An Analytical Expression for the Angular Velocity of Rotation of the Earth

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Abstract

In this paper an analytical expression for the angular velocity of the Earth is determined as function of the orbital elements of the Moon and Sun, in case of the triaxial Earth \((A < B < C)\), a more general expression than in V. P. Dolgachev’s paper *Rotation of the Earth in the gravitational field of the Moon and Sun*, Astron. Zh. 62, 1003-1007, 1985. The equations are obtained by determining the motion of the axis of figure and the instantaneous axis of rotation of the absolutely rigid Earth.

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1 Introduction

The study of the Earth’s rotation includes investigations on the changing position of the instantaneous axis of rotation within the Earth and the motion of the axis of rotation in space. As it is known, the Earth is not an absolutely rigid body. Therefore, to describe the theory of the Earth rotation it is necessary to make use of the geophysical data concerning the mechanical characteristics of the Earth, his internal structure, the mass exchange inside and outside of this. The motion of the Earth is so complicated, that it is not necessary to study his laws of motion, but the deviations of the real model from the standard model of this movement whose laws of motion are known.

The theory of the rotation of the Earth is easier to study when we rely on the theory of the motion of the absolutely rigid bodies. From the point of view of the physical properties, the Earth is, in general, very close to an absolutely
rigid body, hence the true motion is in accordance with the laws of such a body. Woolard’s theory of the rotation of the Earth around its center of mass published in 1953 was taken by IAU as a standard [7].

The study of the variation of the components of the Earth’s angular velocity of rotation is of special importance, because the accuracy of the research in astronomy and geodesy has increased and therefore in many calculations the reality of the polar motion and the non-uniformity of the Earth’s rotation can not be neglected. Also, the connection of the angular velocity components with the mechanical properties of the Earth gives the possibility to study these properties based on the angular velocity variation.

The free and forced oscillations of the components of the instantaneous angular velocity occur. The first possess the mechanical qualities, in particular elasticity, while the forced oscillations have an immediate connection with the Earth’s atmosphere.

The non-uniformity of the Earth’s rotation is due from dynamical point of view to the deviation of the axis of rotation from the axis of the figure, the axis of rotation changing its position in time both within the Earth and in space. This deviation of the axis of rotation from the axis of figure is produced by the Eulerian motion of the Earth, the continuous action of the gravitational forces of the Moon and Sun, as well as the departure of the properties of the actual Earth from the properties of an absolutely rigid body.

In this paper, an analytical expression for the angular velocity of rotation of the Earth is obtained as function of the orbital elements of the Moon and Sun, in case of the triaxial Earth \((A < B < C)\), a more general expression than in work [2] and the equations for the angles which determine the motion of the axis of rotation and axis of the figure as functions of time are given.

\section{An expression of the angular velocity of rotation of the Earth as function of the Euler angles and their derivatives}

To obtain a mathematical representation of the motion of the Earth around its center of mass referred to a reference frame in space, it is advantageous to represent the motion of the Earth as a whole by the motions of the principal axes of inertia relative to a coordinate system with origin at the center of mass and coordinate axes that have fixed directions in space in an inertial frame of reference. When the Earth is considered rigid, the principal axes of inertia are fixed within the Earth and hence move with it. The motion of the absolutely rigid Earth in space around its center of mass is represented by the motion of the principal axes of inertia.
Let $OX, OY, OZ$ be fixed axes associated with our fixed plane of reference, the ecliptic at the initial epoch adopted (1900.0) and $Z$ - the pole of the ecliptic. The $OX$ axis is directed toward the mean point of the vernal equinox for the initial epoch. Let $OA, OB, OC$ be the axes of the ellipsoid of inertia of the Earth and $A, B, C$ the principal moments of inertia. We introduce a moving coordinate system $Oxyz$ connected with the axes of inertia of the Earth and hence rotating together with it relative to the $OXYZ$ system. The position of $Oxyz$ relative to the $OXYZ$ coordinate system is defined by means of three angles $\theta, \phi$ and $\psi$, known as the Eulerian angles, where $\theta$ is the nutation angle, $\phi$ is the angle of proper rotation and $\psi$ is the precession angle.

