Upwards Fired Bullet Turning

at the Trajectory Apex

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Abstract

A six-degree of freedom simulation model is used to investigate a bullet turning at the trajectory apex, particularly when fired about vertically. The bullet model includes an aerodynamic model covering angles of attack up to 180 degrees. The bullet attitude is computed based on the quaternion in order to avoid equation singularity problems. Two separate codes with different body-fixed coordinate systems are used to verify the results obtained. The role of Magnus-phenomena in turning is particularly studied at around the apex. Also some disturbance effects, like atmospheric turbulence on the bullet response, were analyzed. The objective of the study is to find out the terminal velocity of the conceptual 7.62 mm bullet. Based on the results it seems that carelessly upwards fired projectile can be, at least, dangerous to the crowd in the vicinity of the shooter.

Mathematics Subject Classification : 70E15, 76G25

Keywords : Trajectory simulation, Magnus effect, Shooting accident

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1. Introduction

Upwards into the air shooting people is a familiar sight on television news and sometimes one cannot help wondering where the bullets end up. The motivation of the current study is to find out whether those carelessly fired bullets are dangerous to people around the shooter. A fairly complex but standard mathematical model, like the one described in Ref. [5] is needed to accurately enough capture the phenomena of projectile turning at around and after the apex at about 2 000 meters altitude. However, the lack of aerodynamic data typically prevents this kind of simulations. Particularly the aerodynamic model has to cover the angles of attack $\alpha$ (also known as the yaw angle) from 0 to 180 degrees at small Mach numbers.

In this paper, a conceptual 7.62 mm bullet computational model was created and Six Degrees of Freedom (6-DOF) simulations were undertaken to find out some possible trajectories for upwards fired bullets. The study is not covering all the possible bullet geometries, but the results obtained are believed to representative for the typical bullet size and mass considered.

It is obvious that the very complex interaction between the bullet and the air cannot be presented entirely by a few aerodynamic coefficients. However, reasonable estimates for the most important coefficients are included to the model and the model applicability is extended by varying moment coefficients. The moment coefficients are in crucial role while the bullet turning behavior is studied. In addition, some stochastic phenomena are introduced to the study via atmospheric turbulence.

2. Bullet Geometry Model

The simulations were carried out to a conceptual 7.62 mm bullet. The bullet geometry is depicted in Table 1 and Fig. 1. The bullet nose is a blunted cone. The weapon rifle makes one spin while the bullet travels about 240 mm resulting to the initial spin value 2 980 rounds/s.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value</th>
<th>Characteristics</th>
<th>Value</th>
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<tbody>
<tr>
<td>diameter</td>
<td>7.62 mm</td>
<td>weight</td>
<td>8 g</td>
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<tr>
<td>length</td>
<td>38 mm</td>
<td>center of gravity (CG)</td>
<td>24 mm (from the nose)</td>
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<tr>
<td>nose length</td>
<td>18 mm</td>
<td>moment of inertia $I_x$</td>
<td>5E-8 kgm$^2$ (longitudinal)</td>
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<tr>
<td>tip diameter</td>
<td>1.75 mm</td>
<td>moment of inertia $I_y=I_z$</td>
<td>9E-7 kgm$^2$ (transverse)</td>
</tr>
</tbody>
</table>
3. Bullet Aerodynamic Model

The aerodynamic properties were estimated using a simplified engineering method of Ref. [3]. The results obtained are believed to be qualitatively correct for the generic case studied. The bullet aerodynamic model based on the results was further simplified to present the data as closed-form equations. The effect of the simplification is assumed to be negligible on the phenomena investigated in this study.

The aerodynamic force coefficients are given in the body-fixed coordinate system in this study and the corresponding coefficients in the wind coordinate system can be obtained from

\[ C_D(\alpha) = C_N \sin(\alpha) + C_A \cos(\alpha) \]  
\[ C_L(\alpha) = C_N \cos(\alpha) - C_A \sin(\alpha) \]  

The aerodynamic forces and moments are made dimensionless based on the projectile diameter \( d \) and cross section area \( S = \pi d^2/4 \).

Setting up the aerodynamic model up to the yaw angle \( \alpha = 180 \) degrees was performed as follows. At first the static aerodynamic coefficients were obtained up to the yaw angle 90 degrees. Secondly the region 90…180 degrees properties
were estimated by making a reverse bullet geometry model. The reverse backwards flying model obviously has a blunt nose and a long boat-tail. The entire model covering the whole region was finally achieved by joining the results obtained. The procedure described here enables utilizing the software [3] beyond its limits.

