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Abstract

A two-dimension numerical model for pulsed Laser Transformation Hardening (LTH) is developed using the Finite Differences Method (FDM). In this model, the starting temperature of martensite formation is taken into account as a crucial factor in martensite formation. The numerical results are compared to analytical solution and numerical solution of the previous study [2], showing a good agreement. Two experiments are carried out varying two input variables namely, heating duration and laser beam intensity, to investigate their impact on the temperature distribution and martensitic transformation. The results of these experiments will be explained more detail in this paper.

Keywords: Laser Transformation Hardening, Finite Differences Method, Starting temperature of martensite formation; Heating duration; Laser beam intensity.
1. Introduction

There has been increasing interest in the use of lasers in manufacturing applications. The majority of these applications involve surface treatments, in which a surface of the metal is modified in order to improve certain properties. Laser Transformation Hardening (LTH) is an advanced heat treatment method to obtain hard, wear resistant layer on the surface without affecting the bulk material through the interaction of laser and material. LTH can offer distinct advantages in terms of hardness, distortion, treatment speed, and accuracy in comparison to many conventional processes [14, 15].

In LTH processing, a high intensity of laser light is directed to the metal’s surface and it will induce phase transformation inside the metal. The layer on the surface which has characteristics like mentioned before is accomplished by changing the base structure into a harder martensitic phase. In addition, due to its high level of focused energy, only the surface of the metal typically experiences high temperatures. Thus, the process requires no quenching medium, as the piece itself quickly absorbs the bulk of the heat through conduction. Nevertheless, the conduction of heat transfer through the work piece becomes important information in determining how laser surface heating techniques are applied [2].

Knowledge of the temperature distribution provides the prediction of heat-affected zone, phase composition and hardened layer depth. Several studies, either analytically or numerically, have been applied to modeling the laser conduction heating process [2-7]. From these studies, it is known that the heat conduction is more effective in the axial direction than to radial directions, which are only the nodes that directly underneath the laser beam irradiation that undergo high temperature. Though these studies could show good results, none of these models captures the effect of starting temperature of martensite formation in martensitic transformation. Whereas, when formulating the laser transformation hardening, the starting temperature of martensite formation becomes a crucial factor that can influences the simulation results [14, 15]. Therefore, it should be considered to achieve the results that as close possible as to the real condition of physical process that involved in this treatment.

In the present study, a numerical model for pulsed LTH is developed using the Finite Difference Method (FDM). This model has a capability to calculate the starting temperature of martensite formation and the thermal energy generation (some literature calls it as internal heat source) inside the material. This energy form appears when the phase transformation takes place. A series tests are carried out varying two of the operational parameters of LTH process namely, laser beam intensity and heating duration; the impact on the temperature distribution in axial direction is investigated. For the validation, the results of this model are compared with the analytical and numerical results of previous studies [2, 6].
2. Hardening Process Induced by Laser Irradiation

The main purpose of hardening lies in subjecting steel to undergo martensitic transformation. This transformation is determined by the temperature profile and cooling rate, which is occurred in three main steps: austenization, homogenization, and quenching [2, 14, 15]. At room temperature, the steel, which is mainly the mixture of ferrite and pearlite [14, 15], is heated to austenization temperature and below the melting point. The austenization temperature, $T_a$, is dependent on the type of steel alloy. In plain carbon steel, $T_a$ is approximately $725^\circ$C [9, 15]. In this heating course, the base structure of steel will transform into austenite.

![Fig.1. Schematic diagrams of phase transformations when cooling][18]

Typically, a very specific area of the metal work piece is desired to undergo martensitic transformation. It is therefore important to ensure that all of this area (or volume) has reached austenization temperature in a homogenous fashion, and then cooled together [2]. When laser pulse does not irradiate, the austenite will rapidly cool down due to surrounding material acting as an efficient heat sink (self quenching). The rate of cooling should be controlled to avoid transformations of austenite to other phase such as pearlite or bainite. The cooling rate should be fast enough for quenching to take place that will enable the transform of austenite into martensite, which is it should be higher of about 1000°C/s (In Fig.1, $v > v_1$) [14]. This martensitic reaction begins during cooling when the austenite reaches the martensite start temperature ($M_s$) and the parent austenite becomes mechanically unstable [14]. At constant temperature below $M_s$, a fraction of the parent austenite transforms rapidly, and then no further transformation will occurs. When the temperature decreases, more of the austenite...
transform to martensite. Finally, when the martensite finish temperature ($M_f$) is reached, the transformation is complete and 100% martensite is formed.

