Instability of Visco-Elastic Fluid Layer
Heated from Below with Relaxation Time and Presence an Electric Field

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Abstract

The problem of the onset of stability in a horizontal layer of visco-elastic dielectric liquid (Walters’ liquid B’) with relaxation time under simultaneous action of a vertical ac electric field and a vertical temperature gradient is analyzed. Applying linear stability theory an equation of eight order is derived. Under somewhat artificial boundary condition, this equation can be solved exactly to yield the required eigenvalue relationship from which various critical values are determined in detail. The critical Rayleigh heat number and wave number for onset of instability are presented graphically as functions of Rayleigh electric number for various values of Prandtl number, elastic parameters and relaxation time.

Keyword: Stability, Viscoelastic, Temperature gradient, Electrohydrodynamics, Derive, Mathematica and Excel program
Introduction

The study of visco-elastic fluids has become of increasing importance in the last few years. This is mainly due to their many applications in petroleum drilling, manufacturing of foods and paper and many other similar activities. The boundary-layer concept for such fluids is of special importance owing to its application to many engineering problems. The problem of the onset of thermal instability in horizontal layer of a visco-elastic fluid heated from below is both of theoretical and practical interest.

In technological fields there exists an important class of fluids, called non-Newtonian fluids, which are also being studied extensively because of their practical application, such as fluid film lubrication, analysis of polymers in chemical engineering etc. One such fluid is called visco-elastic fluid.

To the author’s knowledge, the first work, which deals directly with this problem, appears in a brief report by Green [28]. His analysis, which is restricted to the case when both bounding surfaces are free, was carried out in terms of a two-time-constant model due to Oldroyd [12,13]. Vest and Arpaci [6] who employed a one-time-constant model due to Maxwell [11] also attacked the same problem in some detail. Takashima [25,26] to the case have recently extended this latter work when the fluid layer is rotating about a vertical axis at a constant rate. Walters [16] and Beard Walters [7] deduced the governing equation for the boundary flow for a prototype viscoelastic fluid which they have designated as liquid B’ when this liquid has a very short memory. Many other authors contributed to the subject. Raptis et al. [2-4] studied the free-convection and mass-transfer flow of a viscous and viscoelastic fluid past a vertical wall. Singh and singh [5] studied the magneto-hydrodynamics flow of a viscoelastic past an accelerated plate. The response of laminar skin friction, temperature and heat transfer to the fluctuations in the stream velocity in the presence of the transverse magnetic field is discussed by Sherief and Ezzat [9]. Ezzat et al. [18] studied the free convective heat transfer in an incompressible viscoelasticity hydromagnetic flow past an infinite vertical plate.

In most of above applications, the method of solution due to Lighthill [19] and stuart [14] is utilized. This method is applicable only to problems of simple harmonic vibrations. This prompted many authors to use other methods of solution when dealing with the problems of a non-vibrating fluid. Gupta [1] and Riley [27] used an approximate Pohlhausen method; Wilks and Hunt [8] used the method of similarity solution. Saponkoff [21] and Vajravelu and Sastri [15] used perturbation methods to solve problems of free convection in hydromagnetic flows.

Gross [20] and Gross and Porter [21] have performed some preliminary experiments in which a horizontal layer of viscous dielectric liquid is heated from above and a vertical dc electric field is applied across the layer. In the absence of the electric field, no convection is produced, as is to be expected since the configuration is gravitationally stable. When the strength of the electric field
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becomes sufficiently large, convection patterns are established which, despite the fact that buoyancy is here stabilizing, are strikingly similar to the familiar Bénard cells.

Othman and Ezzat [23] studied the stability of viscoelastic conducting liquid (Walters’s liquid B′) heated from below in the presence of a magnetic field. Othman [22] studied the problem of the onset of stability in a horizontal layer of viscoelastic dielectric liquid (Walters’s liquid B′) under the simultaneous action of a vertical ac electric field and a vertical temperature gradient. Othman and Zaki [24] studied the effect of thermal relaxation time on a electrohydrodynamic viscoelastic fluid (Oldroyd liquid) layer heated from below.

The purpose of the present investigation is to examine theoretically the problem of the onset of instability in a horizontal layer of viscoelastic dielectric liquid (Walter’s liquid B′) under the simultaneous action of a vertical ac electric field, a vertical temperature gradient, relaxation time and elastic parameters.

