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Optimal Jettison Control for Range Maximization of Gliding Vehicles

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Abstract

The effect of jettison on the range performance of aerial vehicles in gliding flight dealt with. It is shown that the achievable range can be increased by an optimal application of jettisoning. For this purpose, jettisoning is considered a control which is included in the trajectory optimization directed at the range maximization of a gliding vehicle. Furthermore, it is shown that the improvement in the gliding performance by optimal jettisoning is greater at high than at low speed. This effect is demonstrated for two vehicles one of which relates to the high end of the speed spectrum (orbital stage) and the other to the low end (sailplane). Another result is that the performance improvement is greater for the lateral than for the longitudinal maximum range flight.

Keywords: jettison control, gliding flight, range performance, trajectory optimization

1 Introduction

Jettisoning is applied in many cases and for various purposes one of which is to decrease intentionally the mass of the aircraft. This holds for different types of aerial vehicles.

Jettisoning is also used with airplanes in gliding flight which are of interest in the present paper.

Gliding flight involving jettisoning is an issue for a space transportation system consisting of a winged orbital stage and a carrier stage [1]. In nominal flight, the orbital stage is released from the carrier stage to perform an ascent to an orbit. Various abort scenarios for such vehicles can occur due to malfunctions. The ability to cope with abort scenarios is an important issue [2]. With an appropriate abort procedure, a contribution to safety can be achieved [3].

There is a mission abort scenario which is relating to an engine failure immediately after separation from the carrier stage, with the fuel still on board of the orbital stage [4]. After the failure, the orbital stage has to perform a range glide to an emergency landing site. Range maximization is a safety issue for such a case [5, 6]. In this context, it may be of interest to note that a cause of system malfunctions is pertaining to the propulsion system [7 - 9].

Another case for jettisoning in gliding vehicles concerns sailplanes with a water ballast which can be released. The purpose of carrying jettisonable water ballast with sailplanes is to increase the glide performance at greater flight velocities [10, 11]. Using water ballast, the speed polar shifts to higher air speeds [12]. Thus, the cross-country performance from weak to strong thermal conditions can be improved. Optimal ballasting is relating to weather conditions as well as thermal strength and size [13].

A purpose of jettison is to reduce the mass of the vehicle in order to obtain an acceptable landing weight. Jettison may be conducted immediately before landing or at an earlier flight phase, without paying attention to its effect on the performance of the vehicle during the entire flight. It is the purpose of this paper to treat the jettison process as an optimal control problem where the load release is properly timed during the flight for increasing the range performance of the gliding vehicle.

Two trajectory optimization problems concerning the addressed jettison effect will be considered. One problem is pertaining to the high end of the speed spectrum and the other to the low end. The reason for choosing the two cases referenced to the speed spectrum is to show that the range performance improvement possible with jettisoning depends on the speed of the vehicle.

The high speed problem concerns the gliding flight of an orbital stage in case of an abort scenario as illustrated in Fig. 1. This Fig. shows that the orbital stage of a two-stage flight system is released from the carrier stage to perform an ascent to an orbit. If an engine failure occurs immediately after separation the orbital stage has to perform a gliding flight to an emergency landing site (flight cases Nos. 1 and 2 in Fig. 1, termed “Emergency Landing Site Landing”). Since fuel is not consumed in such a case it is jettisoned prior to landing. The jettisoning will be included in the overall optimization of the performance of the gliding vehicle for achieving the maximum range. This is a safety issue because the possibilities to reach an appropriate landing site can be enhanced.

The low speed problem of jettison is pertaining to a sailplane, as graphically illustrated in Fig. 2. This Fig. shows a flight condition where the water ballast is dropped from a sailplane. Evidence of the water ballast dropping is a long streak of white spray behind the vehicle. With regard to the sailplane which is considered to be a representative case of jettisoning at the low end of the speed spectrum, it is investigated as to what extent a range improvement is possible with proper control of dropping the water ballast.

2 Model of Vehicle Dynamics

The motion of the aerial vehicles is described using a realistic modelling which is based on point mass dynamics and on complex aerodynamic characteristics accounting for all relevant dependencies. The equations of motion with reference to a spherical Earth model can be expressed as (for the lateral range maximization as the most general case dealt with in this paper, Fig. 3):

$$\begin{aligned}
\dot{V} &= -\frac{D}{m} - g\sin\gamma + \omega_E^2 (r_E + h)\cos\Delta(\sin\gamma\cos\Delta - \cos\gamma\sin\Delta\cos\chi) \\
\dot{\gamma} &= \frac{L}{mV}\cos\mu_a + \left(\frac{V}{r_E + h} - \frac{g}{V}\right)\cos\gamma + 2\omega_E\cos\Delta\sin\chi \\
&\quad + \frac{r_E + h}{V}\,\omega_E^2\cos\Delta(\cos\gamma\cos\Delta + \sin\gamma\sin\Delta\cos\chi) \\
\dot{\chi} &= \frac{L}{mV\cos\gamma}\sin\mu_a + \frac{V}{r_E + h}\cos\gamma\sin\chi\tan\Delta - 2\omega_E(\tan\gamma\cos\Delta\cos\chi - \sin\Delta) \\
&\quad + \frac{r_E + h}{V\cos\gamma}\,\omega_E^2\sin\Delta\cos\Delta\sin\chi \\
\dot{\Delta} &= \frac{V}{r_E + h}\cos\gamma\sin\chi \\
\dot{\Lambda} &= \frac{V}{(r_E + h)\cos\Delta}V\cos\gamma\cos\chi \\
\dot{h} &= V\sin\gamma \\
\dot{m} &= -\dot{m}_J
\end{aligned} \tag{1}$$

For the longitudinal range maximization cases, the equations of motion simplify due to omission of the variables related to the lateral motion.

