Applied Mathematical Sciences, Vol. 4, 2010, no. 76, 3823 - 3840

# The Stochastic Analysis of Investments in Industrial Plants by Simulation Models with Control of Experimental Error: Theory and Application to a Real Business Case

**Lucia Cassettari**

Department of Production Engineering, Thermoenergetics and Mathematical Models
University of Genoa, Genoa, Italy
cassettari@diptem.unige.it

**Pier Giuseppe Giribone**

Department of Financial Administration (Middle Office)
CARIGE Bank Group
Genoa, Italy

**Marco Mosca**

Department of Production Engineering, Thermoenergetics and Mathematical Models
University of Genoa
Genoa, Italy

**Roberto Mosca**

Department of Production Engineering, Thermoenergetics and Mathematical Models
University of Genoa, Genoa, Italy
mosca@diptem.unige.it

**Abstract**

In literature, the applications of simulation to the stochastic analysis of investments do not often give a satisfactory result to, at least, two problems that can condition in a determinant way the validity of the analysis made. In synthesis:

1. no knowledge of the entity of the experimental error expressed in terms of Mean Square Pure Error (MSpe)
2. not so great importance to variance analysis of the response outgoing from the model

Starting from these considerations the Authors, by using the methodology of the evolution of MSPE in repeated runs developed a specific theory of experimentation for the study of the investments through Monte Carlo Simulation based on:

1. definition of MSPE curve of evolution and consequential choice, both for the mean and for the variance, of the most correct number of experimental runs in order to minimize the influence of the experimental error
2. definition of the scenarios needed in order to have a full vision of the analysed problem

The application to a realistic business case of investment in a new industrial food plant completes the theoretical development.

**Mathematics Subject Classification:** xxxxx

**Keywords:** Monte Carlo Simulation, Experimental Error, Investments

# 1 Introduction

The introduction of Monte Carlo simulation for the analysis of the investment is an approach known since the first half of the 1960's and its first application is ascribed to David Hertz (1964 - "Risk analysis in capital investment"), [1].

It marks a substantial conceptual step compared to the classical techniques (NPV, IRR, Payback Period) usually applied in the certainty context. The deterministic approach is very frequent because it does not require a detailed knowledge about probability of occurrence and it does not imply difficulty in calculation. However, making reference to deterministic systems is a strong limitation to the validity of the response obtained, because the reality of the object of this analysis is stochastic by its own very nature and for this reason, nowadays, this approach is applied to the assessment of investments of lesser and lesser importance.

Compared to the classical approach, this simulation allows the analysis of stochastic economic and financial scenarios and the assessment of the risks correlated to these scenarios (risk analysis). These risks are marked, on the one hand, by a subjective component strictly correlated to the ability and knowledge of the person performing the forecast and, on the other, by an objective dimension related to the random nature of the events having an impact on the company and the way in which these events occur. It must be considered as an integral part of the calculation of the performance and hence of the generation of value and its determination, through the analysis of the causes and a measurement of the effects, becomes essential for the success of a company.

# 2 Methodological Background

The purpose of the Monte Carlo simulation is basically that of extending the applicability and representational power of traditional analysis methods. There are three parameters normally considered in the analysis of industrial and non-industrial investments.

Their origin lies in the need to offer decision-makers several perspectives to assess the investment since none of these seems to be capable of providing a comprehensive view.

In conceptual terms, they are flawed by a fundamental logical contradiction because they provide a deterministic view of an event, which is stochastic by its own very nature.

## 2.1 Classical performance indicators

### 2.1.1 Net Present Value

The net present value is the algebraic sum of the cash flows (OFCF) over several years of the analysis horizon discounted at an interest rate (WACC). It is the incremental wealth generated by the investment posted at the moment of the analysis so that, if its value is positive, wealth is produced. Otherwise it is destroyed. In short, it can be summarized with the following formula:

$$NPV = \sum_{t=0}^{n} \frac{OFCF_t}{(1+WACC)^t}. \tag{1}$$

### 2.1.2 Pay Back Period

The investment's pay-back period is the point of temporal equilibrium of the *cash in* and *cash out* discounted at the WACC rate. It provides information on how much time is needed before the investment is paid back through the cash flows it generated. Therefore, it can be used to validate a project by selecting a priori a *cut-off*. Mathematically, it is expressed as follows:

$$\sum_{t=1}^{PBP} \frac{FCFO_t}{(1+WACC)^t} - FCFO_0 = 0. \tag{2}$$

where what needs to be determined is the PBP index.

### 2.1.3 Internal Rate of Return

It is the interest rate at which the NPV is zero. Therefore, it is the maximum cost of the capital raised that a project can bear without losing its economic convenience. In other words, it is an indicator of the return on an investment including the cost of the resources used. Mathematically, it takes the following form:

$$\sum_{t=1}^{n} \frac{FCFO_t}{(1+IRR)^t} - FCFO_0 = 0. \tag{3}$$

Starting from the three indices described above, the Monte Carlo simulation is proposed as a support instrument for decision-making on investments in conditions of risk, namely those conditions in which a decision-maker deems that deterministic data is rigid and hence too restrictive compared to a reality in which the starting data and the possible relevant events are known in the form of stochastic representations while others are sometimes even unknown.

