.

2- Formulation of the problem and boundary conditions

Let us consider a homogeneous isotropic elastic solid with infinite circular cylinder under initial stress \( P \) and initial temperature \( T_0 \). When the temperature of infinite cylinder is changed incremental thermal stress \( \sigma_{i,j} \) together with incremental strain \( e_{i,j} \) are produced in it. The elastic medium is rotating uniformly with an angular velocity \( \Omega = \Omega \hat{n} \), where \( \hat{n} \) is a unit vector representing the direction of the axis of rotation. All quantities considered will be functions of the time variable \( t \) and of the coordinates \( r \) and \( z \). The displacement equation of motion in the rotating frame has two additional term Schoenberg and Censor [11], centripetal acceleration, \( \frac{2\Omega}{r} \times (\Omega \times \mathbf{u}) \) due to time varying motion only and \( 2\Omega \times \mathbf{u} \) where, \( \mathbf{u} = (u_r, 0, u_z) \) is the dynamic displacement vector and \( \Omega = (0, \Omega, 0) \). These terms do not appear in non-rotating media, and we can neglected limit of \( \frac{2\Omega}{r} \times \mathbf{u} \) is imaginary.

In the absence of body forces the dynamic equation of motion under initial compression \( P \) are given by Abd-Alla [12] as follow:

\[
\frac{\partial \sigma_{rr}}{\partial t} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r} (S_r - S_{\theta \theta}) + P \frac{\partial \omega_0}{\partial z} = \rho \left[ \frac{\partial^2 u_r}{\partial t^2} - \Omega^2 u_r \right], \quad (2.1)
\]

\[
\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{r \theta}}{\partial \theta} + \frac{1}{r} S_r + \frac{P}{r} \frac{\partial}{\partial r} (r \omega_0) = \rho \left[ \frac{\partial^2 u_\theta}{\partial t^2} - \Omega^2 u_z \right]. \quad (2.2)
\]

The generalized equation of heat conduction is given by [4]:

\[
K \nabla^2 T = \rho C_v (T + \tau_1 T) + \gamma T_0 \nabla (u + \delta \tau_1 \mathbf{u}) \quad (2.3)
\]
Effect of rotation and initial stress

where $\rho$ is density of the material, $K$ is thermal conductivity, $C_v$ is specific heat of the material per unit mass, $\alpha$ is coefficient of linear thermal expansion, $\lambda, \mu$ are Lame elastic constants, $S_{rr}, S_{\theta \theta}, S_{zz}$ and $S_{rz}$ are the incremental stresses, $u_r$ and $u_z$ are the displacement components and $\omega_\theta$ is the rotation components.

The stress-strain relations with incremental isotropy under initial stress are given by Abd-Alla [12]:

$$S_{rr} = (\delta_s + \mu + p)e_{rr} + (\delta_s - \mu + p)e_{zz} + (\delta_s - \mu + p)e_{\theta \theta} - \frac{\gamma}{\chi_\theta}(T + \tau_0 \dot{T}),$$  

(2.4)

$$S_{\theta \theta} = (\delta_s + \mu + p)e_{\theta \theta} + (\delta_s - \mu + p)e_{rr} + (\delta_s - \mu + p)e_{zz} - \frac{\gamma}{\chi_\theta}(T + \tau_0 \dot{T}),$$  

(2.5)

$$S_{zz} = (\delta_s + \mu + p)e_{zz} + (\delta_s - \mu + p)e_{rr} + (\delta_s - \mu + p)e_{\theta \theta} - \frac{\gamma}{\chi_\theta}(T + \tau_0 \dot{T}),$$  

(2.6)

$$S_{rz} = 2\mu e_{rz},$$  

(2.7)

$$S_{r\theta} = S_{\theta r} = 0$$  

(2.8)

where, $\delta_s = (\lambda + \mu)$, $\gamma = \alpha_s(3\lambda + 2\mu)$ and $\chi_\theta$ is the isothermal compressibility $\tau_0$ and $\tau_1$ are thermal relaxation parameters.

The incremental strain components and the rotation are given by Bagri and Esami [13] as follow:

$$e_{rr} = \frac{\partial u_r}{\partial r}, \quad e_{\theta \theta} = \frac{u_r}{r}, \quad e_{zz} = \frac{\partial u_z}{\partial z},$$  

$$e_{rz} = \frac{1}{2} \left( \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right), \quad \omega_\theta = \frac{1}{2} \left( \frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right).$$  

(2.9)

By using Eqs. (2.4)-(2.9), Eqs. (2.1) and (2.2) can be written as:

$$(\delta_s + \mu + P)\frac{\partial \Delta}{\partial r} + 2\left( \frac{\mu + P}{2} \right)\frac{\partial \omega_\theta}{\partial z} - \frac{\gamma}{\chi_\theta} \frac{\partial}{\partial r}(T + \tau_0 \dot{T}) = \rho \left[ \frac{\partial^2 u_r}{\partial t^2} - \Omega^2 u_r \right]$$  