The aerodynamic model for the lift and the drag, L and D, can be generally expressed as

$$\begin{aligned}
L &= C_L(\rho/2)V^2 S \\
D &= C_D(\rho/2)V^2 S
\end{aligned} \tag{2}$$

where C_L and C_D are the non-dimensional lift and drag coefficients, S the wing reference area and ρ the air density. The air density is modeled as function of the altitude h

$$\rho = \rho(h) \tag{3}$$

In low speed flight (sailplane), the aerodynamic coefficients can be described by

$$\begin{aligned} C_L &= C_L(\alpha) \\ C_D &= C_D(\alpha) \end{aligned} \tag{4}$$

where α is a control. For the high speed problem (orbital stage), the following relations hold

$$\begin{aligned} C_L &= C_L(\alpha, M) \\ C_D &= C_D(\alpha, M) \end{aligned} \tag{5}$$

where M is the Mach number

$$M = V / a \tag{6}$$

and a the speed of sound

$$a = a(h) \tag{7}$$

The jettison process is modelled as

$$\dot{m}_J = \dot{m}_{J\,\max} \delta_J \tag{8}$$

where δ_J is a control. Maximum jettison rates for the two vehicles under consideration are presented in Table 1.

	Orbital Stage	Sailplane
$\dot{m}_{J\,\max}$ [kg/s]	200.0	1.0

Table 1 Maximum jettison rates

3 Optimization Problem

The optimization problem is to maximize the range in gliding flight from a specified set of initial conditions (altitude, speed, mass and flight path angle) to a corresponding final set. The problem can be formulated as to find a state function

$$\vec{x} = (V, \gamma, \chi, \Delta, \Lambda, h, m) : [t_0, t_f] \to R^7 \tag{9}$$

and a control function

$$\vec{u} = (\alpha, \mu_a, \delta_J) : [t_0, t_f] \to R^3 \tag{10}$$

which minimize the cost function

$$I[u] = -s\,|_{t_f} \to \text{Min} \tag{11}$$

subject to a set of differential equations

$$\dot{x}(t) = f(x(t), u(t))$$

given by Eq. (1) describing the dynamics of the vehicle, to state constraints

$$\begin{aligned} \bar{q}_{\min} &\le \bar{q}(V, h) \le \bar{q}_{\max} \\ l_{\min} &\le l(V, h, \alpha) \le l_{\max} \end{aligned} \tag{12}$$

and to control constraints

$$u_{i,\min} \le u_i \le u_{i,\max}\ ,\ i = 1, 2, 3 \tag{13}$$

The final time of the optimization problem, t_f, is treated as free. For the lateral range case of the orbital stage, the range s of the cost function, Eq. (11), is described in terms of the latitude, Δ.

The quantity l, Eq. (12), is introduced as a measure of the structural load, yielding

$$l = \frac{L \cos \alpha + D \sin \alpha}{S} \tag{14}$$

This quantity can be used to describe the actual load on the vehicle (e.g. wing root bending moment), instead of the often applied load factor $n = L/(mg)$. The reason for selecting l is that it is independent of the vehicle mass which shows significant changes for the orbital stage.

Numerical values for the constraints are given in Tables 2 and 3.

	Orbital Stage	Sailplane
α_{max} [deg]	45.0	15.0
α_{min} [deg]	0	-4.2
$\delta_{J\,max}$	1.0	1.0
$\delta_{J\,min}$	0	0

Table 2 Control variables constraints

	Orbital Stage
$\bar{q}_{max}$ [kPa]	50.0
$\bar{q}_{min}$ [kPa]	5.0
l_{max} [N/m^2]	8000.0
l_{min} [N/m^2]	0

Table 3 State variables constraints for orbital stage

The boundary conditions of the trajectory are presented in Tables 4 and 5.

	Orbital Stage	Sailplane
$h(0)$ [m]	30000.0	1050.0
$V(0)$ [m/s]	2250.0	338.1
$\gamma(0)$ [deg]	–0.3	–1.4
$m(0)$ [kg]	115000.0	515.0

Table 4 Initial boundary conditions

	Orbital Stage	Sailplane
$h(t_f)$ [m]	1000.0	50.0
$V(t_f)$ [m/s]	135.0	24.38
$\gamma(t_f)$ [deg]	–6.0	–1.4
$m(t_f)$ [kg]	31500.0	325.0

Table 5 Final boundary conditions

The mathematical procedure used to solve the range maximization problem is a direct optimization method which is given in [15, 16]. In these references, the optimization method is described in detail. Therefore, focus of the following treatment is on the main steps in applying the procedure.

