Optimal Strategy Analysis of an N-Policy Two-Phase $M/E_k/1$ Queueing System with Server Breakdowns and Gating

V. Vasanta Kumar

K.L.E.F University, Vaddeswaram 522 502
Guntur (Dist), Andhra Pradesh, India
vemuri57@rediffmail.com

B.V.S.N. Hari Prasad

S.R.K. Institute of Technology
Enikepadu, Vijayawada 521 108
Krishna (Dist), Andhra Pradesh, India
bvsnhariprasad@gmail.com

K. Chandan

Department of Statistics
Acharya Nagarjuna University
Guntur (Dist), Andhra Pradesh, India.
kotagirichandan@yahoo.com

K.P.R.Rao

Department of Applied Mathematics
Acharya Nagarjuna University-Dr.M.R.Appa Rao Campus
Nuzvid, Krishna(Dist), A.P., India.
kppr2004@yahoo.com

Abstract. Two-phase $M/E_k/1$ queueing system with $N$-policy for exhaustive batch service with gating, and server startups and breakdowns is studied in this paper. The customers arrive individually according to a Poisson process and waiting customers receive batch service all at a time in the first phase and are served individually in the second phase. The server is turned off each time the system empties, as and when the queue length reaches or exceeds
$N$ (threshold) batch service starts. Before the batch service, the system requires a random startup time for pre-service. As soon as the startup period is over the server starts the batch service followed by individual service to all customers in the batch. It is assumed that the server may breakdown during individual service according to a Poisson process and the repair times has an exponential distribution. Explicit expressions for the steady state distribution of the number of customers in the system are obtained and also derived the expected system length. The total expected cost function is developed to determine the optimal threshold of $N$ at a minimum cost. Numerical experiment is performed to validate the analytical results. The sensitivity analysis has been carried out to examine the effect of different parameters.

**Keywords:** Vacation, N-policy, Queueing System, Two-phase, Startup, Breakdowns, Sensitivity Analysis

1. INTRODUCTION

Queuing models with two phases of service and server vacations have several applications in many areas such as in computer network administration and in telecommunication systems - messages are processed in two phases by a single server and in inventory control processes, due date, quantity and quality are analyzed initially in batch mode followed by individual service of each order in the batch.

Out of several such situations discussed above and in the fitness of the situation that has been discussed in this paper, pharmaceutical industry and computer controlled manufacturing sector are classical examples which can be quoted all the time. All the more so mainly in pharmaceutical sector—formulation of medicines still stands as a classical example. Initially, from the bulk drug, medicines are formulated as per the pre-defined composition at the first stage and thereafter, thus formulated medicines are subjected to thorough quality check. Once, the product satisfies the pre defined quality parameters, it is either packed and sealed in bottles or else in strips and then grouped into lots as per batch number and thereafter released into market. Thus the process of quality check, packing and sealing and releasing into market perfectly satisfies the concept of service in several stages in the second phase which is being termed mathematically as $E_k$.

Another example that can be quoted is in computer controlled manufacturing sector, where the raw material is turned into a product viz: silicon chips, bearings, electronic components at the first phase and then subsequently subjected to quality check, packing into lots as per the batch size in the second stage and then released into market.

However, the above said examples are only few out of many that are encountered in several industrial and real life situations. All such examples provide
an insight for the optimal control policy at every stage of processing. Several attempts have been made by many investigators to provide a valid and a meaningful solution so as to estimate the optimal control policy. Out of several those who had made an attempt to study the system, Krishna and Lee [7], Doshi [3] studied distributed systems where all customers receive batch service in first phase followed by individual service in second phase. Selvam and Sivasankaran [9] introduced a two-phase queuing system with server vacation. For the control policy of vacation queues it is usually assumed that the server becoming available or unavailable depends completely on the number of customers in the system. Every time when the system is empty, the server goes on a vacation. The instance at which the server comes back from a vacation and finds at least \( N \) (threshold) customers in the system, it begins serving immediately and exhaustively. This type of control policy is also called \( N \)-policy queueing system with vacations. Kim and Chae [6] analyzed the Two-Phase queueing system with \( N \)-policy. Vasanta Kumar and Chandan [10] and [11] presented the optimal control policy of two-phase \( M/M/1 \) and \( M^X/E_k/1 \) queueing systems with \( N \)-policy.