(2.10)

$$(\delta_s + \mu)\frac{\partial \Delta}{\partial z} - 2r \left( \frac{\mu - P}{2} \right)\frac{\partial}{\partial r}(r \omega_\theta) - \frac{\gamma}{\chi_\theta} \frac{\partial}{\partial z}(T + \tau_0 \dot{T}) = \rho \left[ \frac{\partial^2 u_z}{\partial t^2} - \Omega^2 u_z \right]$$  

(2.11)

where,

$$\Delta = \nabla \cdot \vec{u} = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z}$$

By Helmholtz's theorem [14], the displacement vector $\vec{u}$ can be written in the form:

$$\vec{u} = \text{grad} \phi + \text{curl} \vec{\psi},$$  

(2.12a)

where, the scalar $\phi$ and the vector $\vec{\psi}$ represent irrotational and rotational parts of the displacement $\vec{u}$. The cylinder being bounded by the curved surface, therefore the stress distribution includes the effect of both $\phi$ and $\vec{\psi}$. It is possible to take only one component of the vector $\vec{\psi}$ to be non-zero, as

$$\vec{\psi} = (0, \psi_r, 0).$$  

(2.12b)

From Eq. (3.12a) and (3.12b) we obtain
Substituting from (2.12a)-(2.12c) into (2.3), (2.10) and (2.11), we get two independent equations for \( \phi \) and \( \psi \) as follows:

\[
K \nabla^2 T = \rho C_v \frac{\partial}{\partial t} \left( 1 + \tau_1 \frac{\partial}{\partial t} \right) T + \gamma T_0 \frac{\partial}{\partial t} \left( 1 + \delta r_1 \frac{\partial}{\partial t} \right) \nabla^2 \phi ,
\]

\[
\nabla^2 \phi = \frac{\rho}{\left( \delta_s + \mu + P \right)} \left[ \frac{\partial^2 \phi}{\partial t^2} - \Omega^2 \phi \right] + \frac{\gamma}{\chi_o (\delta_s + \mu)} (T + \tau_0 \dot{T}) ,
\]

\[
\nabla^2 \psi = \frac{\rho}{\left( \mu + P \right)} \left[ \frac{\partial^2 \psi}{\partial t^2} - \Omega^2 \psi \right] ,
\]

\[
\nabla^2 \psi = \frac{\rho}{\left( \mu - P \right)} \left[ \frac{\partial^2 \psi}{\partial t^2} - \Omega^2 \psi \right] ,
\]

where,

\[
\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} .
\]

Similar results were obtained by Abd-Alla [12] and Bagri and Eslami [13] while, the second reference deriving the constitutive equations for Rayleigh waves in elastic medium under initial stress.

These equations are different in form and in number from those of the classical theory where there is no initial stress. This difference is due to the fact that longitudinal and shear wave velocities are not the same in all directions when the medium is subjected to non-hydrostatic initial stress. Since the initial stress has been taken in the direction of \( r \) only, the velocity of body waves will be different in \( r \) and \( z \) directions. In the absence of \( P \), the Eqs. (2.14)-(2.17) reduce to two equations only.

Now Eqs. (2.14) and (2.15) represent the compressive wave along the \( r \) and \( z \) directions respectively and Eqs. (2.16) and (2.17) represent the shear wave along those directions respectively.

Eq. (2.14) represents the longitudinal wave in the direction of \( r \) with velocity \( C_1 = \left( \left[ \delta_s + \mu + P \right] / \rho \right)^{\frac{1}{2}} \) and Eq. (2.17) represents the velocity of the shear wave in the direction of \( r \) with velocity \( C_2 = \left( \left[ \mu - \frac{P}{2} \right] / \rho \right)^{\frac{1}{2}} \). Equation (2.15) represents the longitudinal wave in the direction of \( z \) with velocity \( C_z = \left( \left[ \delta_s + \mu \right] / \rho \right)^{\frac{1}{2}} \). Equation (2.16) represents the shear wave in the direction of \( z \) with velocity \( C_2 = \left( \left[ \mu - \frac{P}{2} \right] / \rho \right)^{\frac{1}{2}} \).
It must be mentioned that we have been considered compressional and distortional waves along the $r$ direction only. These waves are represented by Eqs. (2.14) and (2.17).

3- Boundary conditions

Consider a homogeneous and isotropic elastic solid with infinite circular cylinder of radius $R$. The axis of the cylinder is taken along the $z$ direction. It is subjected to the boundary conditions which are given as traction free:

\[
\begin{align*}
    s_{r}(R,Z,T) &= 0, & \text{at } r &= R, \\
    s_{z}(R,Z,T) &= 0,
\end{align*}
\]

(3.1)

the thermal boundary condition is:

\[
\frac{\partial T(R,Z,t)}{\partial r} = 0, \quad \text{at } r = R.
\]

(3.2)