The optimization method allows to split the complete problem into several phases (Fig. 4), yielding

$$0 = t_0 < t_1 < \ldots < t_{j-1} < t_j = t_f \tag{15}$$

For each phase, additional grid points are specified

$$t_{j-2} = t_0^{j-2} < t_1^{j-2} < \ldots < t_{i-1}^{j-2} < t_1^{j-2} = t_{j-1} \tag{16}$$

With regard to the time intervals between the grid points, the control vector

$$\vec{u}(t) = (\alpha(t), \mu_a(t), \delta_J(t))$$

is parameterized, yielding

$$\vec{u}(t) = \vec{U}_{ik}^{j}(\vec{p}, t), \quad t_k < t < t_{k+1} \tag{17}$$

where $\vec{U}_{ik}^{j}(\vec{p}, t)$ is an estimation of the control. Various types of approximations can be used, like piecewise constant, piecewise linear and cubical splines. The states are calculated with initial estimations at the left sided node using the differential equations formulating the problem.

$$\vec{\dot{x}}(t) = f(x(t), u(t)), \quad i = 1, \ldots, l \tag{18}$$

The constraints may be formulated as equal or as lower equal conditions. The initial boundary conditions are checked at the respective boundary of the first phase

$$\begin{aligned} x_i(t = t_0) &= x_{i,0}, \quad i = 1, \ldots, n_1 \\ u_i(t = t_0) &= u_{i,0}, \quad i = 1, \ldots, n_2 \end{aligned} \tag{19}$$

The final boundary conditions are dealt with in an analogue manner

$$b(x(t_f), u(t_f)) = 0, \quad i = 1, \ldots, n_3 \tag{20}$$

The path constraints are checked at every point of a specified grid within the phase

$$c(x(t), u(t)) \geq 0, \quad i = 1, \ldots, n_4 \tag{21}$$

The cost function can be formulated in general form as

$$I = \sum_{i=1}^{p} \Lambda_i \cdot g_i \left[\dot{x}(t_i), \dot{u}(t_i), x(t_i), u(t_i) \right] + \sum_{j=1}^{q} \Lambda_j \cdot \int_{t_k}^{t_l} h_j \left[\dot{x}(t_i), \dot{u}(t_i), x(t_i), u(t_i) \right] \mathrm{d}t \quad (22)$$

where Λ_i and Λ_j are weighting factors which have to be chosen in an appropriate manner. The functions g_i and h_j are relating to the optimization criteria.

The described optimization procedure allows the usage of different optimization sequences. For this investigation, the optimizer PROMIS was used. It proofed to be very stable and converges in due time.

A solution of the optimal control problem is obtained as soon as the change of the cost function, as function of the Jacobian matrix, cannot be further improved and is below a specified threshold as well as the alignment between the initial estimation and calculated condition at every single grid point is given and all boundary conditions and path constraints are fulfilled.

4 Numerical Results

4.1 High Speed Problem (Longitudinal Range Maximization)

Results on the high speed problem (orbital stage) for longitudinal range maximization are presented in Figs. 5 and 6 which show the optimal time histories of state and control variables. The case without fuel draining is used as a reference (Fig. 5), yielding a maximum range of 754 km. The results presented in Fig. 5 show that the gliding flight involves a continual deceleration from the high velocity level at the beginning towards the final value. Furthermore, for a large part of the trajectory, the lift coefficient is close to the value at the maximum lift-to-drag ratio denoted by C_L^*.

Introducing jettisoning as a control in the flight path optimization yields a maximum range trajectory as illustrated in Fig. 6. As a main result, there is an increase of 3.1 % when compared with the case without fuel draining. Another result is relating to the trajectory part during which jettisoning is performed. Fig. 6 shows that this takes place at high velocity, in the first part of the trajectory. Further to the optimization results, jettisoning is conducted at the maximum rate. This maximum jettisoning rate is maintained until all fuel is drained.

For comparison purposes, other optimization investigations were performed where fuel draining was not allowed to take place before reaching a lower velocity. In these cases, the range increase due to jettisoning is reduced.

4.2 Low Speed Problem (Longitudinal Range Maximization)

Results on the low speed problem (sailplane) for longitudinal range maximization are presented in Figs. 7 and 8. The sailplane is considered to have a water ballast on board which can be jettisoned. The case without jettisoning illustrated in Fig. 7 is again regarded as a reference. The maximum possible range in this case amounts to 41 km.

Using jettisoning as a control in the trajectory optimization yields a maximum range trajectory as illustrated in Fig. 8. There is an increase in the maximum range of 697 m when compared with the previous case without releasing the water ballast. Comparison between Figs. 7 and 8 shows that the behaviour of the altitude as well as that of the lift coefficient are rather similar. This suggests that the gliding flight is also quite similar in both cases. In particular, the behaviour of the lift coefficient which stays close to the value at the maximum lift-to-drag ratio is indicative for the similarity. This is because gliding flight at the maximum lift-to-drag ratio yields the greatest range, independent of the vehicle mass. The reduction in the velocity corresponds with that of the mass.

Comparison of the two speed cases shows that the performance improving effect of jettisoning is more pronounced with the high velocity vehicle. This manifests especially in the time history of the altitude. In the high speed case, the altitude decrease takes place at a markedly slower rate when jettison is applied.

4.3 High Speed Problem (Lateral Range Maximization)

For mission abort scenarios of an orbital stage in the case of an engine failure immediately after separation from the carrier stage, an emergency landing site may be reached not only by a flight in the longitudinal, but also in a lateral direction. In this case, the gliding vehicle performs a turning flight. With a flight in a lateral direction, the possibility of reaching an emergency landing site is enhanced. Concerning all feasible lateral flight directions, the trajectory that yields the maximum lateral range is of interest here.