The vector $\vec{\omega}$ of the angular velocity of rotation gives both the direction of the instantaneous axis of rotation in space and the absolute value of the angular velocity of rotation around this axis. Projecting this vector along the principal axes of inertia, we obtain the components $\omega_1, \omega_2, \omega_3$ that are connected with $\dot{\theta}, \dot{\phi}, \dot{\psi}$ by the kinematical relations
\[
\begin{align*}
\omega_1 &= \dot{\psi}\sin\theta\sin\phi - \dot{\theta}\cos\phi \\
\omega_2 &= \dot{\psi}\sin\theta\cos\phi + \dot{\theta}\sin\phi, \\
\omega_3 &= \dot{\phi} - \dot{\psi}\cos\theta,
\end{align*}
\]
which are known as Euler’s kinematical equations. From these relations, we can calculate the square of the angular velocity as a function of the Euler angles and their derivatives, namely
\[
\omega^2 = \dot{\psi}^2 + \dot{\theta}^2 + \dot{\psi}^2 - 2\dot{\phi}\dot{\psi}\cos\theta.
\]

Putting in evidence the $\omega_3$ component in the $\omega^2$ expression, we have
\[
\omega^2 = \omega_3^2 + \dot{\psi}^2\sin^2\theta + \dot{\theta}^2.
\]

As in [1], since in the case of the Earth we have that $\frac{\omega_2^2 + \omega_3^2}{\omega_3^2} < 1$, then from (3), neglecting the terms of order of $\frac{(\dot{\psi}^2\sin^2\theta + \dot{\theta}^2)^2}{8\omega_3^2}$, we have the expression of the angular velocity of rotation of the Earth as function of the Euler angles and their derivatives
\[
\omega = \omega_3 + \frac{1}{2\omega_3} \left[ \left( \sin\theta\frac{d\psi}{dt} \right)^2 + \left( \frac{d\theta}{dt} \right)^2 \right].
\]

### 3 An analytical expression for the angular velocity of rotation of the Earth

In paper [2], V. P. Dolgachev determine an analytical expression for the angular velocity of rotation of the Earth as a function of the orbital elements of the
Moon and Sun, in the case of an axisymmetric Earth, i.e., \( A = B \). In the following, in the general case of the triaxial Earth \( A < B < C \), this expression is obtained.

The non-uniformity of the Earth’s rotation in the gravitational field of the Moon and Sun is described by the second term of the equation (4).

Let \( x', y', z' \) be the coordinates of the center of mass of the Moon in the system of the principal axes of inertia of the Earth. In the case \( A \neq B \), we have the following expression for the force function:

\[
U = -\frac{GM'}{r^3} \left\{ \frac{3}{2} \left( C - \frac{A + B}{2} \right) \left( \frac{z'}{r} \right)^2 + \frac{3}{4} (B - A) \left[ \left( \frac{y'}{r} \right)^2 - \left( \frac{x'}{r} \right)^2 \right] \right\}
\]

where \( G \) is the gravitational constant, \( M' \) is the mass of the Moon, \( r \) is the distance between the center of mass of the Moon and Earth, and \( A, B, C \) are the principal central moments of inertia of the Earth. The force function arising from the Sun has a similar form.

The next equations

\[
\frac{d\psi}{dt} = -\frac{1}{C\omega \sin \theta} \frac{\partial U}{\partial \theta} \quad \text{(6)}
\]

\[
\frac{d\theta}{dt} = \frac{1}{C\omega \sin \theta} \left[ \frac{\partial U}{\partial \psi} + \cos \theta \frac{\partial U}{\partial \phi} \right].
\]

describe the motion of the axis of rotation sufficiently precisely.

Without solving the equations (6), it is easy to obtain from here the following expression

\[
\left( \sin \theta \frac{d\psi}{dt} \right)^2 + \left( \frac{d\theta}{dt} \right)^2 = \left[ \frac{\partial}{\partial \theta} \left( -\frac{U}{C\omega} \right) \right]^2 + \frac{1}{\sin^2 \theta} \left[ \frac{\partial}{\partial \psi} \left( -\frac{U}{C\omega} \right) + \cos \theta \frac{\partial}{\partial \phi} \left( -\frac{U}{C\omega} \right) \right]^2. \quad (7)
\]