The server startup corresponds to the preparatory work of the server before starting the service. In some of the practical situations that are encountered in the real life situation involves, the server often requires a startup time before startup of each service period. Concerning to queueing systems with startup time, Baker [2] first proposed the \( N \)-policy \( M/M/1 \) queueing system with startup time. Krishna Reddy et al investigated the bulk arrival queueing system with \( N \)-policy, multiple vacations and setup times. Ke [4] presented the optimal control of an \( M^X/G/1 \) queue with server startup and with a long vacation type and short vacation type. Wang [12] first proposed a Markovian queueing system under \( N \)-policy with server breakdowns. Wang [13] and Wang et al. [14] extended the model proposed by Wang [12] to \( M/E_k/1 \) and \( M/H2/1 \) queueing systems respectively. Ke [5] presented the optimal control policy in batch arrival queue with server breakdowns and multiple vacations. Anantha Lakshmi et al. [1] presented the optimal control strategy of an \( N \)-policy bulk arrival queueing system with server startup and breakdowns. In all the above literature, the concept of queueing systems with server breakdowns and gating has not been examined in detail for two-phase systems.

In this paper, we consider the optimal control policy for two-phase \( M/E_k/1 \) queueing system with \( N \)-policy, server startups and breakdowns. Customers arrive individually according to a Poisson process and waiting customers receive batch service all at a time in the first phase and are served individually in the second phase. The individual service is in \( k \) independent and identically distributed exponential phases. The server is turned off each time the system empties; as and when the queue length reaches or exceeds \( N \) (threshold) batch service starts. Before the batch service, the system requires a random startup time for pre-service. As soon as the startup period is over the server starts the batch service followed by individual service to all customers in the batch. By
gating we mean that the jobs which arrive during pre-service and batch service are not allowed to enter the batch which is already in service, but are served during the next visit of the server to the batch service. This will be after the server has completed its current visit to the individual queue. All the arrivals during the server’s sojourn at the pre-service and batch service are bunched together with the arrivals that occur at the end of the server’s visit.

The four main objectives for which the analysis has been carried out in this paper for the optimal control policy are:

i. To establish the state equations to obtain the steady state probability distribution of the number of consumers in the system.

ii. To derive expected number of consumers in the system.

iii. To formulate the total expected cost function for the system, and determine the optimal value of the control parameter $N$.

iv. To carryout sensitivity analysis on the optimal value of $N$ and the minimum expected cost for various system parameters through numerical illustrations.

2. The System and Assumptions

In the fitness of realistic situations and under the assumptions stated above, it is more appropriate to consider that the customers are assumed to be arriving according to a Poisson process with mean arrival rate $\lambda$ and join the batch queue. When the batch size reaches $N(\geq 1)$ the server will spend a random time $t'$ for pre-service, which is assumed to follow an exponential distribution with mean $1/\theta$. As soon as the period of startup is over, the server begins batch service to all the customers waiting in the queue in the first phase. On completion of batch service, the server proceeds to the second phase to serve all customers in the batch individually. The analysis that has been carried out to be applicable is only when individual queue is served in FIFO mode, while the batch service time is assumed to be exponentially distributed with mean $1/\beta$ and is independent of batch size. Individual service is in $k$ independent and identically distributed exponential phases with mean $1/k\mu$. While serving in individual queue, the server may breakdown at any time with a Poisson breakdown rate $\alpha$. When the server fails it is immediately repaired, where the repair times are exponentially distributed with mean $1/\gamma$. After repair the server immediately resumes service in individual queue. On completion of individual service, the server returns to the batch queue to serve the customers who have arrived. If the customers are waiting, the server restarts the cycle by providing them batch service followed by individual service. If no customer is waiting, the server takes a vacation and return from vacation only after $N$ customers accumulate in the batch queue and start pre-service work.
3. Exhaustive service with N-policy and gating