When addressing investment problems in stochastic terms, it must be borne in mind that the stochastic levels found in this type of reality are usually quite high and, anyhow, not comparable to those normally found in the study of production systems. The variances of the output variables are always such that constitute a range around the mean response amplitude that often makes the model's response scarcely interpretable.

In this regard, it should be noted that normally only the mean value of the responses is taken into account in these models, while, in operating terms, little importance is given to the mean variance, which, owing to the extremely stochastic nature of the input variables, plays a decisive role. As it is widely known, knowledge of the average value of the response alone may lead even to extremely gross errors in the true behaviour of the system studied, Hacura [2], Nawrocki [3], Kelliher et al [4].

By contrast, in these conditions, owing to the high variance values, it can be tempting not to accept the modelling resultants owing to an amplitude of response variability such that the knowledge is almost unusable. However, in these cases, it is possible at least to partly recover the modelling resultants by matching the result obtained by the researcher with the opinion of experts from the sector whose help can give to the response that soundness that would otherwise be lacking.

## 2.2 MSPE as an experimental error measurement

An aspect that is still neglected all too often in experimenting with Monte Carlo simulation models is the control of the quantity defined as experimental pure error. This error affecting the model is generally distributed as an $NID(0, \sigma^2)$, Montgomery [5]. The value of $\sigma^2$, which, according to Cochran's theorem, can be estimated by calculating the Mean Square Pure Error, its unbiased estimator, is an intrinsic characteristic of the model built and it is closely linked to the reality studied since it is directly dependent on the overall stochasticity affecting the system. During the experimental phase what is really important is not the effort to eliminate this "noise", which cannot be eliminated because it is a characteristic element of the system and model used, but rather the effort not to add yet another source of error owing to an inadequate number of extractions of the stochastic variables from their probability distributions. This problem arises each time the number of extractions from the frequency distributions characterizing the model is limited to just one replication or to an inadequate number of samples to obtain a complete statistical description. A typical case is represented by the Corporate Models, used for the construction of possible future economic scenarios, in which specific variables, characterising any following accounting period, are assigned under the form of frequency distributions displaying an uncertainty character growing in the time (costs of raw materials, personnel, services; sale proceeds, transfers, investments etc.) from those, in the experimental phase, it will be selected, at worst, a single descriptive value destined to characterise any specific activity.

The main difference between this methodology and the evolution time methodology, Mosca et al. [6] [7], is that in this case both the variance of the mean response ($MSPE_{MED}$) and the variance of the standard deviation ($MSPE_{STDEV}$) must be kept under control. These two parameters, taken together, allow to choose the numbers of runs needed to obtain an unbiased evaluation of the experimental error affecting the objective function. Many scholars studying these issues recommend a rather large number of replicated runs to the point that even some one thousand replications have been hypothesized. As it is widely known, the larger the sample, the better the statistical inference on the population, Hacura [2]. Thanks to the methodology proposed, the approach to the problem becomes scientific, since it is possible to graphically highlight the evolution in the variance of the experimental error as a function of the size of the sample for which the experimenter is able to choose the best ratio between the experimental cost and the expected results. The methodology allows to identify the number of replications capable of minimizing , as needed by the experimenter, the noise produced by an inadequate overlapping of the probability density function of the variables of interest extracted using the Monte Carlo method, Mosca et al [7]. The knowledge in each point of the values of $MSPE_{MED}$ and $MSPE_{STDEV}$ allows to carry out important inferences on the behaviour of the real experimental response, which, as a result of Cochran's theorem, may vary between a minimum value and a maximum value as specified in the following formula:

$$y^* \geq \overline{y} - 3\sqrt{MSPE_{MED}} - 3\sqrt{\overline{VAR} + MSPE_{STDEV}}$$

$$y^* \leq \overline{y} + 3\sqrt{MSPE_{MED}} + 3\sqrt{\overline{VAR} + MSPE_{STDEV}} \tag{4}$$

where $\overline{VAR}$ is the square of $\overline{stdev_N}$.

For the experimenter, the problem does not lie in obtaining a theoretical $MSPE = 0$, which by result of the central limit theorem can be obtained for a sample of infinite amplitude, but rather in limiting the number $N$ of *runs* through a thorough check of the evolution in the experimental error in terms both of *magnitude* and settlement so as to limit also its impact on $y^*$ to acceptable values.

From what has been said above, the steps necessary for proper experimentation with the Monte Carlo method in case of analyses of investments on industrial plants should be clear.