2. Formulation of the problem

We consider an infinite incompressible and dielectric viscoelastic fluid layer (Walters’s liquid B′) occupying the space between two horizontal rigid bodies, which are at L apart. The lower surface at z = 0 and the upper surface at z = L are maintained at constant temperatures $T_0$ and $T_1$ respectively. In addition to temperature gradient, a vertical ac electric field is also imposed across the layer, the lower surface is grounded and the upper surface at a high (60 HZ) potential whose root value is $\phi_1$.

The basic equations are given as [26, 16]

\[
\nabla \cdot \mathbf{v} = 0,
\]

\[
(1 + \lambda_1 \frac{\partial}{\partial t}) \{ \rho \frac{\partial \mathbf{v}}{\partial t} - \rho \mathbf{g} + \nabla p - f_e + K_o \left( \frac{\partial}{\partial t} \nabla^2 \mathbf{v} + (\nabla \cdot \mathbf{v}) \nabla^2 \mathbf{v} \right) - (\nabla \cdot \mathbf{v}) \nabla^2 \mathbf{v} - 2(\nabla \cdot \mathbf{v}) \nabla^2 \mathbf{v} ) \} = n_o (1 + \lambda_2 \frac{\partial}{\partial t} ) \nabla^2 \mathbf{v}
\]

\[
\rho C_v (1 + \tau_o) \left[ \frac{\partial T}{\partial t} + \mathbf{v} \nabla T \right] = k \nabla^2 T,
\]

\[
\nabla \cdot (\epsilon \mathbf{E}) = 0, \quad (\epsilon \mathbf{E}) = 0 \quad \text{or} \quad \mathbf{E} = -\nabla \phi,
\]

where is $\rho$ the mass density, $\mathbf{v} = (u, v, w)$ is the velocity of the fluid, $P$ is the pressure, $\mathbf{g} = (0, 0, -g)$ is the gravitational acceleration, $k$ is the thermal conductivity, $\eta_o$ is the limiting viscosity, $P$ is the pressure, $K_o$ is the elastic constant of (Walters’s liquid B′), $C_v$ is the specific heat at constant volume, $T$ is the temperature of the fluid, $\tau_o$ is the relaxation time, $\lambda_1$ is the (stress) relaxation
time and $\lambda_2$ is the (strain) retardation time, $\varepsilon$ is the dielectric constant, $E = [0, 0, E_z]$ the electric field, $\varphi$ the root-mean-square value of electric potential and $f_e$ is the force of the electrical origin which may be expressed as Landau [29] in the form,

$$f_e = \rho_e E - \frac{1}{2} E^2 \nabla \varepsilon + \frac{1}{2} \nabla \left( \rho \frac{\partial \varepsilon}{\partial \rho} E^2 \right)$$  \hspace{1cm} (6)$$

taking into account the fact the free charge density $\rho_e$ is zero.

If we replace the pressure by

$$p^* = p - \frac{1}{2} \rho \frac{\partial \varepsilon}{\partial \rho} E^2.$$  \hspace{1cm} (7)$$

The electrostriction term disappears from Eq.(2)

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \left\{ \rho \frac{\partial \varphi}{\partial t} - \rho g + \nabla (p^* + \frac{1}{2} E^2 \text{grad} \varepsilon) + K_0 \left( (\nabla \varphi) \nabla^2 \varphi - (\nabla \varphi) \nabla^2 \varphi + \frac{\partial}{\partial t} \nabla^2 \varphi \right) \right\} = n_o \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \nabla^2 \varphi.$$  \hspace{1cm} (8)$$

The mass density $\rho$ and the dielectric constant $\varepsilon$ are assumed to be linear dependent on temperature as [6]

$$\rho = \rho_o [1 - \alpha (T - T_o)], \quad \alpha > 0, \quad (9)$$
$$\varepsilon = \varepsilon_o [1 - e (T - T_o)], \quad e > 0, \quad (10)$$

where $\alpha$ is the coefficient of volume expansion and $e$ is the coefficient of relative variation of the dielectric constant with temperature.