In case of steady-state the following notations are used.

\[ P_{0,i,0} \equiv \text{The probability that there are } i \text{ service phases in the batch queue when the server is on vacation, where } i = k, 2k, 3k, \ldots \]

\[ P_{1,i,0} \equiv \text{The probability that there are } i \text{ service phases in the batch queue when the server is doing pre-service (startup work), where } i = Nk, (N + 1)k, (N + 2)k, \ldots \]

\[ P_{2,i,0} \equiv \text{The probability that there are } i \text{ service phases in the batch queue when the server is in batch service, where } i = k, 2k, 3k, \ldots \]

\[ P_{3,i,j} \equiv \text{The probability that there are } i \text{ service phases in the batch queue and } j \text{ service phases in individual queue when the server is in individual service, where } i = 0, k, 2k, 3k, \ldots \text{ and } j = 1, 2, 3, \ldots \]

\[ P_{4,i,j} \equiv \text{The probability that there are } i \text{ service phases in the batch queue and } j \text{ service phases in individual queue when the server is in individual queue but found to be broken down, where } i = 0, k, 2k, \ldots \text{ and } j = 1, 2, 3, \ldots \]

The steady-state equations satisfied by the system size probabilities are as follows:

\[(3.1) \quad \lambda P_{0,0,0} = k\mu P_{3,0,1} \]
\[(3.2) \quad \lambda P_{0,i,0} = \lambda P_{0,i-k,0}, \quad i = k, 2k, 3k, \ldots, (N - 1)k. \]
\[(3.3) \quad (\lambda + \theta) P_{1,Nk,0} = \lambda P_{0,(N-1)k,0}. \]
\[(3.4) \quad (\lambda + \theta) P_{1,i,0} = \lambda P_{1,(i-k),0}, \quad i = (N + 1)k, (N + 2)k, (N + 3)k, \ldots \]
\[(3.5) \quad \beta P_{2,i,0} = k\mu P_{3,i,1}, \quad i = k, 2k, 3k, \ldots, (N - 1)k \]
\[(3.6) \quad \beta P_{2,i,0} = k\mu P_{3,i,1} + \theta P_{1,i,0}, \quad i = Nk, (N + 1)k, (N + 2)k, \ldots \]
\[(3.7) \quad (\lambda + \alpha + k\mu) P_{3,0,j} = k\mu P_{3,0,j+1} + \pi_0 \beta P_{2,j,0} + \gamma P_{4,0,j}, \quad j \geq 1 \]
\[(3.8) \quad (\lambda + \alpha + k\mu) P_{3,i,j} = k\mu P_{3,i,j+1} + \lambda P_{3,i-k,j} + \gamma P_{4,i,j} + \pi(i/k)\beta P_{2,j,0}, \quad i \geq k, j \geq 1 \]

\[(3.9) \quad (\lambda + \gamma) P_{4,0,j} = \alpha P_{3,0,j}, \quad j \geq 1. \]
\[(3.10) \quad (\lambda + \gamma) P_{4,i,j} = \alpha P_{3,i,j} + \lambda P_{4,i-k,j}, \quad i \geq k, j \geq 1 \]

where \( \pi(i/k) = \frac{\lambda^{(i/k)} \beta}{(\lambda + \beta)^{i+1}} \)
To determine the probability distribution of the number of customers in the system and hence the expected number of customers in the system the following probability generating functions are defined

\[ G_0(z) = \sum_{i=0}^{(N-1)k} P_{0,i,0} z^i, \]
\[ G_1(z) = \sum_{i=Nk}^{\infty} P_{1,i,0} z^i, \]
\[ G_2(z) = \sum_{i=k}^{\infty} P_{2,i,0} z^i, \]

\[ G_3(z, y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{3,i,j} z^i y^j, \]
\[ G_4(z, y) = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} P_{4,i,j} z^i y^j, \]
\[ R_j(z) = \sum_{i=k}^{\infty} P_{3,i,j} z^i, \]
\[ S_j(z) = \sum_{i=0}^{\infty} P_{4,i,j} z^i, \]

where \(|z| \leq 1\) and \(|y| \leq 1\).