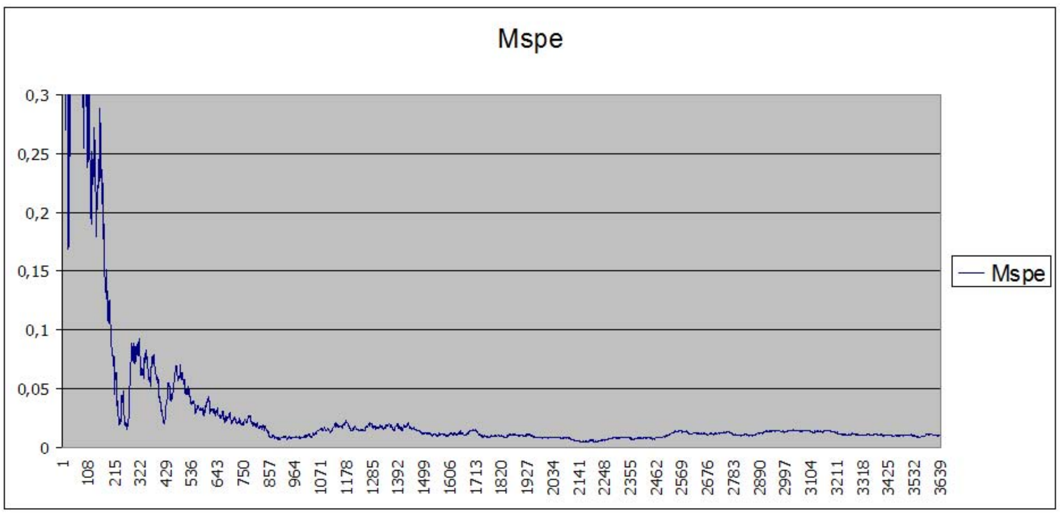


Figure 1: Trend of the pure error (MSPE) in replicated runs

In summary:

1. the quantity of the Pure Error in the form of mean $MSPE$ and variance of the output results ($MSPE_{MED}$ and $MSPE_{STDEV}$) and their evolution in the replicated runs through the construction of suitable knee curves as illustrated in Fig.1. It should be noted that until the curve does not reach an appropriate stabilization phase from one experimental campaign to another experimental campaign the values of the errors can even change in the order of magnitude.

2. The analysis of the two curves determines the number of runs deemed consistent with the experimenter's expectations in terms of impact on the output results generated by the two $MSPE$'s. The output results of the simulation are analysed not on the basis of the punctual values of the dependent variables surveyed but in terms of confidence and/or forecast and/or error interval thus giving a realistic picture of the values that can actually be taken as economic indices for evaluating the investment in

the various scenarios studied. The oscillation amplitude of the responses is to be related to the level of uncertainty expressed in stochastic terms that affects the economic reality in which the investment must be made.

The application to a real investment in a production line for a given food product best shows the difference between the deterministic approach and the designed Monte Carlo simulation.

# 3 Application to an industrial case

## 3.1 Selection of the input variables and construction of the model

In order to apply the proposed methodology, an evaluation will be performed on an industrial investment in the food sector consisting in the insourcing of a production activity previously contracted out to an outsourcer taking into account the combined effect of the behaviour of various parameters with regard to specific future events.

The analysis covers a 5-year horizon and considers the three performance indicators for an investment described in section 2.1. namely PBP, NPV and IRR. The input for the model consists in three families of variables concerning the volume of annual production, the annual production costs and revenues and the time necessary to start up the plant.

For each of these variables, a qualitative-quantitative analysis of the historical data, of the company's internal forecasts and of the know-how of the project managers the time-based statistical distributions over the life of the investment are determined.

With regard to the annual production costs, the elements considered as having an impact were packaging, raw materials, energy, logistics and personnel. Their future mean trends were obtained by carrying out a regression on the data of recent years for similar products and extrapolating their future behaviour as illustrated in Fig.2 and then comparing them to corroborate the forecast with the sector's managers.

With regard to the mean sales price, a process linked to the mean annual inflation rate expected for coming years increased by 50% was hypothesised based on the policies adopted in previous years for similar products. In particular, two separate scenarios were hypothesised depending on the rate in the rise of inflation (constant slow - fast).

With regard to the production volumes, in coordination with the marketing manager and the product manager, depending on the sales data of the previous three-year period for similar products, the level of market saturation, and the latest product launches by competitors, four possible scenarios were hypothesized for the sales of the following five-year period and hence of the realizable production: moderate and constant growth, first rapid and then slow growth, stationary growth, decreasing and progressively worsening growth.

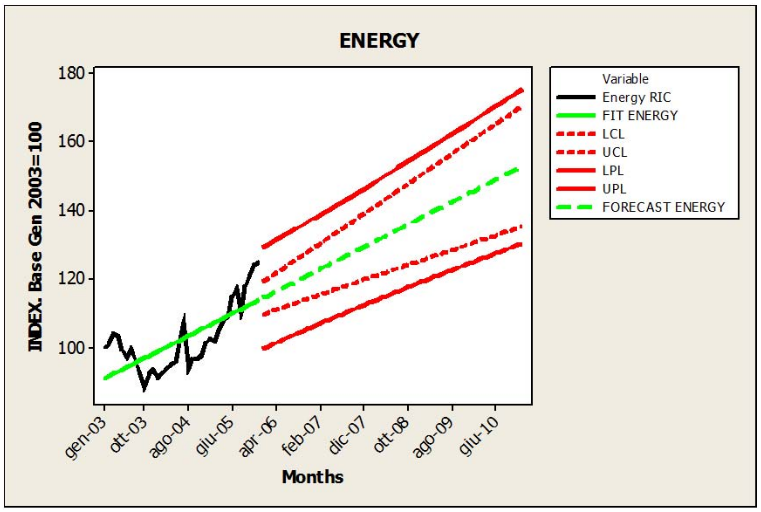


Figure 2: Trend Analysis for energy price index

It was decided to use for each year a triangular distribution function focused on the mean trend associated with growth/decrease by succession. A distinction was made on the amplitude of the distribution with regard to the type of packaging and the sales channel.