In Figs. 9 and 10, results on the lateral range maximization are presented (with the boundary conditions slightly altered when compared with the longitudinal case). The optimal time histories of state and control variables are shown in Fig. 9 for the case with jettisoning. As regards their behaviour, there are some similarities with the longitudinal range maximization. However, the time period for performing the gliding flight is significantly shorter. This is due to the higher drag caused by the turning flight manoeuvre.

From the outcome of the maximum range flight optimization it follows that the performance improvement due to jettisoning is higher in the lateral than in the longitudinal case. For showing this, results on the lateral maximum range flights with and without fuel draining are presented in Fig. 10 where the respective tracks are depicted. The increase in the maximum range achievable due to jettisoning is enlarged by 13.4 % when compared to the case without fuel draining.

6 Conclusions

Jettison is considered a means to increase the range of aircraft in gliding flight. For this purpose, jettisoning is treated as a control and included in the trajectory optimization for maximizing the range of the gliding vehicle. Results are presented which show the improvements in the maximum range due to jettisoning for two vehicles, one pertaining to high speed flight (orbital stage) and the other to low speed flight (sailplane). It turns out that the performance improving effect of jettisoning is greater at high than at low speed. Further to the results, the optimal jettisoning takes place at the beginning of the trajectory, at a rate as high as possible. In the case of a flight in lateral direction, a range increase can also be achieved. The performance improvement due to jettisoning in this case is greater than for the longitudinal maximum range flight.

References

[1] N.N.: Leitkonzept SÄNGER – Referenz-Daten-Buch, Ausgabe 1, Revision 2, Daimler-Benz Aerospace, March 1993.

[2] K. Fujii, S. Ishimoto and H. Kawato: Concept Study of Flight Experiment to Realize Next Generation Launch System. 26th International Symposium on Space Technology and Science (ISTS), Hamamatsu, Japan, 1-8 June 2008, Paper 2008-g-27, 2008.

[3] M. Mayrhofer, M. Wächter, G. Sachs: Safety improvement issues for mission aborts of future space transportation systems. ISA Transactions, Vol. 45, No. 1, pp. 127-140, 2006.

[4] Mayrhofer, M., Sachs, G.: Optimal Abort Trajectory Control for a Winged Orbital Stage, Atmospheric Flight Mechanics Conference, Boston, MA, USA, August 10-12, AIAA 98-4150, 1998.

[5] Sachs, G., Mayrhofer, M.: Gliding Range Performance Improvement by Optimal Control of Ballast Dropping. Proceedings of the International Conference on Mathematical Problems in Engineering and Aerospace Sciences ICNPAA 2008, Cambridge Scientific Publishers, Ed. S. Sivasundaram, 2008.

[6] Sachs, G., Mayrhofer, M.: Reichweitensteigerung bei Hyperschall-Notflugbahnen durch Optimalsteuerung des Treibstoff-Ablaßvorgangs, ZAMM Zeitschrift für angewandte Mathematik und Mechanik, Berlin 81, Suppl. 3, S. 775-776, 2001.

[7] Allred, A.G., Sauvageau, D.R.: Crew Survival and Intact Abort Usixng Solid Rocket Boosters, Joint Propulsion Conference, Lake Buena Vista, FL, USA, July 1-3, 1996, AIAA 96-3156.

[8] Gonzáles, P.: Influence of the Abort Capability in Reusable System Reliability. FESTP-Overview, 9th International Space Planes and Hypersonic Systems and Technologies Conference, Norfolk, VA, USA, November 1-5, 1999, AIAA 99-4928.

[9] Gonzáles, P.: Risk Management Procedures – Application of Technical Risk Assessment in FESTIP, AIAA Atmospheric Flight Mechanics Conference and Exhibit, Portland, OR, USA, August 9-11, 1999, AIAA 99-4253.

[10] Irving, F.: The Paths of Soaring Flight. Imperial College Press, ISBN 978-1-86094-055-2, 1999.

[11] Maughmer, M.D.: Design of Winglets for High-Performance Sailplanes. Journal of Aircraft, Vol. 40, No. 6, pp. 1099-1106, 2003.

[12] Maughmer, M.D., and Kunz, P.J.: Sailplane Winglet Design. Technical Soaring, Vol. XXII, No. 4, pp. 116-123, 1998.

[13] Kubrynski, K.: Aerodynamic design and cross-country flight performance analysis of Diana-2 sailplane. Technical Soaring, Vol. 30, No. 3, July, pp. 79-88, 2006.

[14] www.whiteplanes.com.

[15] N.N.: ALTOS – Software User Manual, Institut für Flugmechanik und Regelung, University of Stuttgart, August 1996.
[16] N.N., GESOP (Graphical Environment for Simulation and Optimization), Softwaresystem für Bahnoptimierung, Institut für Robotik und Systemdynamik, DLR, Oberpfaffenhofen, 1993.