Multiplication of equation (3.2), (3.3) and (3.4) on either side by \(z^i\) and considering summation over \(i = k, 2k, 3k, \ldots, (N-1)k\) and \(i = Nk, (N+1)k, (N+2)k, \ldots\) respectively gives rise to

\[ G_0(z) = \frac{(1-z^{Nk})}{(1-z^k)} P_{0,0,0} \]

\[ G_1(z) = \frac{\lambda z^{Nk}}{(\lambda(1-z^k)+\theta)} P_{0,0,0} \]

Multiplication of equations (3.5) and (3.6) by \(z^i\) and considering summation over \(i = k, 2k, 3k, \ldots\) yields

\[ G_2(z) = \frac{k\mu R_1(z) + \theta G_1(z) - \lambda P_{0,0,0}}{\beta} \]

Multiplying equations (3.8) and (3.10) by \(z^i\) and taking summation over \(i = 0, k, 2k, 3k, \ldots\) and using (3.7) and (3.9) finally yields equations:

\[ [\lambda y(1-z^k) + \alpha y + k\mu (y-1)] G_3(z, y) = \gamma y G_4(z, y) + \beta y \pi(z) G_2(y) - k\mu y R_1(z) \]

\[ G_4(z, y) = \frac{\alpha}{\lambda(1-z^k)+\gamma} G_3(z, y). \]

The total probability generating function \(G(z, y)\) is given by

\[ G(z, y) = G_0(z) + G_1(z) + G_2(z) + G_3(z, y) + G_4(z, y) \]

From equations (3.11) to (3.15)

\[ G_0(1) = NP_{0,0,0}, \]
\[ G_1(1) = (\lambda/\theta) P_{0,0,0}, \]
\[ G_2(1) = \frac{k\mu R_1(1)}{\beta}, \]
Optimal strategy analysis

\[ G_3(1, 1) = \frac{[\theta G'_1(1) + k\mu R_1(1)\pi'_1(1)]\gamma}{k[\mu\gamma - \lambda(\alpha + \gamma)]}, \]

(3.19)

and \( G_4(1, 1) = (\alpha/\gamma)G_3(1, 1), \)

(3.20)

where \( P_{0,0,0} = \left[1 - \frac{\lambda}{\mu} \left(1 + \frac{\alpha}{\gamma}\right) - \frac{\lambda}{\beta}\right] \frac{\theta}{(\lambda + N\theta)} \).

The normalizing condition

\[ G(1, 1) = G_0(1) + G_1(1) + G_2(1) + G_3(1, 1) + G_4(1, 1) = 1 \]

yields \( R_1(1) = \lambda/(k\mu) \) and using this in (3.18), (3.19) and (3.20), we get \( G_2(1) = (\lambda/\beta), G_3(1, 1) = (\lambda/\mu) \) and \( G_4(1, 1) = (\alpha/\gamma)(\lambda/\mu) \).

Under steady state conditions, let \( P_0, P_1, P_2, P_3 \) and \( P_4 \) be the probabilities that the server is in vacation, in startup, in batch service, in individual service and breakdown states respectively. Then,

\[ P_0 = G_0(1) = NP_0, P_1 = G_1(1) = (\lambda/\beta)P_0, P_2 = G_2(1) = (\lambda/\beta), P_3 = G_3(1, 1) = (\lambda/\mu) \text{ and } P_4(1, 1) = (\alpha/\gamma)(\lambda/\mu) \]

3.1. **Expected number of customers in the system.** Let \( L_0, L_1, L_2, L_3, \) and \( L_4 \) be the expected number of service phases in the system when the server is in vacation, in startup, in batch service, in individual service and breakdown states respectively. Then

\[ L_0 = \sum_{i=0}^{(N-1)k} iP_{0,i,0} = G'_0(1) = \frac{N(N-1)k}{2}P_{0,0,0}. \]

(3.22)

\[ L_1 = \sum_{i=Nk}^{\infty} iP_{1,i,0} = G'_1(1) = \frac{\lambda(\lambda + N\theta)}{\theta^2}P_{0,0,0}. \]