Therefore, the searched expression for the angular velocity of rotation will be obtained in explicit form if the force function of the external forces is expressed as a function of the Euler angles and the orbital elements of the Moon and Sun. For this purpose, following the calculation method presented in detail in the work [5] and after laborious calculations, if the leading secular terms and the long-period terms of largest amplitude are retained in the expression for \( U \), then we may write for the final form of \( U \)

\[
-\frac{U}{C\omega} = F \sin^2 \theta + [G(g_1 \cos \psi - g \sin \psi) \sin \theta \cos \theta + H \sin^2 \theta] \cdot t
\]
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\[ +G(h_1 \cos \psi - h \sin \psi) \sin \theta \cos \theta \cdot t^2 + R \sin \theta \cos \theta \cos (N + \psi) \]

\[ - \frac{1}{4} K L s^2 \sin^2 \theta \cos (2N + 2\psi) + \cos 2\phi \{ F \sin^2 \theta - \overline{F} \sin^2 \theta \] 

\[ + \overline{G}(g_1 \cos \psi - g \sin \psi) \sin \theta \cos \theta \cdot t - \overline{G}(h_1 \cos \psi - h \sin \psi) \sin \theta \cos \theta \cdot t^2 \] 

\[ - \overline{R} \sin \theta \cos \theta \cos (N + \psi) + \overline{K} \overline{L} \frac{s^2}{4} \sin^2 \theta \cos (2N + 2\psi) \}

\[ - \sin 2\phi \{ \overline{G}(g \cos \psi + g_1 \sin \psi) \cos \theta \cdot t + \overline{G}(h \cos \psi + h_1 \sin \psi) \sin \theta \cdot t^2 \] 

\[ + \overline{K} \overline{L} \frac{s^2}{2} \sin (2N + 2\psi) \cos \theta + \overline{R} \sin (N + \psi) \sin \theta \} \tag{8} \]

where

\[ K = \frac{3}{2} \cdot \frac{2C - (A + B)}{2C} \cdot \frac{n_1^2}{\omega} \]

\[ = \frac{3}{2} \cdot J_2 \cdot \frac{1}{C} \cdot \frac{n_1^2}{\omega}, \]

\[ L = \frac{M'}{S} \left( \frac{a_\perp}{a} \right)^3, \]

\[ F = K \left[ L \left( \frac{1}{2} + \frac{3}{4} e^2 - \frac{3}{4} s^2 \right) + \frac{1}{2} + \frac{3}{4} e_0^2 \right], \]

\[ G = K(L + 1), \]

\[ H = \frac{3}{2} K e_0 e', \]

\[ R = K L s \left( 1 - \frac{1}{2} s^2 + \frac{3}{4} e^2 \right), \]

\[ \overline{K} = \frac{3 (B - A) n_1^2}{4 C} \cdot \frac{1}{\omega}, \]

\[ \overline{L} = L, \]

\[ \overline{F} = \overline{K} \left[ L \left( -1 + \frac{3}{4} s^2 - \frac{3}{4} e^2 \right) - \frac{1}{2} - \frac{3}{4} e_0^2 \right], \]

\[ \overline{G} = \overline{K}(\overline{L} + 1), \]
\[ \mathcal{H} = \frac{3}{2}Ke_0e', \]

\[ \mathcal{R} = K Ls \left( 1 - \frac{1}{2}s^2 + \frac{3}{2}e^2 \right). \]

Here, \( a_1 \) and \( n_1 \) denote the semi-major axis of the Earth’s orbit and the corresponding mean angular motion, \( a \) is the semi-major axis of the orbit of the Moon, \( e \) and \( e_1 \) are the eccentricities of the orbits of the Moon and Sun, \( M' \) and \( S \) are the masses of the Moon and Sun, \( s = \text{sinc} \), \( c \) is the angle of inclination of the lunar orbit to the instantaneous ecliptic, \( N \) is the longitude of the ascending node of the lunar orbit in the instantaneous ecliptic, \( e_0 \) and \( e' \) are the coefficients of the expansion of the eccentricity of the solar orbit in powers of time, i.e., \( e_1 = e_0 + e't \). The coefficients \( g, h, g_1 \) and \( h_1 \) are the coefficients of the expansions of the quantities \( isin\Omega \) and \( icos\Omega \) in power of time, i.e.,

\[
    isin\Omega = gt + ht^2, \\
    icos\Omega = g_1t + h_1t^2,
\]

where \( i \) is the inclination of the instantaneous ecliptic to the ecliptic of the epoch and \( \Omega \) is the geocentric longitude of the heliocentric ascending node of the instantaneous ecliptic at the ecliptic of the epoch.