The zero yaw drag coefficient \( C_{D_0} \) (\( C_A \) at zero yaw angle) obtained is depicted in Fig. 2 for the forward and backward flying projectile. At subsonic region the backward flying projectile drag is seen to be about 4.5 times larger than the one for the forward flying case.

![Figure 2](image2.png)

**Figure 2.** The 7.62 mm projectile zero yaw drag coefficient \( C_{D_0} \) f(Ma) obtained for the forward and backward flying cases.

The axial force coefficient \( C_A \) covering the whole yaw angle region at subsonic speeds is depicted in Fig. 3.

![Figure 3](image3.png)

**Figure 3.** The axial force coefficient \( C_A \) \( \cos(\alpha) \)^2-fit at subsonic regime as a function of yaw angle.

The axial force coefficient in the simulations was obtained from equations (\( \alpha \) in degrees)
Upwards fired bullet turning at the trajectory apex

\[
C_A(\alpha) = \left[\cos(\alpha)\right]^2 C_{Do} \quad \alpha = 0\ldots90\text{deg} \quad (3)
\]

\[
C_A(\alpha) = -4.5 \left[\cos(\alpha)\right]^2 C_{Do} \quad \alpha = 90\ldots180\text{deg} \quad (4)
\]

The zero yaw drag coefficient \(C_{Do}\) value \(f(Ma)\) used in the simulations is depicted in Fig. 4. The coefficient behavior is taken to be linear between the given data corner points.

![Graph of CDo vs. Ma](image)

**Figure 4.** The 7.62 mm projectile zero yaw drag coefficient \(C_{Do} (Ma)\) used in the simulations for the forward flying projectile.

![Graph of Cm vs. Angle of Attack](image)

**Figure 5.** The 7.62 mm projectile pitching moment coefficient \(C_m (\alpha)\) at Mach = 0.2.

The correspondingly obtained pitching moment coefficient \(C_m\) as a function of yaw angle at Mach 0.2 is depicted in Fig. 5. The moment coefficient is seen to be zero also at yaw angle 40 degrees and reverse negative moment affects above that value. The steep slope at 180 yaw angle is due to the excessive boat-tailing of the backwards flying geometry.
The fits of the moment coefficient in the simulations are ($\alpha$ in degrees)

\[
C_m(\alpha) = C_{m\alpha} \alpha \left( \frac{\pi}{180} \cos(2.25\alpha) \right) \quad \alpha = 0...40 \text{ deg} \quad (5)
\]

\[
C_m(\alpha) = -\frac{C_{m\alpha}}{5} \left[ \sin(\alpha - 40) \right]^2 \quad \alpha = 40...130 \text{ deg} \quad (6)
\]

\[
C_m(\alpha) = -\frac{C_{m\alpha}}{5} - \sin[4.11(\alpha - 130)] \quad \alpha = 130...180 \text{ deg} \quad (7)
\]

Some pitching moment variation influence was also considered in the study. The moment coefficient zero was shifted in the simulations from 40 to 90 degrees yaw angle and $f(\alpha)$-trend was obtained from equations

\[
C_m(\alpha) = C_{m\alpha} \left[ \cos(\alpha) \right]^4 \sin(\alpha) \quad \alpha = 0...90 \text{ deg} \quad (8)
\]

\[
C_m(\alpha) = \alpha \left( \frac{\pi}{180} \right) \cos(\alpha) \left[ \sin(\alpha) \right]^{3/4} \quad \alpha > 90 \text{ deg} \quad (9)
\]

Nevertheless, the moment coefficient general trend was maintained also with the alternative fits of Equations 8 and 9.

Figure 6. The 7.62 mm projectile zero-yaw pitching moment coefficient slope $C_{m\alpha}$ as a function of Mach number ($Ma$). The zero-yaw moment coefficient slope $C_{m\alpha}$ used in simulations is depicted in Fig. 6. The linear trend exists between the corner points.