4-Solution of the problem

Assuming a simple harmonic time dependent factor $e^{i\omega t}$ for all the quantities and omitting the factor $e^{i\omega t}$ throughout the Eqs. (2.13), (2.14) and (2.17) which yield to a set of differential equations for $T^{*}e^{i\omega t}$, $\phi^{*}e^{i\omega t}$ and $\psi^{*}e^{i\omega t}$, i.e.,

\[
\begin{align*}
    \left[ \nabla^{2} - \frac{i\omega \rho C_{i} \tau'_{t}}{K} \right] T^{*} &= \frac{i\omega \gamma \tau_{0}'}{K} \nabla^{2} \phi^{*}, \\
    \left[ \nabla^{2} + \left( \omega^{2} + \Omega^{2} \right) \frac{\tau'_{t}}{C_{i}^{2}} \right] \phi^{*} &= \frac{\gamma \tau_{0}'}{\rho \chi \rho} T^{*}, \\
    \left[ \nabla^{2} + \frac{\rho(\omega^{2} + \Omega^{2})}{\mu - P/2} \right] \psi^{*} &= 0
\end{align*}
\]

(4.1) (4.2) (4.3)

where, $\nabla^{2}$ is Laplace operator, $\nabla^{2} \phi^{*}$ is Laplacian in polar coordinates, $\nabla^{2} \psi^{*}$ is $\theta$-coordinate of the vector. $T^{*}$ can be eliminated from Eq. (4.2) by substituting it in Eq. (4.1), we have

\[
\nabla^{4} \phi^{*} + \left[ \frac{(\omega^{2} + \Omega^{2})}{C_{i}^{2}} - \frac{i\omega \rho C_{i} \tau'_{t}}{K} (1 + \varepsilon \tau'_{t}) \right] \nabla^{2} \phi^{*} - \frac{i\omega(\omega^{2} + \Omega^{2}) \rho C_{i} \tau'_{t}}{KC_{i}^{2}} \phi^{*} = 0
\]

(4.4)

where,

\[
\nabla^{4} = \left[ \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^{2}}{\partial z^{2}} \right]^{2}, \quad \varepsilon = \frac{\gamma \tau_{0}'}{\rho \chi \rho C_{i} \tau'_{t}}, \quad \tau'_{t} = 1 + i\omega \tau_{0}, \tau'_{t} = 1 + i\omega \tau_{t}
\]

(4.5)

General solution of Eqs. (4.3) and (4.4) can be found. If we introduce the inversion of the Hankel transform which is defined by
\[ \phi(r, z, \omega) = \int_0^\infty \phi(\eta, z, \omega) J_0(\eta r) \eta d\eta \]

we obtain

\[ \left(q^2 - \frac{d^2}{dz^2}\right) \phi - \left[\frac{(\omega^2 + \Omega^2)}{C_i^2} - \frac{i\omega \rho C_i \tau' (1 + \varepsilon \tau'_0)}{K}\right]\left(q^2 - \frac{d^2}{dz^2}\right) \phi - \frac{i\omega (\omega^2 + \Omega^2) \rho C_i \tau'_0}{KC_i^2} \phi = 0. \]

(4.6)

By putting \( f^2 = \left(q^2 - \frac{d^2}{dz^2}\right) \), the indicial equation governing (4.6) is:

\[ f^4 - \left[\frac{(\omega^2 + \Omega^2)}{C_i^2} - \frac{i\omega \rho C_i \tau' (1 + \varepsilon \tau'_0)}{K}\right] f^2 - \frac{i\omega (\omega^2 + \Omega^2) \rho C_i \tau'_0}{KC_i^2} = 0 \]

(4.7)

where \( \xi_j^2 = q^2 - f_j^2 \) are the roots of Eq. (4.6), \( R_e(\xi_j) \geq 0, \quad j = 1, 2 \).

If \( \varepsilon = 0 \), then the roots of Eq. (4.7) become:

\[ f_1^{*2} = \frac{(\omega^2 + \Omega^2)}{C_i^2}, \quad f_2^{*2} = -\frac{i\omega \rho C_i \tau'_i}{K}. \]

(4.8)

The above roots correspond to the case in which elastic wave and generalized of heat condition equations are not coupled. For small \( \varepsilon \) (i.e., the first order only is taken), the roots of equation (4.7) take the form:

\[ (f_1^{*2}, f_2^{*2}) = \frac{1}{2} \left[ \frac{(\omega^2 + \Omega^2)}{C_i^2} - \frac{i\omega \rho C_i \tau' (1 + \varepsilon \tau'_0)}{K} \right] \pm \sqrt{\frac{(\omega^2 + \Omega^2)^2}{C_i^2} - \frac{\omega^2 \rho^2 C_i^2 \tau'^2}{K^2} (1 + 2\varepsilon \tau'_0) \left(1 + \frac{2\omega (\omega^2 + \Omega^2) \rho C_i \tau'_0}{C_i^2 K} \frac{C_i^2 K}{C_i^2} (\omega^2 + \Omega^2)^2 - \frac{\omega^2 \rho^2 C_i^2 \tau'^2}{K^2} (1 + 2\varepsilon \tau'_0) \right)} \]

(4.9)

then the solution of Eq. (4.6) is

\[ \phi^*(q, z, \omega) = A(q)e^{-\xi_1^*z} + B(q)e^{-\xi_2^*z} \]

(4.10)

which leads to

\[ \phi(r, z, t) = \int_0^\infty [A(q)e^{-\xi_1^*z + i\omega t} + B(q)e^{-\xi_2^*z + i\omega t}] J_0(qr)qdq. \]

(4.11)

Similarly one can obtain the solution for Eq. (4.3) which leads to

\[ \psi(r, z, t) = \int_0^\infty C(q)e^{-\xi_3^*z + i\omega t} J_0(qr)qdq \]

(4.12)

where, \( \xi_3^2 = q^2 - \frac{(\omega^2 + \Omega^2)}{C_i^2} \) and \( R_e(\xi_3) \geq 0 \).