Finally, with regard to the time for plant start-up, namely the time between the end of the installation of the equipment already inspected and the start of the various phases to obtain a marketable finished product, the project manager indicated three possible scenarios linked to the mean estimated time frames for the start-up of the frying machine and extrusion, which can respectively be 3 and 4 months, 3 and 6 months or 5 and 8 months.

In order to take into account the actual stochastic nature marking this type of works, a series of beta distributions were built around these mean expected values so that the completion time can vary between 10% less and 100% more than the mean estimated time.

The greater the delay in the start-up time, the lesser the investment's capacity to generate profits in the five-year period taken as the investment evaluation time.

The combination of the three different groups of scenarios described above gives a possible combination of 24 different alternatives, which can rise to 96 if a series of growth hypotheses either independent from or dependent on the data of the previous year's volumes and costs were to be taken into account.

Starting from this assumption, the resulting experimental project for a proper estimate of the experimental error would require at least 480 runs, each of which composed of a number of replications in the order of $10^4$ with an overall calculation time of about 72 CPU hours just for the first analysis phase.

The simulator built to study this case is based on a set of $MSExcel^{TM}$ worksheets plus a tool for the reiterated combination of random extractions of stochastic variables called $SimSheet^{TM}$. In order to analyse these historical series, the $Minitab^{TM}$ software package was used to build the frequency distributions and analyse the output results.

Starting from the deterministic business plan developed by the sector's managers, several worksheets were added, each of which for the management of a parameter or an output result. The worksheets regard:

1. the product *cost card*
2. the *setup* of the distributions for the variation in the volumes
3. the *setup* of the distributions of the variation in the costs
4. the *setup* of the industrialization time frames, the line efficiency and the start-up and development costs
5. the master plan receiving the results of the worksheets of items 2, 3, and 4.
6. the profit and loss account
7. the selected investment performance indicators

## 3.2 Experimentation on the model

As described in paragraph 2.2, the procedure steps carried out while experimenting the model were:

- ☐ analysis of the evolution in the pure error as the runs vary in order to ensure the statistical reliability of the model's output variables
- ☐ determination of the number of runs necessary to ensure the best relation between the experimental variance and the experimentation time
- ☐ analysis of the economic indicators for the evaluation of the investment in the various scenarios studied in terms of confidence and/or forecast and/or error interval
- ☐ discussion of the experiment's results with the sector's experts

For self-evident reasons of significance of the 96 possible scenarios only the 16 deemed to be most likely were studied, while those based on extreme hypotheses were set aside. In this study the Authors selected the case deemed by the managers to be closest to reality and compared the results obtained with those derived from the classical deterministic approach. The scenario hypotheses concerning the case selected are illustrated in Table 1.

| INPUT VARIABLES | TREND HIPOTESYS | TYPE OF SCENARIO |
|---|---|---|
| Sales | succession | continuous growth |
| Revenues and Industrial Costs | independence | slow growth |
| Time to Market | TRIANGULAR | 3-6 months |

Table 1: Set-up of the most likely scenario

Fig.3 shows the hypothesized growth trend in the volumes with the relevant confidence intervals divided by packages of the same product solely for GDO (MPK) and traditional packages (OTHERS).

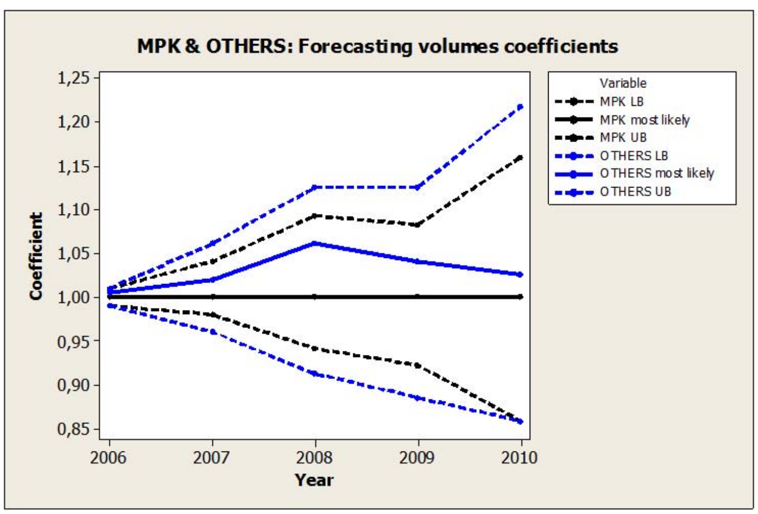


Figure 3: Trend in the volumes for the hypothesis of moderate and constant growth during the project's life

With regard to the variation in the industrial costs in the 5-year period, constant increases were taken for each year. The frequency distribution used to represent each of the years covered by the survey is a normal trimmed mean having a root mean square deviation that increases year after year with the first-year value and the fifth-year value equal to the first and second figure in the brackets respectively (Table 2).