The normal force coefficient $C_N$ as a function of yaw attack at Mach 0.2 is depicted in Fig. 7. The coefficient is obtained from equations
Upwards fired bullet turning at the trajectory apex

\[ C_N(\alpha) = \left[C_{Na} + \cos(\alpha)\sin(\alpha)\right] \alpha \left(\frac{\pi}{180}\right) \quad \alpha = 0...90 \text{ deg} \quad (10) \]

\[ C_N(\alpha) = C_{Na} \alpha \left(\frac{\pi}{180}\right) \left(\frac{180 - \alpha}{90}\right)^{1.5} \quad \alpha = 90...180 \text{ deg} \quad (11) \]

The zero yaw force coefficient slope \( C_{Na} \) is depicted in Fig. 8.

Figure 7. The 7.62 mm projectile normal force coefficient \( C_N(\alpha) \) at Mach = 0.2.

Figure 8. The 7.62 mm projectile zero-yaw normal force coefficient slope \( C_{Na} f(Ma) \).

The Magnus-moment coefficient zero-yaw slope \( C_{npa} \) as function of Mach number is depicted in Fig. 9. The positive value makes the nose to turn to the right in case positive nose up angle of attack and the positive spin (clockwise seen from behind of bullet). The force coefficient slope \( C_{ypa} \) is depicted in Fig. 10. The negative force affects to the left in case of the positive angle of attack and spin.
The Magnus coefficient $C_{np}$ and $C_{yp}$ values as a function of yaw attack are based in this study on the zero yaw slopes $C_{npa}$ (and $C_{ypa}$) value and the Magnus coefficients value zero at 0 and 180 degrees (no asymmetry present). The moment coefficient at Mach number 0.2 as a function of angle of attack is depicted in Fig. 11.

![Figure 9](image1.png)  
**Figure 9.** The 7.62 mm projectile zero-yaw Magnus moment coefficient slope $C_{npa}$ f(Ma).

![Figure 10](image2.png)  
**Figure 10.** The 7.62 mm projectile zero-yaw Magnus force coefficient slope $C_{ypa}$ f(Ma).
The Magnus-phenomena \( f(\alpha) \) trend is uncertain and the coefficients obtained (see Equations 12 and 13) were varied in the simulations by multiplying the values obtained by 0.0, 0.5 and 1.5.

The Magnus-moment is positive or zero in the simulations even though some CFD-results published [1][4] show some negative and also unsteady moment coefficient values at small angles of attack at subsonic and transonic region. However, the moment seems to return back to the positive values at yaw-angles about 5 degrees [1]. This trend is also in good agreement with the results of Ref. [4].

The Magnus-phenomena as a function of yaw attack were obtained in the simulations from equations

\[
C_{np}(\alpha) = k \left[ C_{n \rho \alpha} \cos(\alpha / 2) \right] \left( \frac{\pi}{180} \right) \alpha = 0...180 \text{ deg} \tag{12}
\]

\[
C_{YP}(\alpha) = k \left[ C_{Y \rho \alpha} \cos(\alpha / 2) \right] \left( \frac{\pi}{180} \right) \alpha = 0...180 \text{ deg} \tag{13}
\]

The variation coefficient \( k \) is 1 in case of the basic aerodynamic model.

In addition two moment damping coefficients were included into the bullet aerodynamic model. The spin damping moment coefficient \( C_{lp} f(Ma) \) is obtained from

\[
C_{lp} = -0.045 + 0.013Ma - 0.0015Ma^2 \tag{14}
\]
The pitch damping moment coefficient $C_{mq}(Ma)$ for the forward and backward flying bullets is depicted in Fig. 12. Since no data was available for the coefficient at higher yaw-angles the trend is simply estimated from

$$C_{mq} = C_{mq, Forward} - (C_{mq, Forward} - C_{mq, Backward}) \sin(\alpha / 2)$$  \hspace{1cm} (15)

![Figure 12. The 7.62 projectile pitch damping moment coefficient $C_{mq}(Ma)$ for the forward and backward flying cases.](image)

In the simulations, the aerodynamic forces and moments were at first obtained based on the total angle of attack value and the body mass center fixed coordinate system components were obtained based on the bullet velocity components. For example the normal force $N$ lateral components are

$$N_{yb} = C_N(\alpha)qS \frac{v - W_{yb}}{\sqrt{(v - W_{yb})^2 + (w - W_{zb})^2}}$$  \hspace{1cm} (16)

$$N_{zb} = C_N(\alpha)qS \frac{w - W_{zb}}{\sqrt{(v - W_{yb})^2 + (w - W_{zb})^2}}$$  \hspace{1cm} (17)

### 4. Trajectory Simulation Model

Two separate 6-DOF simulation codes were used in order to simulate the bullet flight. The mathematical model needed to accurately enough capture the phenomena is described in many text books (see for example in Ref. [5]). The projectile body-fixed coordinate system used is depicted in Fig. 13.
The projectile body-fixed coordinate system was defined in two different ways in two separate simulation codes used to verify the results obtained. The spinning mass center fixed coordinate system requires a very short integration time step compared with the so-called zero-spin coordinate system also used. However the former approach is needed in case some projectile asymmetry effects should be investigated.