The temperature deviation \( T \) can be obtained by substituting Eq. (4.11) into Eq. (4.2) we get
The stress components \( S_{rr} \) and \( S_{rz} \) in terms of the potential functions \( \phi \) and \( \psi \) are given by:

\[
S_{rr} = (\lambda + p) V^2 \phi + 2 \mu \frac{\partial^2 \phi}{\partial r^2} - 2 \mu \frac{\partial^3 \psi}{\partial r^2 \partial z} - \frac{\psi}{\chi_0} (T + \tau T),
\]
\[
S_{rz} = \mu \left[ 2 \frac{\partial^2 \phi}{\partial r \partial z} - \frac{\partial^3 \psi}{\partial r^3} + \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} - \frac{1}{r^2} \frac{\partial \psi}{\partial r} \right].
\]  

Substituting from Eqs. (4.11)-(4.13) into Eqs. (2.12c) and (4.14), we get:

\[
u_r = - \int_0^\infty \left\{ \left[ A(q) e^{-\zeta_z t} + B(q) e^{-\zeta_z t} \right] + C(q) \xi \xi e^{-\zeta_z t} \right\} q^2 J_1(qr) dq, \]  

\[
u_z = - \int_0^\infty \left\{ \left[ A(q) \xi^2 e^{-\zeta_z t} + B(q) \xi^2 e^{-\zeta_z t} \right] + C(q) q^3 \xi e^{-\zeta_z t} \right\} J_0(qr) dq, \]

\[
S_{rr} = A(q) \int_0^\infty \left\{ (\xi_1^2 - q^2)(\lambda + p) J_0(qr) - 2 \mu (q^2 J_0(qr) - \frac{1}{r} q J_1(qr)) - \rho((\omega^2 + \Omega^2) - C_1 f_1^2) J_0(qr) \right\} q e^{-\zeta_z t} dq,
\]

\[
+ B(q) \int_0^\infty \left\{ (\xi_2^2 - q^2)(\lambda + p) J_0(qr) - 2 \mu (q^2 J_0(qr) - \frac{1}{r} q J_1(qr)) \right\} q e^{-\zeta_z t} dq,
\]

\[
- \rho((\omega^2 + \Omega^2) - C_1 f_2^2) J_0(qr) \right\} q e^{-\zeta_z t} dq,
\]

\[
S_{rz} = \mu \int_0^\infty \left\{ A(q) (2 \xi_1 J_1(qr)q^2) e^{-\zeta_z t} + B(q) (2 \xi_1 J_1(qr)q^2) e^{-\zeta_z t} \right\} dq.
\]

5-Frequency equation

In this section, the frequency equation (applying the boundary conditions in Eqs. (3.1) and (3.2) on Eqs. (4.13), (4.17) and (4.18) is obtained. Using these conditions we get three homogeneous linear Eqs. in \( A(q) \), \( B(q) \) and \( C(q) \):
\[ A(q) \left\{ (\xi^2 - q^2) q J_\nu(qR)(\lambda + P) - 2\mu [q^3 J_\nu(qR) - \frac{1}{R} q^2 J_1(qR)] \right\} e^{-\xi z + \text{int}} + B(q) \left\{ (\xi^2 - q^2) q J_\nu(qR)(\lambda + P) \right\} e^{-\xi z + \text{int}} \]  
\[ -\rho((\omega^2 + \Omega^2) - C_i f_i^2) q J_\nu(qR) \right\} e^{-\xi z + \text{int}} \] 
\[ +C(q) \left\{ 2\mu \xi^3 \left[ \frac{1}{R} q^2 J_1(qR) - q^3 J_\nu(qR) \right] \right\} e^{-\xi z + \text{int}} = 0, \] 
\[ A(q) (2\xi_1 e^{-\xi z + \text{int}}) + B(q) (2\xi_2 e^{-\xi z + \text{int}}) + C(q) (\xi^2 + q^2) e^{-\xi z + \text{int}} = 0, \] 
\[ A(q) ((\omega^2 + \Omega^2) - C_i f_i^2) e^{-\xi z + \text{int}} + B(q) ((\omega^2 + \Omega^2) - C_i f_i^2) e^{-\xi z + \text{int}} = 0. \] 