3. Numerical model

Heat conduction plays an important role in laser surface hardening process. The relaxation time in metals is of the order of $10^{-13}$ s and the laser pulse duration used in this paper is of the order of $10^{-2}$ s. Hence, the classical Fourier’s law of heat conduction can still describe this heating process [1]. Several simplifications are made in order to arrive at a solution that can appropriately define the temperature distribution. The following simplifications are made in Von Allmen’s and Roger’s works [2,13]:

1. The power density of heat production equals the absorbed light
2. Material properties (see Table 1) are taken to be constant
3. The initial temperature is assumed equal to the environment’s temperature, and the slab is thermally insulated
4. The laser beam is cylindrical and perpendicular to the slab.

The heat transfer equation for a laser heating pulse in 2-D domain (see Figure 2) can be written from the relationship between temperature, internal energy and enthalpy as follow:

$$\frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial z^2} + 4 \pi I_a = \frac{\partial H}{\partial T}$$  \hspace{1cm} (3.1)

where $U = \frac{1}{2} T$ is defined as internal energy, $H = \rho c T$ is defined as enthalpy, $T$ is defined as temperature, $r$ is the radial direction (normal to the laser beam axis) and $z$ is the axial direction (along the laser beam axis). $I_a$ is defined as the heat distribution through the depth of the metal slab [3-6]:

$$I_a = \rho I_p (1 - r_p) e^{-r_p}$$  \hspace{1cm} (3.2)

Fig. 2. Nodal network (mesh) [9]
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Where $I_o$ is the initial laser power intensity, $\varrho$ is the absorption depth, $r_f$ is the reflection coefficient (assumed to be 0 in this analysis). The explicit FDE scheme for Eq. 3.1 in 2D domain in the FTCS form is written as:

$$\frac{H_{ij}^{n+1} - H_{ij}^n}{\Delta t} = \left[ \frac{U_{i+1,j}^n - 2U_{i,j}^n + U_{i-1,j}^n}{(\Delta z)^2} + \frac{U_{i,j+1}^n - 2U_{i,j}^n + U_{i,j-1}^n}{(\Delta r)^2} \right] + I_o + \varrho \quad (3.3)$$

with T.E of $O(\Delta t, \Delta z^2, \Delta r^2)$. Where $n$ is the current time step and $n+1$ is the next time step. The latent heat per time unit, expressed as the internal heat source in the heat conduction equation is usually calculated by the following equation [14]:

$$\varrho = \Delta H \frac{\Delta V}{\Delta t} \quad (3.4)$$

where $\Delta V$ is the change of transformation fraction in the time step $\Delta t$. By substituting Eq. 3.4 into Eq. 3.3 and rearrange it, the value of current enthalpy can be calculated as follows:

$$H_{ij}^{n+1} = H_{ij}^n + \frac{\Delta V}{(1 - \Delta V)} \left[ \frac{U_{i+1,j}^n - 2U_{i,j}^n + U_{i-1,j}^n}{(\Delta z)^2} + \frac{U_{i,j+1}^n - 2U_{i,j}^n + U_{i,j-1}^n}{(\Delta r)^2} \right] + I_o \quad (3.5)$$