| | MEAN | STANDARD DEVIATION |
|---|---|---|
| Raw materials | +2.0% | (5.2 - 7)% |
| Packaging | +3.2% | (9 - 12)% |
| MOVD | +4.7% | (2 - 4.3)% |
| Energy | +5.0% | (4 - 6)% |
| Finished Good | +4.5% | (1 - 5)% |
| Logistics | +2.0% | (1- 5)% |

Table 2: Increases in the annual costs in the future

The value of the intermediate years is obtained by linear interpolation. According to the methodological steps described above, an experimental campaign was carried for each performance indicator in order to determine the error variance calculated by means of the contributions of the variance in the mean response ($MSPE_{MED}$) and the variance in the mean deviation ($MSPE_{DEV}$). The experimental campaign for each indicator, an operation repeated on each of the 16 scenarios considered, was organized on 5 replications, each consisting of 10000 runs. The observation of the curves illustrated in Fig.4 relating to the NPV indicator for the most likely scenario shows that the 4000 replicated runs give an $MSPE_{DEV}$ value in the order of a few units while the $MSPE_{MED}$ value is in the order of $10^2$. This makes it possible to state that the output result obtained for the NPV after the 4000 runs is affected by an almost negligible pure error, since, in the light of a mean value of $10^3$ k€, the pure error in terms of standard deviation, is smaller by two orders of magnitude.

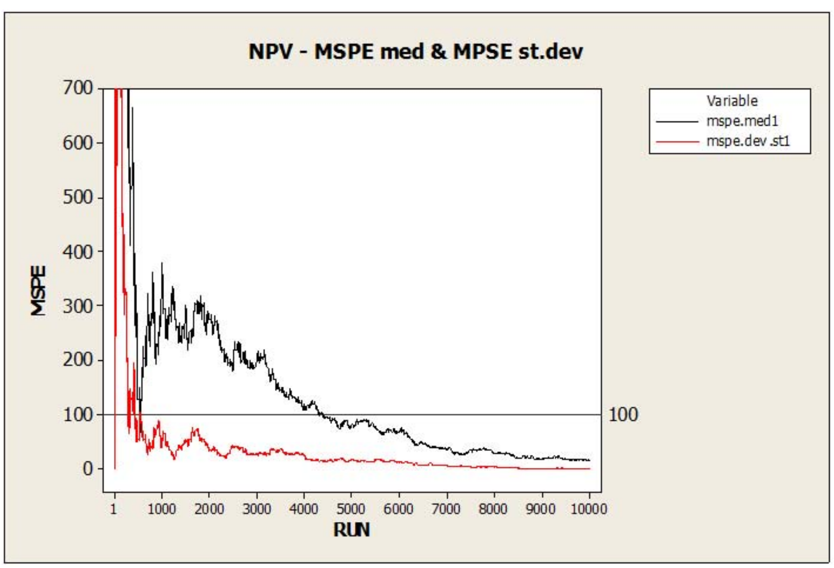


Figure 4: $MSPE_{MED}$ and $MSPE_{STDEV}$ of the NPV for the most likely scenario

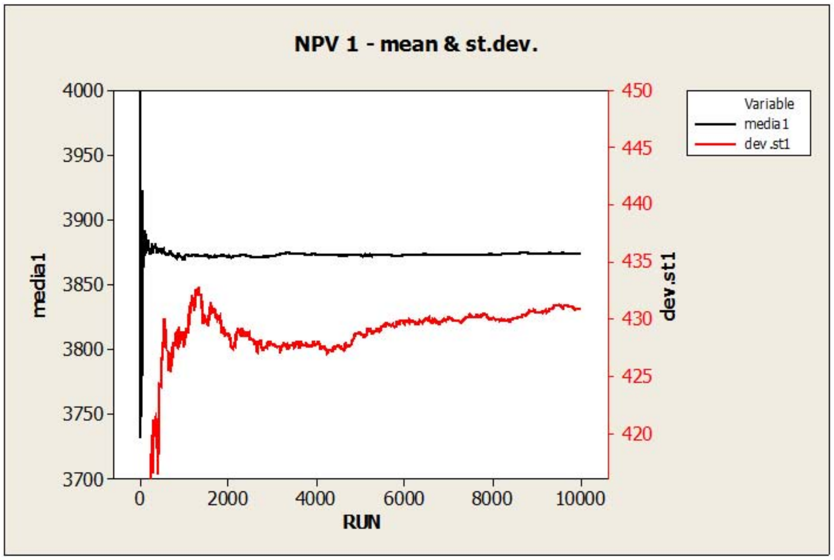


Figure 5: Trend in the mean value and standard deviation replicated runs for NPV of the most likely scenario

It can be noted in Fig.5 that, in the light of a pure error made negligible by an appropriate choice of the number of replicated runs, the actual system shows a high level of inherent stochasticity through a mean value of the standard deviation, which is all but negligible and which will play a decisive role in the amplitude of the confidence intervals on the response and hence in the range in which the indicator will tend to vary.