Eliminating the constants \( A(q), B(q) \) and \( C(q) \) we obtain the frequency equation in the form of a third order determinant as

\[
\begin{vmatrix}
D_{11} & D_{12} & D_{13} \\
D_{21} & D_{22} & D_{23} \\
D_{31} & D_{32} & 0
\end{vmatrix} = 0
\] (5.4)

where,

\[ D_{11} = \left\{ (\xi_1^2 - q^2) q J_\nu(qR)(\lambda + P) - 2\mu [q^3 J_\nu(qR) - \frac{1}{R} q^2 J_1(qR)] - \rho((\omega^2 + \Omega^2) - C_i f_i^2) q J_\nu(qR) \right\}, \]
\[ D_{12} = \left\{ (\xi_2^2 - q^2) q J_\nu(qR)(\lambda + P) - 2\mu [q^3 J_\nu(qR) - \frac{1}{R} q^2 J_1(qR)] - \rho((\omega^2 + \Omega^2) - C_i f_i^2) q J_\nu(qR) \right\}, \]
\[ D_{13} = \left\{ 2\mu \xi_3 \left[ \frac{1}{R} q^2 J_1(qR) - q^3 J_\nu(qR) \right] \right\}, \]
\[ D_{21} = 2\xi_1, \quad D_{22} = 2\xi_2, \quad D_{23} = (\xi_3^2 + q^2), \]
\[ D_{31} = ((\omega^2 + \Omega^2) - C_i f_i^2), \quad D_{32} = ((\omega^2 + \Omega^2) - C_i f_i^2), \]
\[ D_{33} = 0. \]

The transcendental Eq. (5.4), in the determinant form, has complex roots. The real part (Re) gives the velocity of Rayleigh waves and the imaginary part (Im) gives the attenuation due to the granular nature of the medium. We discuss this case and special cases in two models as the following:

(I) \textbf{LS}-model (\( \tau_0 = 0, \quad \tau_1 > 0, \quad \delta = 1 \)) and (II) \textbf{GL}-model (\( \tau_0 \geq \tau_1 > 0, \quad \delta = 0 \)).

The discuss is clear up from Figs. (1a-h).

(I) \textbf{LS}-model (\( \tau_0 = 0, \quad \tau_1 > 0, \quad \delta = 1 \)).
Effect of rotation and initial stress

Fig. (1.a): $P = 1.5$, $\tau = 0.1$, $r = 2$ with varies values of $\Omega$.

Fig. (1.b): $\Omega = 0.5$, $\tau = 0.1$, $r = 2$ with varies values of $P$. 
Fig (1.c): $\Omega = 0.5$, $P = 1.5 \times 10^{11}$, $r = 2$ with varies values of relaxation times.
Effect of rotation and initial stress

Fig (1.d): $\Omega = 0.5$, $p = 1.5 \times 10^1$ and $\tau_1 = 0.1$.

(II) GL-model ($\tau_0 \geq \tau_1 > 0$, $\delta = 0$), $r_0 = 2\tau_1$. 
Fig (1.e): \( P = 1.5, \tau_i = 0.1, r = 2 \) with varies values of \( \Omega \).

Fig (1.f): \( \Omega = 0.5, \tau_i = 0.1, r = 2 \) with varies values of \( P \).
Fig (1.g): $\Omega = 0.5, \, P = 1.5 \times 10^3, \, r = 2$ with varies values of relaxation times.
6- Special cases

(a) -It is extremely difficult to obtain the roots of transcendental equation. However, if the coupling factor $\epsilon$ is assumed to be small, approximate solution to this equation can be found. Substituting from Eq. (4.9) into Eq. (5.4), we get

$$
\begin{bmatrix}
D'_{11} & D'_{12} & D'_{13} \\
D'_{21} & D'_{22} & D'_{23} \\
D'_{31} & D'_{32} & 0
\end{bmatrix} = 0
$$

(6.1)

where,

$$
D'_{11} = \left\{ -f'^1 qJ_o(qR)(\lambda + P) - 2\mu \left[ q^3 J_o(qR) - \frac{1}{R} q^2 J_1(qR) \right] \right. \\
- \rho((\omega^2 + \Omega^2) - C^2 f'^2 J_o(qR)) \right\},
$$

$$
D'_{12} = \left\{ f'^2 qJ_o(qR)(\lambda + P) - 2\mu \left[ q^3 J_o(qR) - \frac{1}{R} q^2 J_1(qR) \right] \right. \\
- \rho((\omega^2 + \Omega^2) - C^2 f'^2 J_o(qR)) \right\},
$$

$$
D'_{13} = 2\mu(q^2 - \frac{(\omega^2 + \Omega^2)}{C^2})^{1/2} \left[ \frac{1}{R} q^2 J_1(qR) - q^2 J_o(qR) \right],
$$

$$
D'_{21} = 2(q^2 - f'^2)^{1/2}, \quad D'_{22} = 2(q^2 - f'^2)^{1/2}, \quad D'_{23} = 2q^2 - \frac{(\omega^2 + \Omega^2)}{C^2},
$$

$$
D'_{31} = ((\omega^2 + \Omega^2) - C^2 f'^2), \quad D'_{32} = ((\omega^2 + \Omega^2) - C^2 f'^2), \quad D'_{33} = 0.
$$

Equation (6.1) is the frequency equation of a generalized thermo-elastic Rayleigh wave which has not yet been studied. It is clear from this frequency equation that the \
phase velocity of Rayleigh wave depends on the initial stress $P$ and rotation $\Omega$, (which are present in the medium at the beginning of the process).

The frequency equation (6.1) contains an initial value stress and rotation. When $\Omega = 0$ this case discussed by Abd-All [5].

The discussions is clear up from Figs. (2a-h).

(I) LS-model ($\tau_0 = 0, \tau_1 > 0, \delta = 1$).