It is obvious that there exist the following steady solution (denoted by an over bar):

$$\varphi = 0,$$  \hspace{1cm} (11)$$
$$T = T_o - \beta z,$$  \hspace{1cm} (12)$$
$$\rho = \rho_o (1 + \alpha \beta z), \quad (13)$$
$$\varphi = \varepsilon_o (1 + e \beta z), \quad (14)$$

$$E_x = E_y = 0, \quad E = \frac{E_o}{1 + e \beta z},$$  \hspace{1cm} (15)$$
$$\varphi = -\frac{E_o}{e \beta} \ln(1 + e \beta z),$$  \hspace{1cm} (16)$$

where

$$\beta = \frac{T_o - T_1}{L} \quad (17)$$

is the adverse temperature gradient and

$$E_o = -\frac{\varphi \alpha \beta}{\ln(1 + \alpha \beta L)} \quad (18)$$
is the root-mean-square value of the electric field at \( z = 0 \). The modified pressure \( \tilde{P}^* \) can be determined, if necessary, from the equation

\[
\text{grad} \tilde{P}^* = \rho \tilde{E} - \frac{1}{2} \tilde{E}^2 \text{grad} \tilde{E}.
\]  

Let this initial steady-state be slightly perturbed. Following the classical linear of stability theory, we obtain the following main equation:

\[
1 + \lambda_1 \frac{\partial}{\partial t} \left[ \frac{\partial}{\partial t} \nabla^2 w' - \left( \frac{\alpha g + \varepsilon_o E_o^2 \varepsilon^2 \beta}{\rho_o} \right) \nabla^2 \Theta' - \left( \frac{\varepsilon \varepsilon_o E_o}{\rho_o} \right) \nabla^2 \left( \frac{\partial \phi'}{\partial z} \right) + K_o \frac{\partial}{\partial t} \nabla^4 w' \right]
= \nu \left( 1 + \lambda_2 \frac{\partial}{\partial t} \right) \nabla^4 w',
\]

\[
(1 + \tau_0 \frac{\partial}{\partial t}) \left[ \frac{\partial \Theta'}{\partial t} - \beta w' \right] = K \nabla^2 \Theta',
\]

\[
\nabla^2 \phi' + \varepsilon E_o \frac{\partial \Theta'}{\partial z} = 0,
\]

where \( \nu = \frac{\eta_o}{\rho_o} \) is the kinematic viscosity, \( K = \kappa/(\rho_o C_v) \) the thermal diffusivity, the prime refers to perturbed quantities, \( \nabla^2 = \left( \frac{\partial^2}{\partial x^2} \right) + \left( \frac{\partial^2}{\partial y^2} \right) \) is the two-dimensional Laplacian, and \( \nabla^2 = \nabla^2 + \frac{\partial^2}{\partial z^2} \). Here small terms have been neglected using the fact that \( |\alpha \beta z| << 1 \).

Eqs. (20)-(22) are first rendered dimensionless by choosing \( L, L^2/K, K/L, \beta L \) and \( \varepsilon \varepsilon_o E_o L^2 \) as the units of length, time, velocity, temperature and electric potential respectively, and are then simplified in the usual manner by decomposing the solution in terms of normal modes, so that

\[
[w', \Theta', \phi'](x, y, z, t) = [W(z), \Theta(z), \Phi(z)] \exp[\iota c t + \iota (ax + by)]
\]

where \( a, b \) are the (real) wavenumber in \( x \)- and \( y \)-directions and \( c \) the time constant (which is complex in general). Thus, we arrive at

\[
(1 + c F)[p^{-1}_r c (D^2 - \lambda^2) W(z) + \lambda^2 (R_h + R_e) \Theta(z) + \lambda^2 R_e D \Phi(z) + p^{-1}_r K_o c (D^2 - \lambda^2)^2 W(z) = (1 + \gamma Fe) (D^2 - \lambda^2)^2 W(z),
\]

\[
[c (1 + c \tau_o) - (D^2 - \lambda^2)] \Theta(z) = (1 + c \tau_o) W(z)
\]

\[
(D^2 - \lambda^2) \Phi(z) + D \Theta(z) = 0
\]

where, \( \lambda = \sqrt{a^2 + b^2} \) is the horizontal wavenumber, \( D = \frac{d}{dz} \), \( \gamma = \frac{\lambda_1}{\lambda_2} \) is the ratio of
the (strain) retardation to the (stress) relaxation time, \( F = \frac{\lambda_1 K}{L^2} \) is an elastic parameter which may be interpreted as a Fourier number in terms of \( \lambda_1 \), \( P_r = \frac{v}{\kappa} \) is the Prandtl number, \( R_H = \frac{a g \beta L^4}{v K} \) is the Raleigh heat number, \( R_E = \frac{\varepsilon_0 e^2 E^2 \beta^2 L^4}{\rho_0 v K} \)
is the Raleigh electric number and \( K_o^* = \frac{K_o}{L^2} \) is the elastic parameter of (Walters's liquid B') .