The traditional Eulerian angles are useful while the bullet attitude is illustrated but inconvenient as a part of equation system due to the singularity while projectile nose is pointing straight up or down. The quaternion approach is used in this study and the Eulerian angles only utilized to illustrate the simulation results are obtained based on the quaternion system. The quaternion was renormalized like explained in the Ref. [5] to maintain the system orthonormality.

The trajectories were integrated using the traditional RK 4th order method and the trajectory launch angle was varied between 10…90 degrees. However, the high angles were particularly studied. Also some aerodynamic moments were varied in the simulations. The atmosphere model used was the ISO/ICAO standard one.

Some stochastic phenomena were introduced to the trajectory model via atmospheric conditions. Turbulence velocity fluctuations were evoked utilizing the model based on the Dryden spectra [5]. The one sigma value was 2 m/s and the effect on turning around the apex was particularly investigated.

![Figure 13. The projectile body-fixed coordinate system. The positive moments and angular velocities are also depicted. The total angle of attack \( \alpha \) is the angle between the \( xb \)-axis and the velocity vector \( V \). The applied aerodynamic forces presented in the Figure are those in the wind coordinate system (\( D = \text{Drag}, L = \text{Lift and } S = \text{side force} \).)](image)

5. Results and Discussion

Some aerodynamic moment is always needed to evoke fast spinning bullet (gyro)
turning and make the bullet centre line to follow the velocity vector. It turned out in the simulations that the aerodynamic moment and particularly the Magnus moment is an important factor for projectile turning at the apex (max. altitude about 2200 m) when fired about straight upwards, with initial velocity 715 m/s.

The positive Magnus-moment obtained turns the projectile nose to the direction of the normal coning motion (clockwise seen from behind in case the projectile is also spinning clockwise) and makes the turning take place easier at small velocities. The Magnus-moment behavior at large yaw angles is difficult to estimate and the moment values obtained were varied in the simulations (NoMagnus k=0, 0.5Magnus k=0.5, Basic k=1 and 1.5Magnus k=1.5, see Equations 12 and 13).

The pitching moment coefficient \( f(\alpha) \) -trend is expected to be predicted with better accuracy than Magnus-moment. However, the moment cross-over (\( C_m = 0 \)) yaw angle might be somewhat uncertain and in the computations the moment zero was shifted also to a value 90 degrees to find out its effect on turning. The slope values at both ends and the general trend was still maintained.

The conceptual 7.62 mm bullet investigated was able to turn around in most of simulations undertaken. The larger the Magnus moment was the more fluently turning took place. However, the early turning did not lead to higher terminal velocities due to the more severe bullet instability at the end of the descending part of trajectory.

The terminal velocities (TV) obtained including the effects of aerodynamic model variations are depicted as a function of the launch angle in Fig. 14. The bullet TV depends less on the launch angle than it was expected. For the basic bullet model the TV value increases from 70 m/s to 100 m/s while the elevation angle decreases from 90 to 20 degrees. The Magnus-phenomena variations make the TV to drop from 70 m/s to 50 m/s at the most at very large launch angles. Otherwise the variation effect is not dramatic from the TV point of view. The pitching moment variation also carried out and depicted has minor influence to TV.

The less serious instability associated with the small Magnus-moment (NoMagnus or 0.5Magnus) resulted to the largest terminal velocities when the launch angle was below 80 degrees. On the other hand, the small or no Magnus-moment at all could not turn the bullet at the larger elevation angles resulting to very high yaw angles (Fig. 15) all the way through the descending trajectory part and minimum terminal velocities around 50 m/s (see Fig. 14).
Figure 14. The 7.62mm projectile terminal velocity (TV) as a function of the launch angle.

The angle of attack and velocity time histories of 90 degrees launch angle simulations are shown in Figures 15 and 16. Besides the NoMagnus- and 0.5Magnus-cases the time histories show decreasing velocities associated with abruptly increasing yaw angles at the end of trajectories. The instable angle of attack growth appears with the increasing air density and bullet velocity and also the decreased spin.