(3.23)

\[ L_2 = \sum_{i=k}^{\infty} iP_{2,i,0} = G'_2(1) = \frac{\lambda k}{\beta}. \]

\[ L_3 = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} (i + j)P_{3,i,j} = G'_3(1, 1) \]

\[ = \rho \left[ 1 + \left( \frac{k + 1}{2} \right) \frac{\rho_1}{1 - \rho_1} + \frac{\lambda k \rho}{\gamma^2(1 - \rho_1)} + \frac{\lambda k}{\beta(1 - \rho_1)} + \frac{\lambda(k - 1)}{2}\beta(1 - \rho_1) + \frac{\lambda^2 k}{\beta^2(1 - \rho_1)} \right] \frac{1}{1 - \rho_1} \]

\[ + \frac{1}{\theta^2} \left( \frac{\lambda(\lambda + N\theta)k}{\theta^2} + \frac{N(Nk - 1)}{2} + \left( \frac{k - 1}{2} \right) \frac{\lambda}{\theta} \right) P_{0,0,0} \]

(3.25)

\[ L_4 = \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} (i + j)P_{4,i,j} = G'_4(1, 1) = \frac{\alpha}{\gamma} \left[ G'_3(1, 1) + \frac{\lambda k \rho}{\gamma} \right]. \]

(3.26)

Expected number of service phases in the system is given by

\[ L(P) = L_0 + L_1 + L_2 + L_3 + L_4 \]
\[ L(N) = \frac{N(N-1)k}{2} P_{0,0,0} + \frac{N(Nk-1)\rho_1}{2(1-\rho_1)} P_{0,0,0} + \frac{\lambda(\lambda + N\theta)k}{\theta^2(1-\rho_1)} P_{0,0,0} + \rho_1 \left[ 1 + \frac{\rho_1(k+1)}{2(1-\rho_1)} \right] \]

\[ (3.27) \]

\[ \rho = \frac{\lambda}{\mu} \text{ and } \rho_1 = \frac{\lambda}{\mu}(1 + \alpha/\gamma). \]

Expected number of customers in the system is given by

\[ (3.28) \]

3.2. Characteristic features of the system. Let \( E_0, E_1, E_2, E_3 \) and \( E_4 \) denote the expected length of vacation period, startup period, batch service period, individual service period and breakdown period respectively. Then the expected length of a cycle is given by

\[ (3.29) \]

The long run fractions of time the server is in vacation, in startup, in batch service, in individual service and breakdown states are respectively given by

\[ (3.30) \]

\[ (3.31) \]

\[ (3.32) \]

\[ (3.33) \]

\[ (3.34) \]

\[ \text{and } E_4/E_C = P_4 = \rho(\alpha/\gamma). \]

Expected length of vacation period \( E_0 = N/\lambda. \)

Substituting this in equation (3.30).

\[ (3.35) \]

4. Determination of the optimal policy

We develop a steady state total expected cost function per unit time for the \( N \)-policy two phase \( M/E_k/1 \) queueing system with server startup and breakdowns, in which \( N \) is a decision variable. With the cost structure being constructed, the objective is to determine the optimal operating \( N \)-policy so as to minimize this function. Let

\[ C_h \equiv \text{holding cost per unit time for each customer present in the system}, \]

\[ C_o \equiv \text{cost per unit for keeping the server on and in operation}, \]

\[ C_m \equiv \text{startup cost per unit time per cycle}, \]

\[ C_s \equiv \text{setup cost per cycle}, \]

\[ C_b \equiv \text{breakdown cost per unit time}, \]
$C_r \equiv$ reward per unit time for the server being on vacation.