In seeking solution of these equations we must impose certain boundary conditions at the lower surface \( z = 0 \) and the upper surface \( z = 1 \). The most realistic boundary conditions may be written as

\[
W = DW = \Theta = \Phi = 0 \quad \text{at} \quad z = 0, 1
\]

In this paper, however, we shall use somewhat different boundary conditions given by

\[
W = D^2 W = \Theta = D \Phi = 0 \quad \text{at} \quad z = 0, 1. \tag{27}
\]

In this case, although admittedly an artificial one to consider, is importance since its exact solution is readily obtained. Eqs. (24)-(26) together with the boundary conditions (27) constitute an eigenvalue system of eighth order. It is evident that for an ordinary viscous fluid \( (F = 0, K_o^* = 0) \) [28] and that for a Maxwell fluid \( (\gamma = 0, K_o^* = 0) \) [6].

3. Solution

The eigenvalue system defined by Eqs. (8), (9) and (10) can be combined to yield

\[
\{(1 + c F)[p^{-1}_r c (D^2 - \lambda^2)]^2 \{c (1 + c \tau_o) - (D^2 - \lambda^2)\} + \lambda^2 (1 + c \tau_o) \} (R_H + R_E) (D^2 - \lambda^2) \\
- (1 + c \tau_o) \lambda^2 R_E D^2 \} + \{p^{\gamma}_r c (1 + c F) K_o^* - (1 + \gamma F c)\} \{c (1 + c \tau_o) \}
\]

\[
- (D^2 - \lambda^2) \} (D^2 - \lambda^2)^3 \Theta(z) = 0, \tag{28}
\]

together with

\[
\Theta = D^{(2m)} \Theta = 0 \quad \text{at} \quad z = 0, 1 \quad (m = 1, 2, 3, 4). \tag{29}
\]

Examination of (28) and (29) indicates that relevant solution for \( \Theta \) (characterizing the lowest mode) is given by

\[
\Theta(z) = \Theta_0 \sin(\pi z). \tag{30}
\]
where $\Theta_0$ is a constant. Substituting of this solution for $\Theta$ in equation (28) leads to required eigenvalue equation

$$B \left( c + \frac{B}{(1 + c \tau_0)} \right) \left\{ \frac{(1 + \gamma F_c)}{(1 + F_c)} B + p_r^{-1} c (1 - K_o^* B) \right\} - \lambda^2 R_H - \left( \frac{\lambda^4}{B} \right) R_E = 0,$$

then,

$$R_H = \frac{B \left( c + \frac{B}{(1 + c \tau_0)} \right) \left\{ \frac{(1 + \gamma F_c)}{(1 + F_c)} B + p_r^{-1} c (1 - K_o^* B) \right\} - \left( \frac{\lambda^2}{B} \right) R_E}{\lambda^2},$$

where it must be remembered that $c$ can be complex and

$$B = \pi^2 + \lambda^2.$$

4. Oscillatory instability

Let us now separate the right hand-side of Eq.(32) into the real and imaginary parts after setting $c = i \omega$ with $\omega$ being real. Then, we have

$$R_H = X + i \omega Y.$$  

There, $X$ and $Y$ are real-value functions of $P_r, R_E, F, \tau_o, \lambda, \gamma$ and $\omega$, and explicit expansions for these functions are as follows :

$$X = \frac{1 + \omega^2 \gamma F^2 - \omega^2 \tau_o \left[ (p_r^{-1} K_o^* - \gamma F + F + \omega^2 K_o^*) \right] B^3}{\lambda^2 (1 + \omega^2 F^2)} \left\{ \left[ (K_o^* + \tau_o) p_r^{-1} F(\gamma - 1) \right] \omega^2 + \left[ p_r^{-1} \tau_o K_o^* + p_r^{-1} F^2 K_o^* \right] \omega^4 \right\} B^2$$