Next, we denote

\[
    \left( \sin\theta \frac{d\psi}{dt} \right)^2 + \left( \frac{d\theta}{dt} \right)^2 = E(t). \tag{9}
\]

Taking into account the relations (7) and (8) and using the notation adopted in the Nautical Almanac, namely \( N + \psi = \Omega \), we’ll obtain for \( E(t) \) the following expression:

\[
    E(t) = t^2[(Gg_1cos2\theta_0 + Hsin2\theta_0)^2 + h_1 \left( FG - \frac{1}{2}FG' \right) sin4\theta_0 \\
    +\frac{1}{2}(Hsin2\theta_0 + CGcos2\theta_0)^2 + \frac{1}{2}G^2 g_1 + g^2(G^2 + 2G'G)cos^2\theta_0 \\
    +2FG'h_1sin2\theta_0sin^2\theta_0 - G^2 g^2 sin2\theta_0cos\theta_0 \\
    +\frac{1}{2}(H + CG_1cos2\theta_0)^2 sin^22\theta_0 \\
    -4Gg_1 \left( Hsin^2\theta_0 + \frac{G}{2} g_1sin2\theta_0 \right) ctg\theta_0 \\
    +h_1(2GRCos^22\theta_0 + CG Rcos4\theta_0 + G K - 4G Rcos^2\theta_0) cos\Omega \\
    +h_1 s^2 \left[ -\frac{1}{4} (GKL + \frac{1}{2} G K L) sin4\theta_0 - GL Lcos2\theta_0 ctg\theta_0 \right].
\]
\[+\frac{1}{2} \mathcal{G} \mathcal{K} L \sin^2 \theta_0 \sin 2\theta_0 \cos 2\Omega + 2h(GR + \mathcal{G} R) \cos^2 \theta_0 \sin \Omega\]
\[+ hs^2 \left( \mathcal{G} \mathcal{K} L \cos^2 \theta_0 \cot \theta_0 - \frac{1}{2} GKL \right) \sin 2\Omega\]
\[+ t \{ 2F(Gg_1 \cos 2\theta_0 + H \sin 2\theta_0) + F \sin 2\theta_0 [2Gg_1 (1 - \cos 2\theta_0) - \mathcal{H} \sin 2\theta_0] + [g_1 (2GR + \mathcal{G} R) \cos^2 2\theta_0 + (RH + \frac{1}{2} R \mathcal{H}) \sin 4\theta_0 + \mathcal{G} \mathcal{R} g_1 (\sin 2\theta_0 + 4\cos^4 \theta_0 - 2\sin^2 2\theta_0)] \cos \Omega\]
\[+ [- \frac{s}{2} (GKL + \frac{1}{2} \mathcal{G} \mathcal{K} L) \sin 4\theta_0\]
\[- \frac{s^2}{2} (HKL + \mathcal{H} \mathcal{K} L + \mathcal{K} L) \sin^2 2\theta_0 + \mathcal{G} \mathcal{K} L g_1 s^2 (1 + \cos \theta_0) \cot \theta_0\]
\[- \frac{1}{2} \mathcal{G} g_1 (K L s^2 - 1) \sin 2\theta_0 - 4 \mathcal{H} \mathcal{K} L s^2 \cos \theta_0 \cos 2\Omega\]
\[+(3 \mathcal{G} \mathcal{R} g + \mathcal{G} \mathcal{R} + 2GRg) \cos^2 \theta_0 \sin \Omega - \frac{1}{2} GKLgs^2 \sin 2\theta_0 \sin 2\Omega\}
\[+ \left[ F^2 + \mathcal{F}^2 + \frac{s^4}{32} (K^2 L^2 + \mathcal{K}^2 L^2) \right] \sin^2 2\theta_0 + \frac{1}{2} (R^2 + \mathcal{R}^2) \cos^2 2\theta_0\]
\[+ \frac{s^2}{8} \left[ \frac{1}{2} \mathcal{K}^2 L^2 (1 + s^2) + K^2 L^2 s^2 \right] \sin^2 \theta_0\]
\[+ \frac{1}{2} \left( R^2 - 9 \mathcal{R}^2 + \frac{3}{2} \mathcal{K}^2 L^2 s^4 \right) \cos^2 \theta_0\]
\[+ \frac{1}{4} \mathcal{K}^2 L^2 s^4 \cot^2 \theta_0 (1 + \cos^2 \theta_0) + \mathcal{R}^2 \left( \frac{1}{4} + \cos^4 \theta_0 \right)\]
\[+ \{ [R(F - \frac{1}{8} KLs^2) - \frac{1}{2} \mathcal{R}( \mathcal{F} + \frac{1}{8} \mathcal{K} \mathcal{L} s^2)] \sin 4\theta_0\]
\[+ [- \frac{3}{8} \mathcal{K} \mathcal{L} \mathcal{R} s^2 - \frac{1}{4} KLRs^2 + 2R \mathcal{F} + \frac{1}{2} \mathcal{K}^2 L^2 s^3\]
\[+ \frac{1}{4} \mathcal{K} \mathcal{L} \mathcal{R} ctg^2 \theta_0 - \mathcal{R} \cos^2 \theta_0 (2F + \frac{1}{4} \mathcal{K} \mathcal{L} s^2)] \sin 2\theta_0 \} \cos \Omega\]
\[+ \{ \left[ \frac{s^2}{2} \left( \frac{1}{2} F \mathcal{K} \mathcal{L} - FK L \right) - \frac{1}{4} \mathcal{R}^2 + \frac{1}{2} F \mathcal{K} \mathcal{L} s^2 \right] \sin^2 2\theta_0\]
\[+ \frac{1}{2} (R^2 + \frac{1}{2} \mathcal{R}^2) \cos^2 2\theta_0 - \left( \frac{5}{2} \mathcal{R}^2 + \frac{1}{2} R^2 - 4F \mathcal{K} \mathcal{L} s^2 \right) \cos^2 \theta_0\]
\[+ \frac{1}{4} \mathcal{R}^2 \} \cos 2\Omega + \{ - \frac{s}{8} \left( R^2 + \frac{1}{2} \mathcal{R}^2 \right) \sin 4\theta_0\]
\[+ \frac{s^2}{4} \left( \frac{3}{2} \mathcal{K} \mathcal{L} \mathcal{R} + KLR \right) \sin 2\theta_0\]
\[+ \frac{1}{4} \mathcal{K} \mathcal{L} \mathcal{R} \cot \theta_0 [2 + \cos^2 \theta_0 (5 - 2\sin^2 \theta_0)]] \} \cos 3\Omega\]
\[+ \left\{ \frac{s^4}{32} (K^2 L^2 + \mathcal{K}^2 L^2) - \frac{s^2}{8} \left[ \frac{1}{2} \mathcal{K}^2 L^2 (s^2 + 1) + K^2 L^2 s^2 \right] \sin^2 \theta_0 \right\} \]
\[ + \frac{1}{4}K^2 L^2 s^4(1 + \cos^2\theta_0)\cos 4\Omega - 2KL R s^2 \cos^2\theta_0 \cot \theta_0 \sin \Omega \]
\[ - 2KL R s^2 \cos^2\theta_0 \cot \theta_0 \sin 3\Omega. \quad (10) \]