By proceeding in the same identical way for the NPV, the statistical parameters of the PBP and IRR were calculated starting from the $MSPE_{MED}$ and $MSPE_{VAR}$ evolution curve in the replicated runs. These parameters are the mean value, the standard deviation, the lower and upper boundary of the error interval encompassing about 99.8% of the data in Table 3.

| STATISTICAL | NPV | PBP | IRR |
|---|---|---|---|
| μ | 3874 | 21,18 | 92,8% |
| σ | 434,9 | 1,63 | 9,20% |
| LPL | 2570 | 16,28 | 65,17% |
| UPL | 5179 | 26,09 | 120,4% |

Table 3: Results of the most likely scenario

It should be noted that the upper boundary and the lower boundary of each indicator were obtained considering also the $MSPE_{MED}$ and $MSPE_{STDEV}$ according to the following formula:

$$y^* \geq \overline{y} - 3\sqrt{MSPE_{MED}} - 3\sqrt{\sigma^2 + MSPE_{STDEV}} \tag{5}$$
$$y^* \leq \overline{y} + 3\sqrt{MSPE_{MED}} + 3\sqrt{\sigma^2 + MSPE_{STDEV}}$$

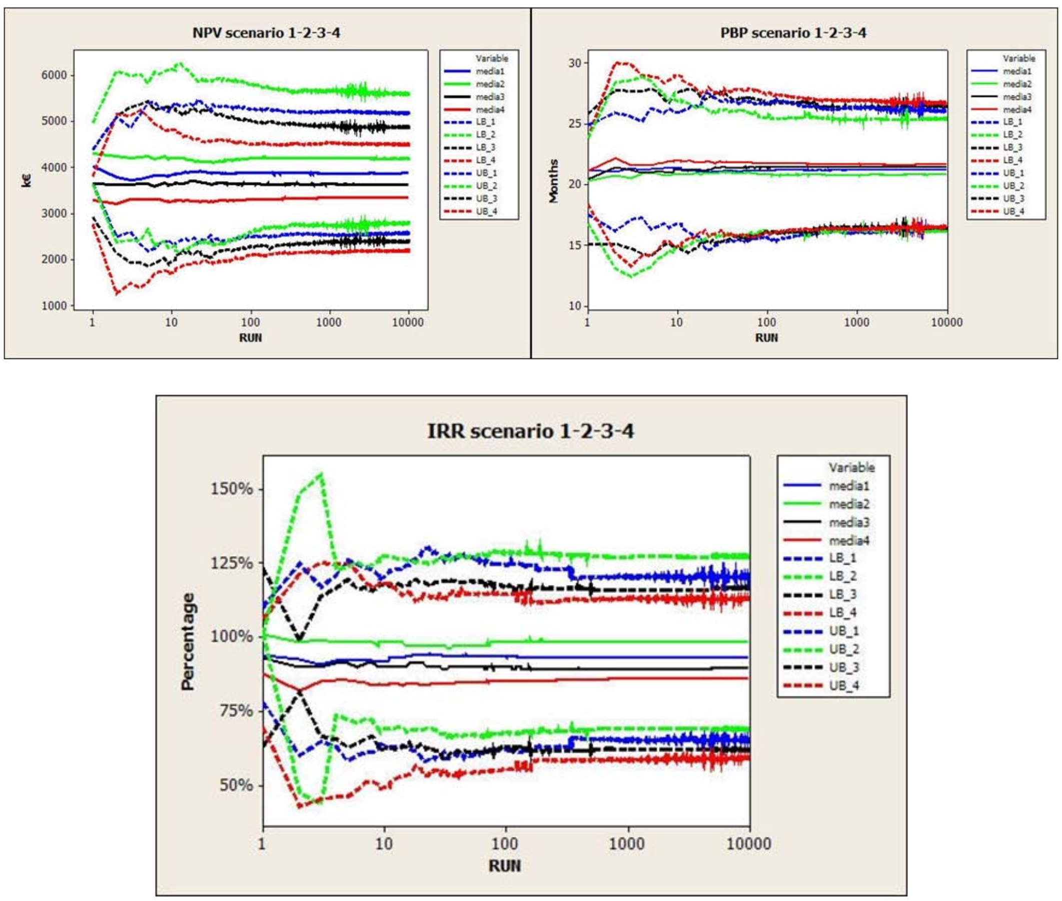


Figure 6: Error intervals for the NPV, PBP and IRR in the 4 volume scenarios considered

Fig. 6 shows the trend in the error intervals for the NPV, PBP and IRR in the 4 volume scenarios considered. As it can be noted for instance in the case of the NPV, owing to the stochasticity affecting the input variables for a mean value of 3874 k€, a variation field of this value is obtained by which the minimum is equal to 2570 k€ and the maximum to 5179 k€. Now, for the decision-maker, owing to the amplitude of the possible responses, it is important to determine the likelihood of an event of a certain value or, rather, the likelihood that the NPV and other indices fall within a given interval.

Since, as confirmed by the Kolmogorov-Smirnov, Anderson-Darling and Chi Squared tests carried out automatically by the software used for the analysis of the output results, the values of the three indicators are distributed according to the NID's ($\mu$, $\sigma^2$), as illustrated in Fig. 7, it is possible to obtain for each the likelihood associated with value intervals of interest to the decision-makers (see Table 4, Table 5, Table 6).