$$+ \frac{\left[ \frac{1}{\lambda^2} + \omega^2 F(\gamma - 1) \right] \omega^4}{\lambda^2 (1 + \omega^2 F^2)} \left[ \lambda^2 + \left( \frac{\lambda^2}{B} \right) R_E \right],$$

$$Y = \frac{1 + \omega^2 \gamma F^2 - \omega^2 \tau_o \left[ (p_r^{-1} K_o^* - \gamma F + F + \omega^2 K_o^*) \right] B^3}{\lambda^2 (1 + \omega^2 F^2)} \left\{ \left[ (F(\gamma - 1) - p_r^{-1} K_o^* \right] - \left[ (K_o^* + \tau_o F^2) \right] \omega^2 \right\} B^3$$

$$+ \frac{\left[ (1 + p_r^{-1}) + (p_r^{-1} F^2 + \tau_o F^2) \right] \omega^4}{\lambda^2 (1 + \omega^2 F^2)} \left[ \lambda^2 + \left( \frac{\lambda^2}{B} \right) R_E \right].$$
It is apparent from Eq. (31) that for arbitrary assigned values of $p_r, R_E, F, \tau_o, K^*_o, \lambda, \gamma$ and $\omega, R_H$ will be complex, but the physical meaning of $R_H$ requires to be real.

Consequently, from the condition that $R_E$ must be real, so we have either

$R_H = X$ and $\omega = 0, \quad (37)$

Or

$R_H = X$ and $Y = 0. \quad (38)$

from equation (37) we have obtain the eigenvalue equation for stationary instability,

$$R_H = \frac{B^3}{\lambda^2} - \left(\frac{\lambda^2}{B}\right) R_E \quad (39)$$

For a Newtonian viscous fluid, when the electric field is absent, i.e $R_E = 0$, Eq. (39) reduces to

$$R_H = \frac{B^3}{\lambda^2} \quad (40)$$

which agrees with the classical result (Chandrasekher[28]).

Equation (37) will give the critical Rayleigh heat number $R_{HC}$ for the onset of stationary instability.

On the other hand, Eq.(38) leads, after some rearrangement, to

$$R_H = \frac{B^3}{\lambda^2} \left\{ \left(1+\omega^2 \gamma F^2 \right) + F \omega^2 \tau_o \left(1-\gamma\right)-\omega^2 K^*_o \tau_o \left(p_r^{-1} + \omega^2 \right) \right\}$$

$$\lambda^2 \left(1+\omega^2 \tau_o^2 \right) \left(1+\omega^2 F^2 \right) \lambda$$

$$+ \frac{B^2 \left\{ \omega^2 F \left(1-\gamma\right) \left(1+\omega^2 \tau_o^2 \right) + p_r^{-1} \omega^2 \tau_o \left(1-\omega^2 F^2 \right) + \omega^2 K^*_o \left(p_r^{-1} + \omega^2 F^2 \right) \right\}}{\lambda^2 \left(1+\omega^2 \tau_o^2 \right) \left(1+\omega^2 F^2 \right)}$$

$$+ \frac{p_r^{-1} \omega^2 \left(1-\omega^2 F^2 \right)}{\lambda^2 \left(1+\omega^2 F^2 \right)} B \left(\frac{\lambda^2}{B}\right) R_E, \quad (41)$$

and

$$\gamma \tau_o^2 F^2 B^2 \omega^4 + \left\{ \left[F \left(p_r^{-1} + \gamma\right) + \tau_o^2 \right] B^2 - \left(K^*_o + \tau_o \gamma F^2 \right) B^3 \right\} \omega^2$$

$$+ \left\{ \left(1+p_r^{-1}\right) B^2 + \left[F (\gamma - 1) - p_r^{-1} K^*_o - \tau_o \right] B^3 \right\} = 0. \quad (42)$$

For assigned values of $p_r, \tau_o, F, K^*_o, \gamma$ and $R_E$ Eq. (41), (42) define $R_H$ as a function of $\lambda$; the minimum of this function determines the critical Rayleigh heat number $R_{HC}$ for the onset of oscillatory instability (i.e. overstability) should be compared with that for the onset of stationary instability (i.e. ordinary convection). The type of instability, which takes place in practice, will be that corresponding to the lower value of the critical Rayleigh heat number.
5. Numerical results and conclusion

In order to determine the conditions under which instability sets in a overestability \( \rho_r, \tau_0, F, K_0^*, \gamma, \lambda \) and \( R_E \) where assigned fixed values, and the value of \( \omega \) was evaluated numerically from (42). Using this value of \( \omega \), the value of \( R_H \) was evaluated numerically from (41). The procedure was repeated for various values of \( \lambda \) to locate the minimum of \( R_H \).