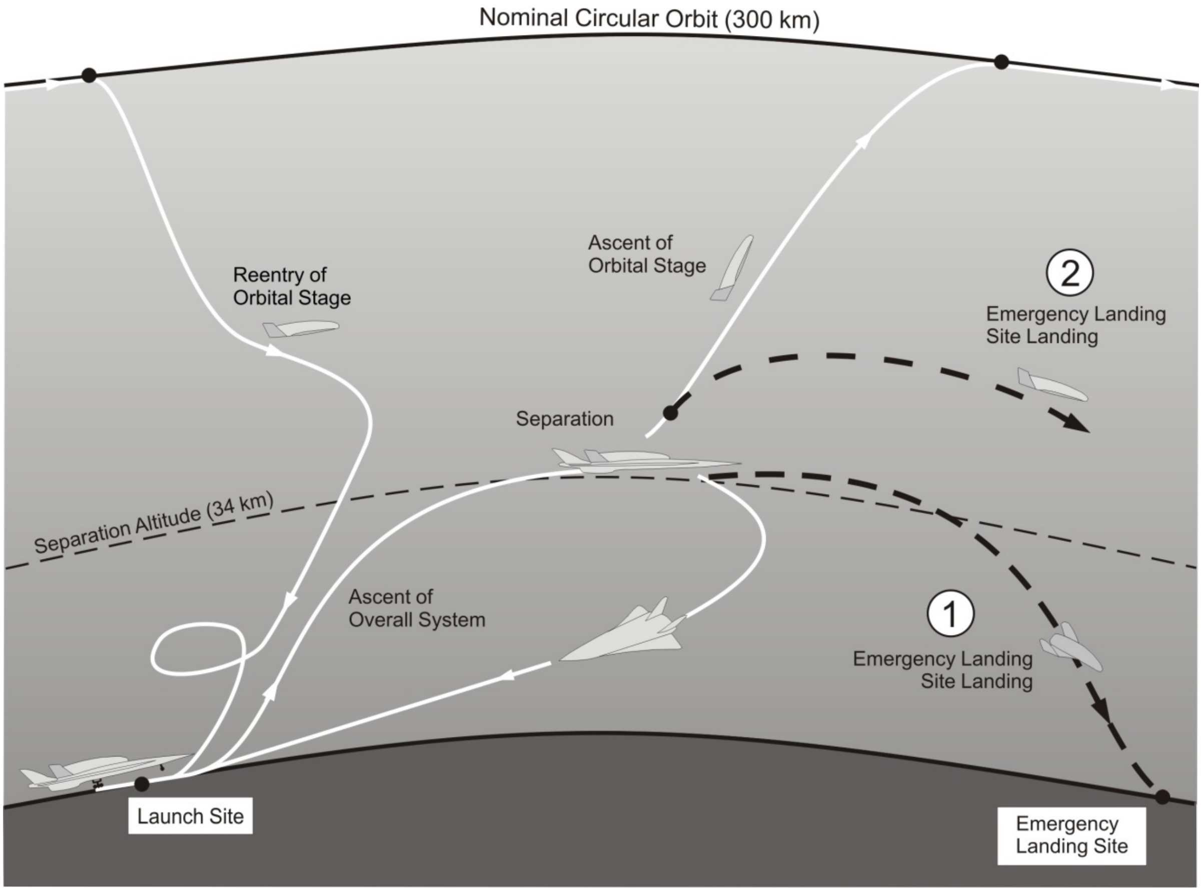

Fig. 1 Abort scenarios of orbital stage with requirement for maximum range glide and fuel draining possibility (flight cases Nos. 1 and 2)

Fig. 2 Water ballast dropping of sailplane (from [14])