Fig (2a): $P = 1.5, r = 2, \tau_1 = 0.1$ with varies values of $\Omega$.
Fig. (2.b): $\Omega = 0.5$, $r = 2, \tau_i = 0.1$ with varies values of $P$.

Fig (2.c): $\Omega = 0.5, P = 1.5 \times 10^{11}, r = 2$ with varies values of relaxation times.
Effect of rotation and initial stress

Fig (2.d): \( \Omega = 0.5, P = 1.5 \times 10^4 \) and \( \tau_i = 0.1 \).

(II) GL-model (\( \tau_0 > \tau_i > 0, \delta = 0 \)), \( \tau_0 = 2 \tau_i \).
Fig. (2.e): $P=1.5$, $r=2$, $\tau_i = 0.1$ with varies values of $\Omega$.

Fig. (2.f): $\Omega = 0.5$, $r=2$, $\tau_i = 0.1$ with varies values of $P$. 
Effect of rotation and initial stress

Fig (2.g): $\Omega = 0.5$, $\rho = 1.5 \times 10^{11}$, $r = 2$ with varies values of relaxation times.
Fig (2.h): \( \Omega = 0.5, \; P = 1.5 \times 10^{11} \) and \( \tau_i = 0.1 \)

(b) - When, \( P = 0 \) and \( \Omega = 0 \) equation (6.1) yields a new formula given by:

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & 0
\end{bmatrix} = 0
\]  \hspace{1cm} (6.2)

where,

\[
A_{11} = -\lambda f_1^{(2)} qJ_\nu(qR) - 2\mu \left[ q^3 J_\nu(qR) - \frac{1}{R} q^3 J_1(qR) \right] - \rho (\omega^2 - C_1^{(2)} f_1^{(2)}) q J_\nu(qR),
\]

\[
A_{12} = \lambda f_2^{(2)} qJ_\nu(qR) - 2\mu \left[ q^3 J_\nu(qR) - \frac{1}{R} q^3 J_1(qR) \right] - \rho (\omega^2 - C_1^{(2)} f_2^{(2)}) q J_\nu(qR),
\]

\[
A_{13} = \left[ 2\mu (q^2 - \frac{\omega^2}{C_2^{(2)}}) \right] \left[ \frac{1}{R} q^2 J_1(qR) - q^3 J_\nu(qR) \right],
\]

\[
A_{21} = 2(q^2 - f_1^{(2)}) \frac{1}{R}, \quad A_{22} = 2(q^2 - f_2^{(2)}) \frac{1}{R}, \quad A_{23} = 2q^2 \frac{\omega^2}{C_2^{(2)}}, \quad A_{31} = (\omega^2 - C_1^{(2)} f_1^{(2)}),
\]

\[
A_{32} = (\omega^2 - C_1^{(2)} f_2^{(2)}), \quad A_{33} = 0
\]

where, \( C_1^{(2)} = (\lambda + 2\mu)/\rho \), \( C_2^{(2)} = \mu/\rho \).

Equation (6.2) represents the frequency equation of Rayleigh wave in a generalized thermo-elastic in an infinite cylinder. Nowaki [15], Bagri and Eslami [13] have found similar formula in thermo-elastic case only. The discussions this case under effect the relaxation times is clear up from Figs. (3a-b)

(I) **LS-model** (\( \tau_0 = 0, \; \tau_i > 0, \; \delta = 1 \)).
Fig. (3.a): $\Omega = 0$, $P = 0$ and $r = 2$ if $\varepsilon$ is small.

(II) GL-model ($\tau_0 \geq \tau_1 > 0$, $\delta = 0$), $\tau_0 = 2\tau_1$. 
Fig. (3.b): $\Omega = 0, P = 0, \tau_i = 0.1$ and $r = 2$ if $\varepsilon$ is small.

(c) - For initial stress $P = 0$, $\Omega = 0$ and if $\varepsilon$ vanishes, i.e., when there is no coupling between the temperature and the strain fields, in case $\varepsilon = 0$ the mechanical relaxation time $\tau_0$ always equal zero and this case independent only $\tau_1$, clear up from Fig. (4.a), the frequency equation (5.4) takes the form:

$$
\begin{bmatrix}
A_{11}' & A_{12}' & A_{13}' \\
A_{21}' & A_{22}' & A_{23}' \\
A_{31}' & A_{32}' & 0
\end{bmatrix} = 0
$$

(6.3)

where,

$$
A_{11}' = \left\{-\lambda f^{1*}_1 q J_0(qR) - 2\mu \left[ q^2 J_0(qR) - \frac{1}{R} q^2 J_1(qR) \right] - \rho (\omega^2 - C_{11}^t f^{2*}_1) q J_0(qR) \right\},
$$

$$
A_{12}' = \left\{\lambda f_2 q J_0(qR) - 2\mu \left[ q^2 J_0(qR) - \frac{1}{R} q^2 J_1(qR) \right] - \rho (\omega^2 - C_{12}^t f^{2*}_2) q J_0(qR) \right\},
$$

$$
A_{13}' = \left\{2\mu \left[ q^2 - \frac{\omega^2}{C_{33}^t} \right] \left[ \frac{1}{R} q^2 J_1(qR) - q J_0(qR) \right] \right\},
$$

$$
A_{21}' = 2(q^2 - f^{1*}_1)^{\frac{1}{2}}, \quad A_{22}' = 2(q^2 - f^{2*}_2)^{\frac{1}{2}}, \quad A_{23}' = 2q^2 - \frac{\omega^2}{C_2^t}, \quad A_{33}' = (\omega^2 - C_{11}^t f^{2*}_1),
$$

$$
A_{32}' = (\omega^2 - C_{12}^t f^{2*}_2), \quad A_{33} = 0, \quad C_{11}^t = (\lambda + 2\mu)/\rho, \quad C_{22}^t = \mu/\rho.
$$