Figure 15. The 7.62mm projectile total yaw angle as a function of time (launch angle 90 degrees).
The bullet velocity history graphs shown in Fig. 16 reach the minimum values at the apex and also when the bullet is flying about sidewise (in cases the turning has occurred). In the Basic aerodynamics case the sidewise flight i.e. when the yaw angle is about 90 degrees takes place at around 50 s of travel (see Fig. 15).

The dynamic stability analysis was also performed in this study like described in Ref. [2]. The precession (slow oscillation mode) and nutation (fast oscillation mode)
mode) stability parameters are depicted in the Fig. 17 for the launch angle case 90 degrees. Only results for the Basic aerodynamics bullet are presented. The stability parameter is defined to be the inverse of disturbance amplitude time-to-half $t_{1/2}$ (or $t_2$ time-to-double value in case instability). The negative value stands for the mode stable behavior. The stability parameter values are about 0 most of the flight time indicating bullet neutral dynamic response when disturbed. The fast mode instability evokes the angle of attack growth after 58.5 s flight.

![Figure 18](image1.png)  \textbf{Figure 18. } The 7.62mm projectile total yaw angle as a function of time (launch angle 80 degrees).

![Figure 19](image2.png)  \textbf{Figure 19. } The 7.62mm projectile flight velocity as a function of time (launch angle 80 degrees).

The yaw angle and velocity histories for the 80 degrees elevation angle case are shown in Figures 18 and 19. The bullet is now able to turn around if some
Magnus moment is present. The nose-first falling bullet terminal velocity (see also Fig. 14) is determined by the degree of instability at the trajectory end. The 0.5Magnus case turns very late but has the maximum TV due to the smallest instability issue and hence the angle of attack. The NoMagnus base first flying bullet has high drag all the way down and also the smallest TV.

The turning process at around the apex was also investigated by letting wind gusts and the Dryden spectra based turbulence velocity fluctuations with one sigma value 2 m/s to affect on bullet flight. The turbulence also introduced some stochastic phenomenon into the simulations otherwise not included to the study. However, the preliminary results obtained indicate that the bullet is fairly insensitive to disturbances due to the small kinetic pressure and neutral stability. The neutral stability observed is seen also in the results of dynamic stability analysis in Fig 17.

6. Conclusions

The conceptual 7.62 mm (38 mm length) bullet turning around the apex was investigated particularly in case it was fired about straight up to the air. The terminal velocities were determined as a function of the elevation angle with the Magnus phenomena and pitching moment also varied. The computational study undertaken shows that the bullet terminal velocity is about 60…80 m/s (~215…290 km/h) when the launch angle is in the region of 50…80 and decreases to values between 50…70 m/s (~180…250 km/h) in case fired at launch angles 80…90 degrees. The terminal velocities obtained when shooting about straight upwards may not always be lethal but the careless shooting can at least be considered dangerous despite the firing elevation angle.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$C_A$</td>
<td>axial force coefficient</td>
</tr>
<tr>
<td>$C_D$</td>
<td>drag coefficient</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>zero yaw drag coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift force coefficient</td>
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<tr>
<td>$C_{l_p}$</td>
<td>spin damping moment coefficient</td>
</tr>
<tr>
<td>$C_m$</td>
<td>overturning (pitch) moment coefficient</td>
</tr>
<tr>
<td>$C_{m_q}$</td>
<td>pitch damping moment coefficient</td>
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</table>
Upwards fired bullet turning at the trajectory apex

$C_{m\alpha}$ overturning (pitch) moment coefficient slope
$C_{n_p}$ Magnus moment coefficient
$C_{n_{p\alpha}}$ Magnus moment coefficient slope
$C_N$ normal force coefficient
$C_{N\alpha}$ normal force coefficient slope
$C_{Y_p}$ Magnus force coefficient
$C_{Y_{p\alpha}}$ Magnus force coefficient slope
$d$ projectile diameter
$I_x, I_y, I_z$ inertia moment
$Ma$ Mach number
$k$ Magnus force and moment adjustment factor
$N$ normal force
$q$ kinetic pressure
$S$ cross section area (reference area)
$TV$ terminal velocity
$v, w$ transverse velocity components in body mass
center fixed coordinate system (see Fig 13)
$W$ wind velocity
$\alpha$ angle of attack, yaw angle

References


Received: August, 2010