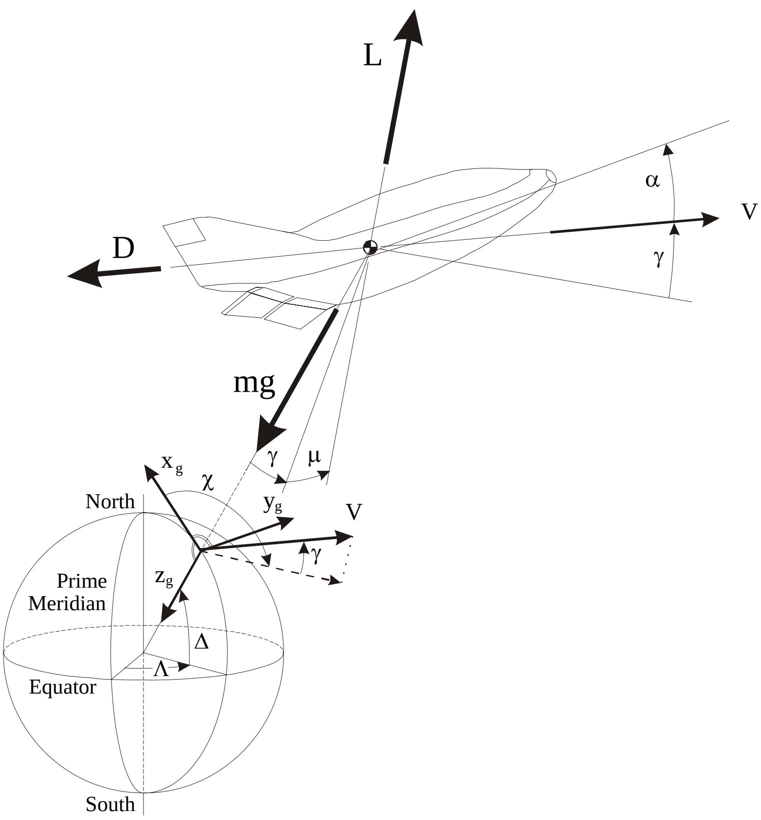

Fig. 3 Forces on vehicle and coordinate systems

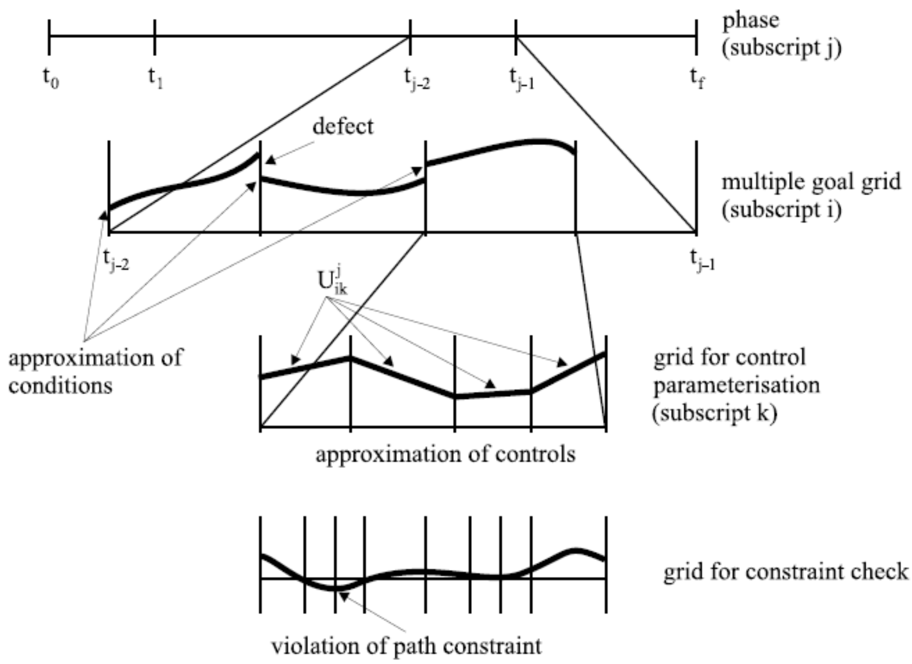

Fig. 4 Phases for optimization computation

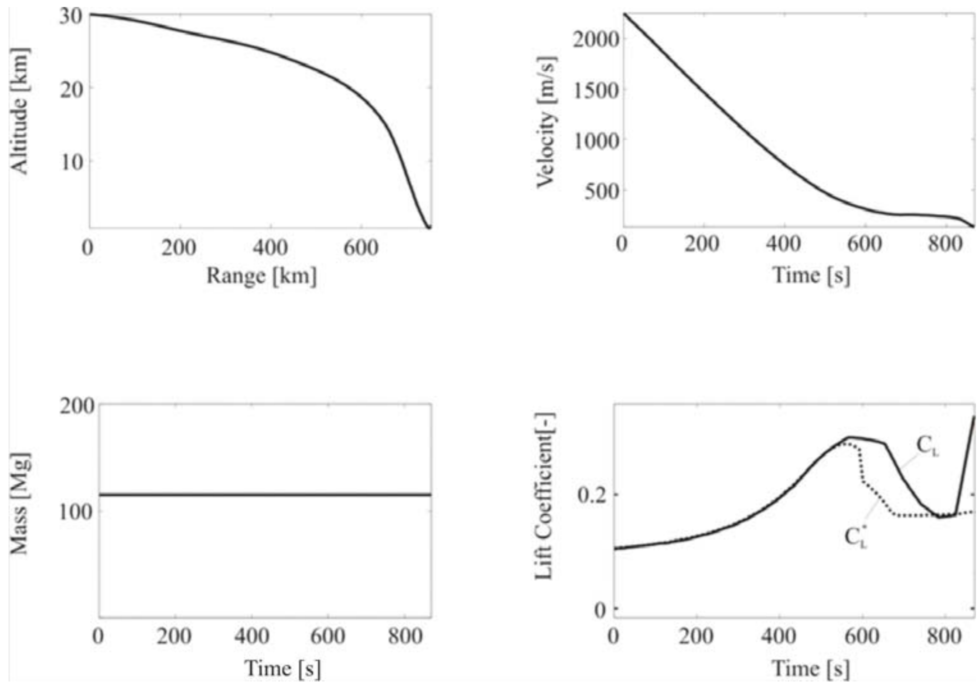

Fig. 5 Maximum longitudinal range glide of orbital stage without fuel draining