The total expected cost function per unit time is given by

\begin{equation}
T(N) = C_h L(N) + C_0 \left( \frac{E_2 + E_3}{E_C} \right) + C_m \left( \frac{E_4}{E_C} \right) + C_b \left( \frac{E_4}{E_C} \right) + C_s \left( \frac{1}{E_C} \right) - C_r \left( \frac{E_0}{E_C} \right)
\end{equation}

From (3.32) to (3.34), it is observed that $E_2/E_C$, $E_3/E_C$ and $E_4/E_C$ are not a function of decision variable $N$. Hence for the determination of the optimal operating $N$-policy, minimizing $T(N)$ in (4.1) is equivalent to minimizing

\begin{equation}
T_1(N) = C_h L(N) + C_m \left( \frac{E_4}{E_C} \right) + C_s \left( \frac{1}{E_C} \right) - C_r \left( \frac{E_0}{E_C} \right)
\end{equation}

Differentiating $T_1(N)$ with respect to $N$ and setting the result to zero, we obtain the optimal value $N^*$ of $N$. Hence,

\begin{equation}
N^* = \left[ \sigma^2 + \frac{\sigma}{k} ((2k - 1) - (k - 1)\rho_1) + \frac{2\sigma(1 - \rho_1)}{C_h} (C_m + \theta C_s + C_r) \right]^{1/2} - \sigma,
\end{equation}

where $\sigma = \lambda/\theta$ (mean number of arrivals during startup times).

5. Sensitivity analysis

In the course of analysis, sensitivity analysis has been carried out on the optimum value $N^*$ based on changes in specific values of the system parameters. In order to arrive at the conclusions, the following arbitrary cost elements are considered.

Case 1: $C_h=5, C_0=100, C_m=300, C_b=125, C_r=25, C_s=500$
Case 2: $C_h=5, C_0=200, C_m=500, C_b=250, C_r=50, C_s=1250$
Case 3: $C_h=5, C_0=400, C_m=800, C_b=500, C_r=100, C_s=2500$
Case 4: $C_h=10, C_0=400, C_m=800, C_b=500, C_r=100, C_s=2500$
Case 5: $C_h=50, C_0=400, C_m=800, C_b=500, C_r=200, C_s=2500$

The optimal value of $N$, $N^*$ and its minimum expected cost $T(N^*)$ for the above five cases are shown in Table 1 for $(\mu, \beta, \alpha, \gamma, k)=(3.5, 5, 0.1, 4, 3)$ and for various values of $(\lambda, \theta)$. We observe from Table 1 that (i) $N^*$ and $T(N^*)$ increase as $\lambda$ increases in every case, (ii) $N^*$ shows decreasing trend when $\theta$ changes from 2 to 4 for cases 1, 2, 3 and insensitive for cases 4 and 5, (iii) $T(N^*)$ decreases as $\theta$ changes from 2 to 4 in all five cases.

The optimal value of $N$, $N^*$ and its minimum expected cost $T(N^*)$ for the five cost cases are shown in Table 2 for $(\lambda, \theta, \beta, \alpha, k)=(0.6, 4, 5, 0.1, 3)$ and for various values of $(\mu, \gamma)$.

From Table 2 we find that (i) $N^*$ shows an increasing trend and $T(N^*)$ decreases as $\mu$ increases from 1.5 to 3.5 in every case, $N^*$ does not change at all and $T(N^*)$ decreases as $\gamma$ increases from 2 to 5.

The optimal value of $N$, $N^*$ and minimum expected cost $T(N^*)$ for the five cost cases are shown in Table 3 for $(\lambda, \theta, \mu, \gamma, k)=(0.6, 4, 2, 4, 3)$ and for various of $(\beta, \alpha)$. One observes from Table 3 that (i) $N^*$ is insensitive and $T(N^*)$
decreases as $\beta$ increases from 3 to 7 in every case, (ii) $N^*$ is insensitive and $T(N^*)$ increases as $\alpha$ increases from 0.05 to 0.15.

The optimal value of $N, N^*$ and its minimum expected cost $T(N^*)$ for the five cost cases are shown in Table 4 for $(\lambda, \theta, \mu, \beta, \alpha, \gamma) = (0.6, 1, 2, 5, 0.05, 4)$ and for different values of $k$. From Table 4 we observe that (i) $N^*$ is insensitive in every case and $T(N^*)$ increases marginally as $k$ increases for any case.