There are six possibilities that modeled in this analysis for two successive states in a step of temperature varying from $T_{ij}$ to $T^*_{ij}$ involving the possibility of steel to melt during the heating process as shown in Figure 3. The corresponding transformation fraction values of the five cases for melting and solidification can be determined as follows:

$$\Delta V = \begin{cases} 
0 & \text{Condition (1)}, \\
\frac{T_i - T_x}{T_i - T_x} & \text{Condition (2)}, \\
\frac{T_i - T_{i-1}}{T_i - T_x} & \text{Condition (3)}, \\
\frac{T_x - T_{i-1}}{T_i - T_x} & \text{Condition (4)}, \\
0 & \text{Condition (5)}, 
\end{cases} \quad (3.6)$$

where $T_i$ is the temperature of current time and $T_{i-1}$ is the temperature of previous time. The change of volume fraction in martensite transformation region is calculated from the empirical equation proposed by Koistenen and Marburger [16]:

$$V = 1 - e^{-0.011[\mu - T_i]} \quad (3.7)$$

So the change of volume fraction when temperature varying from $T_{ij}$ to $T^*_{ij}$ can be represented as follow:

$$\Delta V = \left[ 1 - e^{-0.011[\mu - T_{ij}]} \right] - \left[ 1 - e^{-0.011[\mu - T^*_{ij}]} \right] \quad (3.8)$$
where $M_s$ is the starting temperature of martensite formation. The temperature and internal energy value is calculated by using relationship between their values with the enthalpy in every time step.

$$T(z, r, 0) = T_{\text{environment}} \quad \forall (z, r, 0) = H_{\text{environment}} \quad 0 \leq (z, r) \leq (z_{\text{max}}, r_{\text{max}}) \quad (3.9)$$

At the top of surface ($z = 0$):

The laser irradiates in this surface, and then pulsing of the laser is achieved in MATLAB by creating a switch,”SWITCH” (an on/off for the laser beam), that is multiplied by $I_{\text{Total}}$ term that describes the total absorbed intensity of the laser [2, 6]. This switch is then turned on and off at specific times by setting it equal to “1 (on)” or “0 (off)”.

$$I_{\text{Total}} = E_{\text{Flux}} \times \text{SWITCH} - Q_{\text{conv}} - Q_{\text{rad}} \quad r \leq r_2$$

Where,

$$E_{\text{Flux}} = \frac{I_0 (1 - r_2)}{\pi r_2^2} \quad (3.11)$$

$$Q_{\text{conv}} = \frac{M(T_{\text{ef}}^n - T_{\text{environment}})}{\text{conv}^2} \quad (3.12)$$

$$Q_{\text{rad}} = \frac{\text{rad} (T_{\text{ef}}^n)^4 - T_{\text{environment}}^4}{\text{rad}^2} \quad (3.13)$$

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**Fig.3. Six conditions of temperature**
$Q_{conv}$ and $Q_{rad}$ represent the heat lose due to convection and radiation respectively. However, for experiments carried out in this analysis, both the heat loses via convection and radiation are neglected, and $r_b$ is the beam radius. The finite difference equation for nodes at top of surface can be presented based on the average value between nodes at $i=1$ and $i=2$, presented as follow:

$$H_{z+1}^{n+1} = H_{z}^{n} + \frac{\Delta z}{\Delta v} \left[ \frac{U_{z+1/2}^{n} - U_{z+1/2}^{n}}{\Delta z} + l_{total} \right]$$

at the centerline ($r = r_0$):

$$T(z_n, r_n, t) = T(z, r_n + \Delta r_n, t); U(z_n, r_n, t) = U(z, r_n + \Delta r_n, t);$$

$$H(z_n, r_n, t) = H(z, r_n + \Delta r_n, t)$$

for $z_b \leq z \leq z_{max}$ and $t > 0$ (3.14)

$$T(z_n, r_n, t) = T_{initial}; U(z_n, r_n, t) = U_{initial}; H(z_n, r_n, t) = H_{initial}$$

at $z = z_{max}$ and $r = r_{max}$:

$$T(z_n, r_n, t) = T_{initial}; U(z_n, r_n, t) = U_{initial}; H(z_n, r_n, t) = H_{initial}$$

The stability of Eq. 3.5 and 3.14 can be achieved by Von Neumann [7, 8] analysis and taken form as follow:

$$\alpha = \left[ \frac{1}{\Delta t} \frac{1}{\Delta z^2} \right]^{\frac{1}{2}} \leq \frac{1}{2}$$

The following physical properties are used in this analysis. They are app properties of standard carbon steel in Table 1 [14, 15].