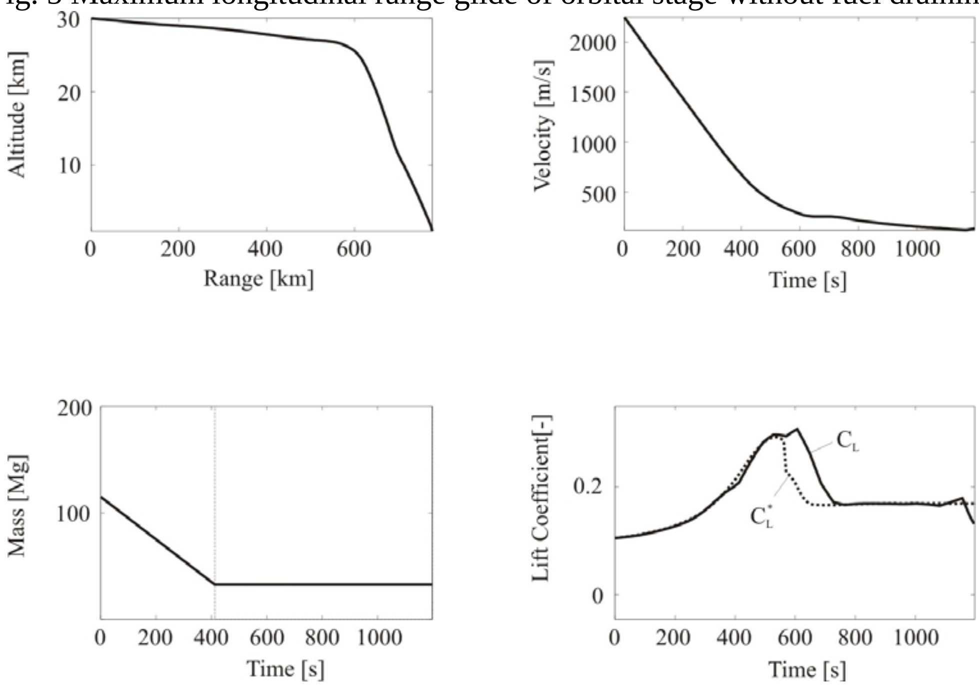

Fig. 6 Maximum longitudinal range glide of orbital stage with fuel draining

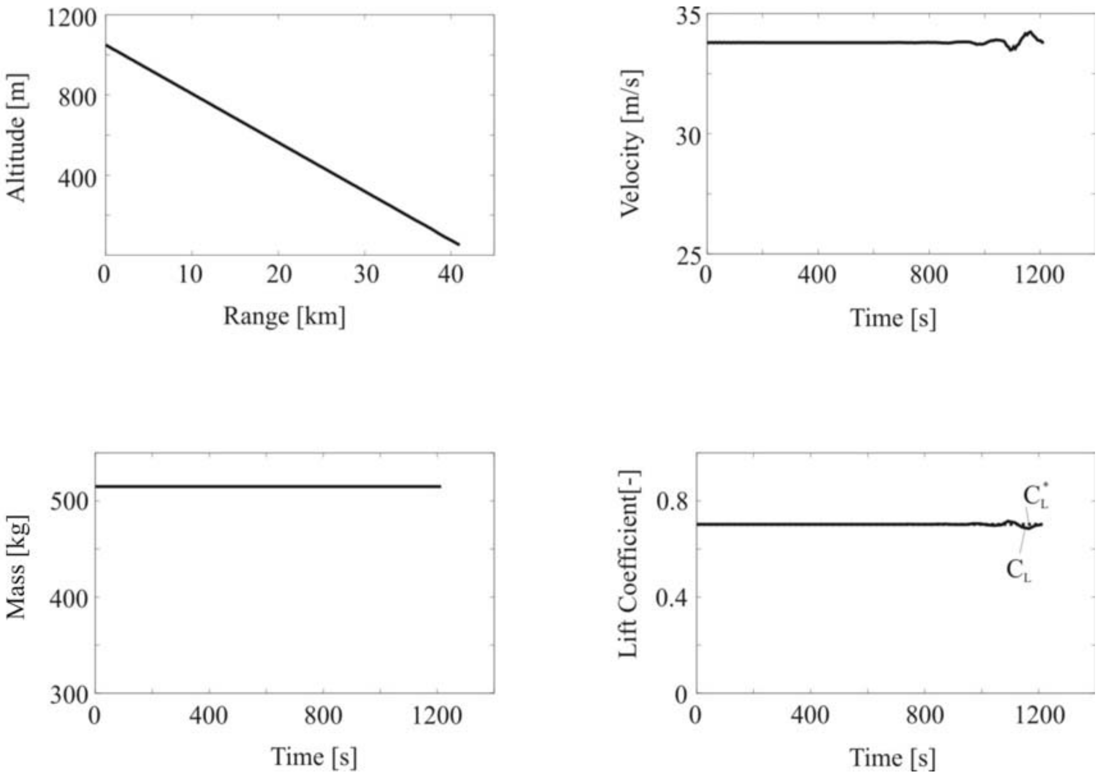

Fig. 7 Maximum longitudinal range glide of sailplane without water ballast release