It is clear that equation (6.3) is the familiar frequency equation of Rayleigh wave in a generalized thermo-elastic in an infinite cylinder in classical case as obtained by Nowakl [15]. The discusses this case under effect the relaxation times is clear up from Fig. (4.a), also discuss numerical this case if $P \neq 0, \Omega \neq 0$ clear up from Fig. (4.b) and under effect the rotation clear up from Fig (4.c).

(I) LS-model ($\tau_0 = 0, \tau_i > 0, \delta = 1$).
Effect of rotation and initial stress

Fig (4.a): $\Omega = 0$, $P = 0$ and $r = 2$ if $\varepsilon = 0$
Fig. (4.b): $\Omega = 0.5$, $P = 1.5 \times 10^{11}$ and $r = 2$ if $\varepsilon = 0$

Fig. (4.c): $P = 1.5$, $r_i = 0.1$ and $r = 2$ if $\varepsilon = 0$
7- Numerical results and discussion

Numerical calculation were carried out of the frequency $\omega$ with the aid of an electronic computer for both initial stress $P = 0$ and $P \neq 0$, and both rotation $\Omega = 0$ and $\Omega \neq 0$.

In order to illustrate theoretical results obtained in the preceding section, we now present some numerical results. The material chosen for this purpose of Carbon steel, the physical data for which is given below [16].

\[
\rho = 7.9 \times 10^3 \text{kgm}^{-3}, \quad \lambda = 9.3 \times 10^{10} \text{Nm}^{-1}, \quad \mu = 8.4 \times 10^{10} \text{Nm}^{-1}, \quad \varepsilon = 0.34, \quad T_0 = 293.1 \text{k},
\]

\[
C_v = 6.4 \times 10^7 \text{JK}^{-1} \text{deg}^{-1}, \quad \alpha = 13.2 \times 10^{-6} \text{deg}^{-1}, \quad K = 50 \text{Wm}^{-1} \text{K}^{-1}, \quad \chi_0 = 0.321 \text{JK}^{-1} \text{deg}^{-1}.
\]

- General case

For LS-model: From Fig.(1.a), that clarify of effect of rotation, we find that the velocity of Rayleigh waves (Re) decreased with increased value of $\Omega$, contrary the attenuation coefficient (Im) increased with increased $\Omega$ while by division the velocity of Rayleigh waves (Re) on the attenuation coefficient (Im), or contrary by division the attenuation coefficient (Im) on the velocity of Rayleigh waves (Re) this value is equal with different value of $\Omega$, whereas from this fig show the value of (Re/Im) become one curve decreased with increased $\omega$, while the value of (Im/Re) obsolete with the change in value $\omega$.

From Fig. (1.b), that clarify of effect of initial stress, we find from increased value of $P$ decreased values of (Re) and (Im), while the value of (Re/Im) become obsolete with the change in value $\omega$, also, the value of (Im/Re) with change value of $P$ become one curve increased with increased $\omega$.

From Fig. (1.c), that clarify the effect of relaxation times, we find decreased (Re) and (Re/Im) with increased the values of $\tau_i$, while values of (Im) and (Im/Re) increased with increased the values of $\tau_i$.

From Fig. (1.d), that clarify of effect of $r$, we find increased (Re) with increased $r$, while (Im) decreased with increased $r$, also (Re/Im) become one curve decreased with increased $\omega$, while the value of (Im/Re) obsolete with the change in value $\omega$.

For GL-model

From Fig. (1.e), that clarify of effect of $\Omega$, we find decreased (Re) with increased $\Omega$, while increased (Im) with increased $\Omega$, while values of (Re/Im) and (Im/Re) obsolete with the change in values $\Omega$ and $\omega$.

From Fig. (1.f), that clarify of effect of initial stress, we found the values of (Re) and (Im) decreased with increased $P$, while obsolete (Im/Re) with the change in values $P$ and $\omega$, also (Im/Re) become one curve increased with increased $\omega$.

From Fig. (1.g), that clarify the effect of relaxation times, we find decreased (Re) and (Re/Im) with increased the values of $\tau_i$, while values of (Im) and (Im/Re) increased with increased the values of $\tau_i$. 
From Fig. (1.h), that clarify of effect of $r$, we find decreased (Re) with increased $r$, while (Im) decreased with increased $r$, also (Re/Im) become one curve increased with increased $\omega$, while the value of (Im/Re) obsolete with the change in value $\omega$.

-Special cases

-Case (a): If the coupling factor $\varepsilon$ is assumed to be small, in this case discussed the effect of relaxation times with effect $P$ and $\Omega$, $\tau_0 = 2\tau_1$.