Overall we conclude that

- $\alpha, \beta, k,$ and $\gamma$ do not effect $N^*$
- $\theta$ rarely affects $N^*$
- $\lambda$ affects $N^*$ significantly
- $C_h$ and $C_s$ have much stronger effect on $N^*$ than $\lambda, \theta, \mu$ and $\alpha$.

Table 1. The optimal value of $N$ and its minimum expected cost for $(\mu, \beta, \alpha, \gamma, k) = (3.5, 5, 0.1, 4, 3)$

<table>
<thead>
<tr>
<th>$(\lambda, \theta)$</th>
<th>(0.4,4)</th>
<th>(0.6,4)</th>
<th>(0.8,4)</th>
<th>(0.6,2)</th>
<th>(0.6,4)</th>
<th>(0.6,8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>$N^*$</td>
<td>8</td>
<td>10</td>
<td>11</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>40.16</td>
<td>57.80</td>
<td>72.56</td>
<td>60.87</td>
<td>57.80</td>
</tr>
<tr>
<td>Case2</td>
<td>$N^*$</td>
<td>13</td>
<td>16</td>
<td>18</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>62.20</td>
<td>94.70</td>
<td>122.61</td>
<td>98.21</td>
<td>94.78</td>
</tr>
<tr>
<td>Case3</td>
<td>$N^*$</td>
<td>19</td>
<td>23</td>
<td>25</td>
<td>23</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>87.87</td>
<td>147.31</td>
<td>200.76</td>
<td>151.41</td>
<td>147.31</td>
</tr>
<tr>
<td>Case4</td>
<td>$N^*$</td>
<td>13</td>
<td>16</td>
<td>18</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>123.18</td>
<td>188.26</td>
<td>243.89</td>
<td>194.13</td>
<td>180.26</td>
</tr>
<tr>
<td>Case5</td>
<td>$N^*$</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>192.00</td>
<td>291.77</td>
<td>370.30</td>
<td>306.73</td>
<td>291.77</td>
</tr>
</tbody>
</table>

Table 2. The optimal value of $N$ and its minimum expected cost for $(\lambda, \theta, \beta, \alpha, k) = (0.6, 4, 5, 0.1, 3)$

<table>
<thead>
<tr>
<th>$(\mu, \gamma)$</th>
<th>(1.5,3)</th>
<th>(2.5,3)</th>
<th>(3.5,3)</th>
<th>(2.5,2)</th>
<th>(2.5,4)</th>
<th>(2.5,5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>$N^*$</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>80.17</td>
<td>64.48</td>
<td>57.80</td>
<td>64.91</td>
<td>64.33</td>
</tr>
<tr>
<td>Case2</td>
<td>$N^*$</td>
<td>13</td>
<td>15</td>
<td>16</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>141.26</td>
<td>100.93</td>
<td>94.78</td>
<td>100.43</td>
<td>108.73</td>
</tr>
<tr>
<td>Case3</td>
<td>$N^*$</td>
<td>19</td>
<td>22</td>
<td>23</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>246.57</td>
<td>177.40</td>
<td>147.31</td>
<td>180.65</td>
<td>177.40</td>
</tr>
<tr>
<td>Case4</td>
<td>$N^*$</td>
<td>13</td>
<td>15</td>
<td>16</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>281.44</td>
<td>216.61</td>
<td>188.26</td>
<td>219.61</td>
<td>216.61</td>
</tr>
<tr>
<td>Case5</td>
<td>$N^*$</td>
<td>6</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$T(N^*)$</td>
<td>391.01</td>
<td>324.37</td>
<td>291.77</td>
<td>327.51</td>
<td>324.37</td>
</tr>
</tbody>
</table>
Optimal strategy analysis of $N$-policy twp-phase gated $M/E_k/1$ queuing system with startup times and server breakdowns is analysed. System performance measures are derived in explicit form by using generating functions approach. A cost function is formulated to determine the optimal value of $N$. Furthermore, we have performed sensitivity analysis for the optimal value of $N$ with various system parameter values and cost elements. These numerical values may be helpful to improve the grade of service by selecting appropriate system descriptors.

6. Conclusions
REFERENCES


Received: April, 2010