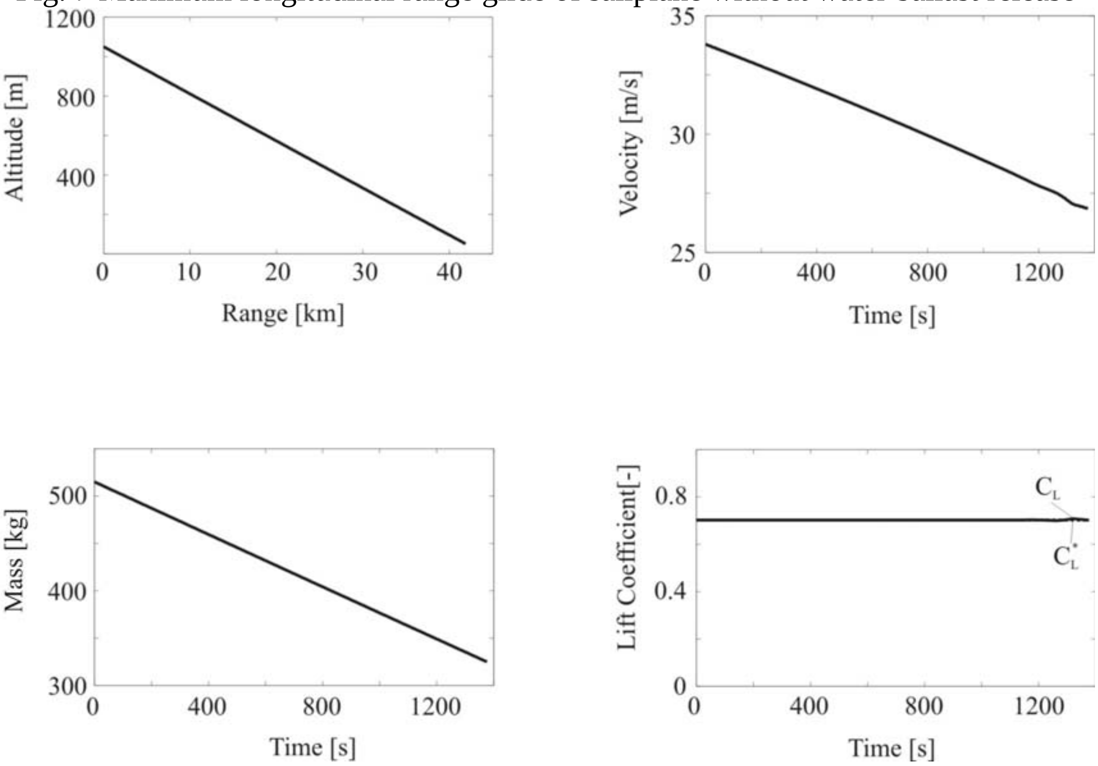

Fig. 8 Maximum longitudinal range glide of sailplane with water ballast release

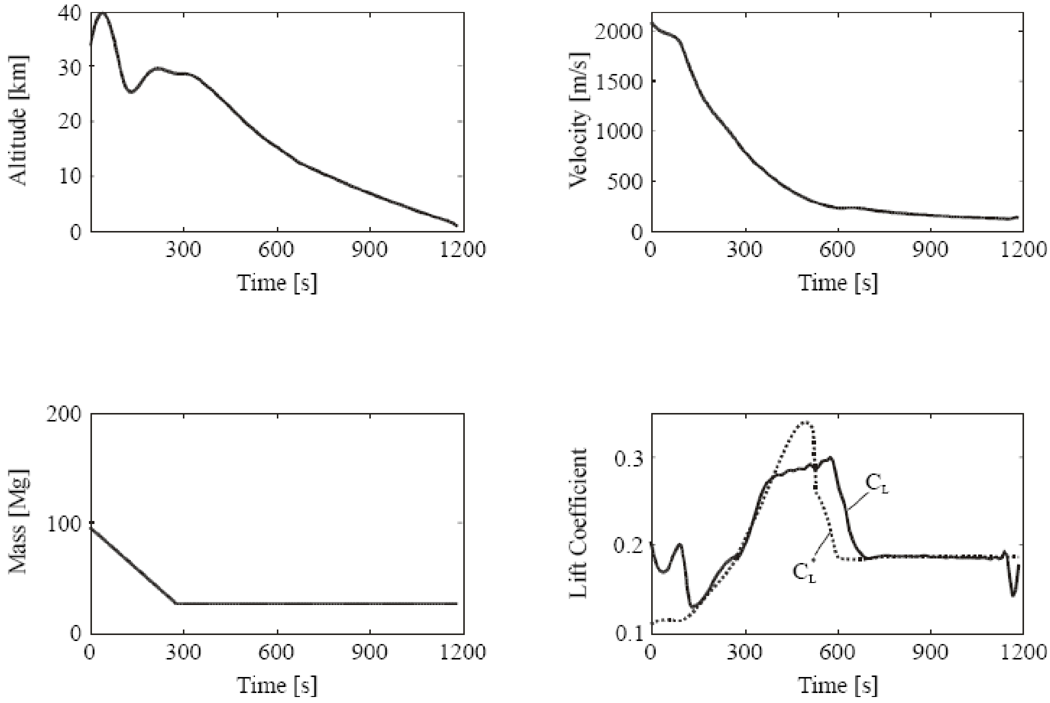

Figure 9 Maximum lateral range glide of orbital stage with fuel draining

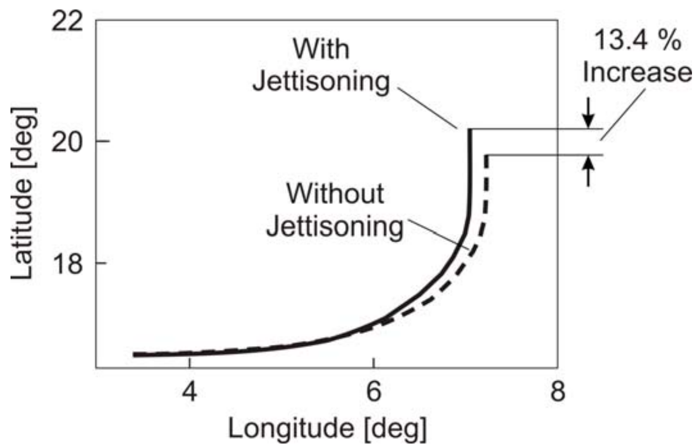

Fig. 10 Maximal lateral range of orbital stage in gliding flight (tracks of optimal flight paths)

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