Therefore, the expression of the absolute value of the angular velocity of the rotation of the Earth is expressed as

\[ \omega = \omega_3 + \frac{1}{2} \frac{E(t)}{\omega_3}, \quad (11) \]

with \( E(t) \) defined by (10).

If January 0, 1900.0 is taken as the initial time, then following Newcomb [4], we adopt in (10)

\[ \psi_0 = 0, \quad \theta_0 = 23^0 27' 08''.26. \]

We use the system of astronomical constants adopted by the IAU in 1964 [3]. For the principal moments of inertia \( A \) and \( C \), we consider the mean values of the normalized principal moments of inertia of the Earth obtained in the paper [6], namely \( \overline{A} = 0.329619261 \) and \( \overline{C} = 0.330705446 \) (with \( \overline{A} = A/\overline{M}a_e^2 \), where \( M \) and \( a_e \) are the mass and the equatorial radius of the Earth). The value of \( J_2 \) is 0.001082628. We obtain the following numerical expression for \( E(t) \):

\[ E(t) = t^2(3.453 \cdot 10^{-9} + 2.041 \cdot 10^{-12} \cos \Omega - 4.899 \cdot 10^{-14} \cos 2\Omega + 1.309 \cdot 10^{-10} \sin \Omega) + t(-2.090 \cdot 10^{-5} - 1.782 \cdot 10^{-7} \cos \Omega - 0.657 \cos 2\Omega + 4.105 \cdot 10^{-7} \sin \Omega - 6.531 \cdot 10^{-9} \sin 2\Omega) + 1.819 + 0.410 \cos \Omega - 0.095 \cos 2\Omega + 7.834 \cdot 10^{-4} \cos 3\Omega + 4.684 \cdot 10^{-6} \cos 4\Omega - 1.914 \cdot 10^{-7} \sin \Omega - 1.914 \cdot 10^{-7} \sin 3\Omega). \quad (12) \]

In the above relation, keeping the measuring units used in the paper [2], \( t \) is the time in years and the coefficients are expressed in seconds of time.