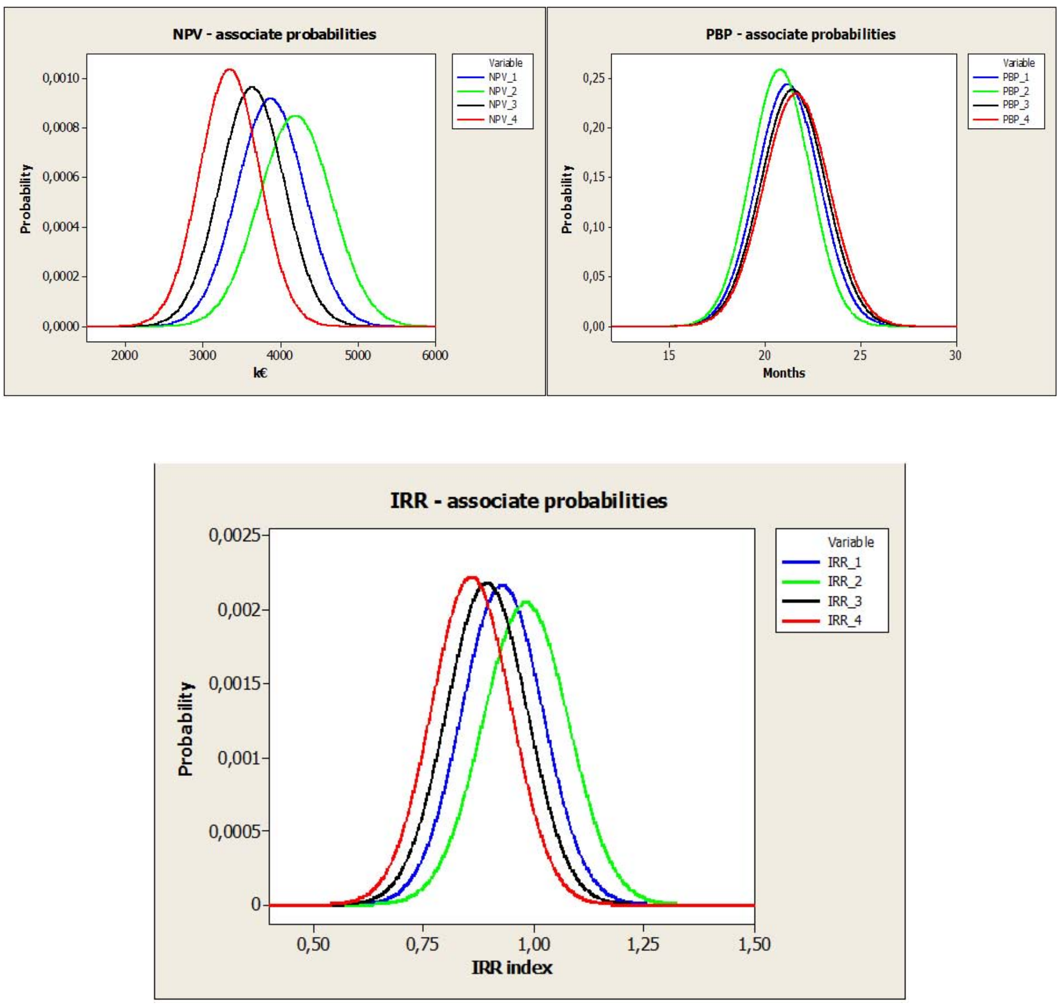


Figure 7: Distribution of the likelihood of the response for NPV, PBP and IRR for scenarios 1-2-3-4 on the sales volumes

It can be noted that in 59.14% of cases the value of NPV falls within the 3000 and 4000 k€ range, while in 38.16% it falls within the 4000 and 5000 k€ range. Once the NPV is known, it is also possible to calculate with formula (6) another evaluation parameter of great interest for the decision-maker, namely the net percentage return of the discounted investment (ROR), i.e., what rate of return is available after covering all the fixed investment expenses.

| x1 | | x2 | P(NPV<x1) | P(x1<NPV<x2) | P(NPV>x2) |
|---|---|---|---|---|---|
| 0 | - | 1000 | 0.00% | 0.00% | 100.00% |
| 1000 | - | 2000 | 0.00% | 0.00% | 100.00% |
| 2000 | - | 3000 | 0.00% | 2.22% | 97.78% |
| 3000 | - | 4000 | 2.22% | 59.14% | 38.64% |
| 4000 | - | 5000 | 61.36% | 38.16% | 0.48% |
| 5000 | - | 6000 | 99.52% | 0.48% | 0.00% |

Table 4: Likelihood associated with the NPV, most likely scenario

$$ROR[\%] = \frac{NPV}{CAPITAL\_EXPENDITURE} \cdot 100 \tag{6}$$

Through the inverse formulation, formula (7), it is also possible to estimate the desired likelihood by choosing a priori the mean $ROR^*$ to be achieved.