For LS-model

From Fig. (2.a), that clarify of effect of $\Omega$, we found that (Re) become one curve decreased with increased $\omega$, also, the values of (Im) and (Re/Im) increased with increased $\Omega$, while the value of (Im/Re) decreased with increased $\Omega$.

From Fig. (2.b), that clarify of effect of $P$, we find (Re), (Im) and (Im/Re) decreased with increased $P$, while the value of (Re/Im) obsolete with the change in value $\omega$.

From Fig. (2.c), that clarify the effect of relaxation times, we find decreased (Re) and (Re/Im) with increased the values of $\tau_1$, while values of (Im) and (Im/Re) increased with increased the values of $\tau_1$.

From Fig. (2.d), that clarify of effect of $r$, we find decreased (Re) with increased $r$, while (Im) and (Re/Im) decreased with increased $r$, while (Im/Re) become one curve take almost one value with change in value $\omega$.

For GL-model

From Fig. (2.e), that clarify of effect of $\Omega$, we found that (Re) and (Im) becomes one curve decreased with increased $\omega$, while the value of (Re/Im) obsolete with the change in value $\omega$, and the value of (Im/Re) decreased with increased value of $\Omega$.

From Fig. (2.f), that clarify of effect of $P$, we find the values of (Re) and (Im) decreased with increased $P$, while the value of (Re/Im) obsolete with the change in value $\omega$, also, the value of (Im/Re) increased with increased $P$.

From Fig. (2.g), that clarify the effect of relaxation times, we find decreased (Re) and (Re/Im) with increased the values of $\tau_1$, while values of (Im) and (Im/Re) increased with increased the values of $\tau_1$.

From Fig. (2.h), that clarify of effect of $r$, we find decreased (Re) with increased $r$, while (Im) increased with increased values of $r$, also, the values of (Re/Im) and (Im/Re) obsolete with the change in value $\omega$.

-Case (b): If the coupling factor $\varepsilon$ is assumed to be small, $P=0$ and $\Omega = 0$.

For LS-model

From Fig. (3.a), that clarify of effect of $\tau_1$, we find the values of (Re), (Im) and (Re/Im) decreased with increased $\tau_1$, while the value of (Im/Re) increased with increased in value $\tau_1$.

For GL-model
From Fig. (3.b), that clarify of effect of $\tau_1$, we found the values of (Re) and (Im) increased and decreased with increased $\tau_1$, respectively, while values of (Re/Im) and (Im/Re) obsolete with the change in value $\omega$.

- Case (c): If $\varepsilon=0$, $P=0$, $\Omega=0$ and $p \neq 0$, $\Omega \neq 0$.

For LS-model

From Figs. (4.a) and (4.b), at $P=0$, $\Omega=0$ and $p \neq 0$, $\Omega \neq 0$, that clarify of effect of $\tau_1$, we find the value of (Re) increased with increased $\tau_1$, and the value of (Im) become one curve decreased with increased $\omega$, while (Re/Im) decreased with increased $\tau_1$, and value of (Im/Re) obsolete with the change in value $\tau_1$.

From Fig(4.c), that clarify of effect of $\Omega$, we find the value of (Re) increased with increased $\Omega$, and the value of (Im) decreased with increased $\Omega$, while (Re/Im) become one curve decreased with increased $\omega$, while, and value of (Im/Re) obsolete with the change in value $\omega$.

8-Conclusions

The aim of this paper is to estimate the effects of the rotation, initial stress, and relaxation times on wave propagation in generalized thermoelastic problem in tow models: (I) LS-model ($\tau_0=0$, $\tau_i>0$, $\delta=1$) and (II) GL-model ($\tau_0 \geq \tau_i>0$, $\delta=0$).

The frequency equation for this problem has been obtained. From the calculations and numerical results obtained, we concluded that: in two models clear the effect of the rotation, initial stress, and relaxation times on wave propagation whereas the Rayleigh wave velocity (Re) and Attenuation coefficient (Im) take pendulous values (i.e., increased or decreased with increased $\omega$), but (Re/Im) and (Im/Re) take different values (i.e., increased, decreased, become one curve, or take almost one value obsolete with change in value $\omega$). The exist clear different of effect of rotation in two models on in general case and special cases onto (Re), (Im), (Re/Im) and (Im/Re). The initial stress effect into (Re) and (Im) in two models are decreased with increased values of $\omega$, also, the values of (Im/Re) increased with increased $\omega$, put (Re/Im) obsolete with change in value $\omega$ in two models. The relaxation times effect in (Re), (Im) and (Im/Re) are increased put (Re/Im) decreased with increased values of $\omega$ in two models in general and special case (a), while in cases (b-c) effect of initial stress give wobbly values with values of $\omega$. In effect of $r$ we have in LS model the values of (Re) and (Re/Im) decreased with increased values of $\omega$ but in GL models the value of (Re) increased with increased values of $\omega$ and (Re/Im) decreased or obsolete with value of $\omega$, while the value of (Im) increased and decreased with value of $\omega$ in LS and GL models respectively. The values of (Im/Re) in two models are obsolete with increased values of $\omega$.

References


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