4 The expressions of the angles that determine the motion of the axis of figure and the axis of rotation as functions of time

The Earth’s rotation around its center of mass consists of instantaneous rotation around an axis that occupies different positions within the Earth at different moments of time, an axis inclined at an angle \( \gamma \) to the axis of the figure and passing through the center of mass. This variation of the rotational motion and the resulting difference between the motion of the axis of rotation
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and the motion of the axis of the figure relative to the fixed coordinate system depends on the positions of both these axes with respect to the angular momentum vector $\mathbf{K}$. Both these axes and the angular momentum vector $\mathbf{K}$ constantly pass through the center of mass, $\mathbf{K}$ being located in the plane passing through the axis of the figure $OC$ and $\omega$, between $OC$ and $\omega$ [7].

The angle $\gamma$ characterizes not only the geometrical properties of the rotation, but also serves as a measure of the asymmetry in the mass distribution about the axis of rotation. This asymmetry determines the amount $\nu$ by which the angular momentum vector $\mathbf{K}$ deviates from the angular velocity vector $\mathbf{\omega}$. Having an analytical expression for the absolute value of the angular velocity of rotation of the Earth, it is easy to obtain an equation for $\gamma$. From the expression (11), we have

$$\omega = \omega_3 \sqrt{1 + \frac{E(t)}{\omega_3^2}},$$

and consequently

$$\cos \gamma = \frac{\omega}{\omega_3} = \left[ 1 + \frac{E(t)}{\omega_3^2} \right]^{\frac{3}{2}} = 1 - \frac{1}{2} \cdot \frac{E(t)}{\omega_3^2} + \ldots$$

Comparing this last expression with the standard power-series expansion of the cosine, one can obtain immediately that

$$\gamma = \frac{\sqrt{E(t)}}{\omega_3}.$$  \hspace{1cm} (13)

To estimate the size of this angle, we keep only the first free term in the expression for $E(t)$ and taking for $\omega_3$ the value $0.316440 \cdot 10^8$ sec/yr, one found $\gamma = 0''.009$.

In [7], the equation for the angles that determine the orientation of the axes of the figure and the rotation about angular momentum vector were obtained as function of $\sin \gamma$ and $\cos \gamma$. Using (13) and keeping the terms up to $\gamma^3$ in the power-series expansions of the $\sin \gamma$ and $\cos \gamma$, we obtain

$$\sin(\mathbf{K},OC) = \frac{\sqrt{E(t)}}{\omega_3} \cdot \frac{C - A}{A} \cdot \left( 1 - \frac{E(t)}{2\omega_3^2} \cdot \frac{C - A}{A} \cdot \frac{A + C}{C} \right),$$  \hspace{1cm} (14)

respectively, for the angle $\nu$ between the angular momentum vector $\mathbf{K}$ and the axis of rotation, we have

$$\sin \nu = \frac{\sqrt{E(t)}}{\omega_3} \cdot \frac{C - A}{A} \cdot \left( 1 - \frac{E(t)}{2\omega_3^2} \cdot \frac{C - A}{A} \cdot \frac{A + C}{C} \right).$$  \hspace{1cm} (15)

From the equation (14) one obtains that the angle between $\mathbf{K}$ and $OC$ has the same order as the angle $\gamma$, while from the equation (15), for the angle $\nu$ one obtain $\nu = 0''.3 \cdot 10^{-4}$. One can remark that the values obtained for $\gamma$ and $\nu$ practically coincide with the values determined for an axisymmetric Earth ($A = B$) [2].
In conclusion, in the case of the absolutely rigid Earth the axis of rotation practically coincides with the direction of the angular momentum vector.

The difference between the properties of the actual Earth and those of an absolutely rigid body has a definite effect on the quantity $\gamma$. The size of such a contribution can be determined only from observations. Total deviation of the axis of rotation from the axis of the figure is about $0''\cdot5$, it being almost entirely of geophysical nature, but obviously a small lunisolar component of about $0''\cdot01$ which overlaps the constant deviation $\gamma_0$ from the Eulerian motion is present. The presence of such minor deviations generates the non-uniformity observed in the Earth’s rotation.

References


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