We have plotted the variation of Rayleigh heat number \( R_H \) with wave number \( \lambda \) using (41) satisfying (42) for onset of the overstable case for values of the dimensionless parameters \( \rho_r = 0.001, 0.01, \tau_0 = 0.005, 0.01 \) and \( \gamma = 0.3 \). Figs. 1-4 corresponds to \( K_0^* = 0.1, 0.3, F = 0.5, 1, \rho_r = 0.001, 0.01 \) and \( \gamma = 0.3 \). Figs. 1-4 show that for the onset of the overstability as \( K_0^* \) increases the value of \( R_H \) increases, however as \( \tau_0 \) increases the value of \( R_H \) decreases, while the Rayleigh heat number decreases as Prandtl number increases, i.e. the onset of stability is hastened as \( R_E \cdot \tau_0 \) and \( \rho_r \) increase, while it is delayed as \( K_0^*, F \) increase. The critical Rayleigh heat number \( R_{HC} \) obtained in the manner for both stationary instability and overestability is shown in Figs. 5-10 as a function of \( R_E \) for values of the dimensionless parameters \( \rho_r = 0.001, 0.01, \gamma = 0.3, F = 1, K_0^* = 0.005, 0.01, 0.5, \tau_0 = 0.005, 0.01, 0.5 \). From Figs. 5-8 we notice that the critical Rayleigh heat number \( R_{HC} \) decreases as the relaxation time \( \tau_0 \) increases and increases as the frequency \( \omega \) increases. Also from the Figs. 9, 10 we notice that the critical Rayleigh heat number \( R_{HC} \) is increases as elastic parameter \( F \) increases.

From the above findings, we conclude that the stabilizing effects of the relaxation times and elastic parameters \( K_0^*, F \) are controlled by presence of electric field.

References


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Fig. 1. Variation of $R_H$ with $\lambda$ for various values of $K_o^*$, $F$ and $\tau_o$ at $p_r = 0.01$ and $R_E = 100$. 
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Fig. 2. Variation of \( R_H \) with \( \lambda \) for various values of \( K_o^* \), F and \( \tau_o \) at \( p_r = 0.001 \) and \( R_E = 100 \).
Fig. 3. Variation of $R_H$ with $\lambda$ for various values of $K_o^*$, $F$ and $\tau_o$ at $p_r = 0.001$ and $R_E = 4000$. 
Fig. 4. Variation of $R_H$ with $\lambda$ for various values of $K_0^*$, $F$ and $\tau_o$ at $p_r = 0.01$ and $R_E = 4000$. 
Fig. 5. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $K^*_o$ at $p_r = 0.001$, $F = 1$, $\gamma = 0.3$, $\tau_o = 0.05$ and $\omega = 10$. $\omega = 0$ represents the onset stationary convection.
Fig. 6. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $K^*_{o}$ at $p_r = 0.001$, $F = 1$, $\gamma = 0.3$, $\tau_o = 0.05$ and $\omega = 4$. $\omega = 0$ represents the onset stationary convection.
Fig. 7. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $K_\phi^*$ at $p_r = 0.01$, $F = 1$, $\gamma = 0.3$, $\tau_\phi = 0.005$ and $\omega = 10$. $\omega = 0$ represents the onset stationary convection.
Fig. 8. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $K_0^*$ at $\rho=0.01$, $F=1$, $\gamma=0.3$, $\tau=0.005$ and $\omega=4$. $\omega=0$ represents the onset stationary convection.
Fig. 9. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $F$ at $p_r = 0.001$, $K_o^* = 0.5$, $\gamma = 0.3$, $\tau_o = 0.005$ and $\omega = 10$. 
Fig. 10. Representation the critical Rayleigh heat number $R_{HC}$ as a function of $R_E$ for various values of $F$ at $p_r = 0.001$, $K^*_o = 0.5$, $\gamma = 0.3$, $\tau_o = 0.01$ and $\omega = 10$.

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