$$NPV^* = \frac{ROR^* \cdot CAPITAL\_EXPENDITURE}{100} \tag{7}$$

$$P(ROR) = P(NPV \geq NPV^*) = 1 - P(NPV \leq NPV^*) =$$

$$= 1 - cdf_{NID(\mu_{NPV},\sigma^2)}(NPV^*)$$

where $cdf_{NID(\mu_{NPV},\sigma^2)}$ is a probability density function for a $NID(\mu_{NPV}, \sigma^2_{NPV})$ calculated at the point NPV

| x1 | | x2 | P(PBP<x1) | P(x1<PBP<x2) | P(PBP>x2) |
|---|---|---|---|---|---|
| 0 | - | 12 | 0.00% | 0.00% | 100.00% |
| 12 | - | 15 | 0.00% | 0.01% | 99.99% |
| 15 | - | 18 | 0.01% | 2.56% | 97.43% |
| 18 | - | 20 | 2.57% | 20.86% | 76.57% |
| 20 | - | 22 | 23.43% | 45.67% | 30.90% |
| 22 | - | 24 | 69.10% | 26.65% | 4. 25% |
| 24 | - | 30 | 95.75% | 4.25% | 0.00% |

Table 5: Likelihood associated with the PBP, most likely scenario

| x1 - x2 | P(IRR<x1) | P(x1<IRR<x2) | P(IRR>x2) |
|---|---|---|---|
| 0% - 50% | 0.00% | 0.00% | 100.00% |
| 50% - 70% | 0.00% | 0.67% | 99.33% |
| 70% - 80% | 0.67% | 7.59% | 91.74% |
| 80% - 90% | 8.26% | 29.91% | 61.83% |
| 90% - 100% | 38.17% | 40.24% | 21.60% |
| 100% - 120% | 78.40% | 21.44% | 0.15% |
| 120% - 160% | 99.85% | 0.15% | 0.00% |

Table 6: Likelihood associated with the IRR, most likely scenario

Once the economic indicators with the relevant error intervals have been determined using the proposed Monte Carlo approach, the Authors did a comparison with the original business plan shown in Table 7. The results were obtained considering the total stability of the volumes, prices and full compliance with the project deadlines.

| INDICATOR | VALUE |
|---|---|
| NPV | 2,763 |
| PBP | 20.5 |
| IRR | 89.5% |

Table 7: Results according to the business plan using the traditional method

The differences between the deterministic and the designed Monte Carlo approaches are now clear. The net present value (NPV) of the traditional approach has a likelihood of just 2.22% (Table 4) in the light of the hypothesized stochasticity. The reason is that in the original analysis both the volumes and prices were considered stable, but these are bound to change in the course of time. By contrast, the mean PBP is higher in the Monte Carlo model because the plant start-up time can suffer delays not envisaged by the deterministic model generating liquidity problems owing to the extension of the production's outsourcing. It should also be taken into account that the impact of the delays may even make the PBP rise up to a maximum of 26 months as shown in the Table 3.

With regard to the IRR, the mean value in the two experiments is more or less the same, but the Monte Carlo simulation gives a range comprised between 65% and 120%. It should be noted, however, that from a technical point of view values of this size do not negatively affect the decision-makers judgement.

# 4 Conclusions

The traditional approach faces in the deterministic regime a typical stochastic problem as the one of the evaluation of industrial investments. This fact often leads to wrong decision cause based on values not so very representative of the object reality as it has been highlighted in the industrial case previously described.

The advanced approach, being stochastic by its own very nature, allows to highlight the variability of the responses resulting from the stochastic nature typical of the input variables, thus providing intervals of response with a known degree of probability. This is possible thanks to the check of the pure error carried out through the study of the evolutions curves for the two quantities called $MSPE_{MED}$ and $MSPE_{STDEV}$, which allow the experimenter to know a priori the size of the error overlapping the background noise of the real system.

Thanks to this methodology the decision maker is able to evaluate the risk related to the different alternatives, that could condition the future of the Company in which he operates.

# References

[1] D. Hertz, Risk Analysis in Capital Investment, *Harvard Business Review,* **42:95/106** (1964).

[2] A. Hacura, M. Jadamus-Hacura, A. Kocot, Risk Analysis in investment appraisal based on the Monte Carlo simulation technique, *Eur. Phys. J.,* **B20/551-553** (2001).

[3] D. Nawrocki, The Problems with Simulation, *Journal of Financial Planning, 14,* **11:92/108** (2001).

[4] C.F. Kelliher, L.S. Mahoney, Using Monte Carlo simulation to improve long-term investment decisions, *The Appraisal Journal Vol.68,* **44-56** (2000).

[5] Douglas C. Montgomery, Design and Analysis of Experiment, *John Wiley and Sons, Inc.* (1997).

[6] R. Mosca, P. Giribone, Optimal Length in O.R. Simulation Experiments of Large Scale Production System, *Proc. IASTED International Symposium "Applied Modelling and Simulation" - AMS'82, Paris, F,* **78:82** (1982).

[7] R. Mosca, P. Giribone, R. Revetria, L. Cassettari, S. Cipollina, Theoretical Development and Applications of the MSPE Methodology in Discrete and Stochastic Simulation Models Evolving in Replicated Runs, *Journal of Engineering, Computing and Architecture Volume 2, Issue 1,* **1934-7197** (2008).

**Received: June, 2008**