### Table 1. Physical properties of steel.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal conductivity</td>
<td>$k$</td>
<td>25</td>
<td>W/m$^0$C</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>8000</td>
<td>Kg/m$^3$</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$c_p$</td>
<td>400</td>
<td>J/kg$^0$C</td>
</tr>
<tr>
<td>Solidus temperature</td>
<td>$T_s$</td>
<td>1500</td>
<td>$^0$C</td>
</tr>
<tr>
<td>Liquidus temperature</td>
<td>$T_l$</td>
<td>1525</td>
<td>$^0$C</td>
</tr>
<tr>
<td>Latent heat of fusion</td>
<td>$H_m$</td>
<td>$1.93 \times 10^9$</td>
<td>J/m$^3$</td>
</tr>
<tr>
<td>Absorption coefficient</td>
<td>$\alpha$</td>
<td>$1 \times 10^5$</td>
<td>m$^{-1}$</td>
</tr>
<tr>
<td>Staring temperature of Martensite</td>
<td>$M_s$</td>
<td>350</td>
<td>$^0$C</td>
</tr>
<tr>
<td>Austenization temperature</td>
<td>$T_A$</td>
<td>750</td>
<td>$^0$C</td>
</tr>
</tbody>
</table>
Two experiments are performed with a single pulse and summarized collectively in Table 2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Heating Duration, sec</th>
<th>Thickness, m</th>
<th>Laser intensity, W</th>
<th>Laser diameter, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05; 0.1; 0.15; 0.2</td>
<td>0.005</td>
<td>300</td>
<td>0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.05</td>
<td>0.005</td>
<td>300; 600; 900</td>
<td>0.002</td>
</tr>
</tbody>
</table>

5. Results and discussions

The first step should be done after developing a numerical solution is to check its compatibility with the analytical solution. In order to be acceptable, any numerical solution needs to meet the condition of convergence. It is said to satisfy the condition of convergence if it meets the condition of consistency and stability [7, 8]. Figure 4 shows temperature profile of centerline surface obtained from the new numerical solution that is compared with the analytical and numerical solutions from the previous studies [2, 6].

It was found that the present numerical model is more convergence than the previous model due to its better accuracy when the mesh size was increased. In addition, it was discovered that the mesh size of 61 x 21 is the best size that produces minimum total error. Although there are still errors in heating and cooling portions, the temperature profile obtained from the present numerical model has better compatibility with the analytical solution than the previous model, at least until the starting temperature of martensite formation (\(M_s = 350^\circ C\)) is reached, which is a crucial part in the martensitic transformation.

In the first experiment, the heating duration is varied (see Table 2) in order to investigate its influences to the temperature profile and the martensite
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formation inside the steel slab. Figure 5 illustrates the temperature profile of the centerline surface for different heating durations with single pulse. It can be seen there that the maximum temperatures increase with increasing the heating duration namely, 690°C for the duration of 0.05 seconds, 930°C for the duration of 0.1 seconds, 1100°C for the duration of 0.15 seconds, and 1224°C for the duration of 0.2 seconds. When the duration increase, the heat transfer via conduction from the surface to the deeper thickness enhances, and eventually increasing the possibility of austenite to be formed.

Figure 6 illustrates temporal variation of time derivative of temperature at the surface of centerline for different heating durations. It is interesting to note that, in the beginning of heating, the temperature of each heating duration rises rapidly with same rate namely, around 60000°C/s, and then the temperature rise reduces slightly with the same rate. In the beginning of cooling period, the cooling rates rise with increasing the heating duration namely, around of 56000°C/s for the duration of 0.05 s, around of 59000°C/s for the duration of 0.1 s, around of 60000°C/s for the duration of 0.15 s, and around of 60700°C/s for the duration of 0.2 s. Then as the cooling progresses, the cooling rates reduce slightly with different rates for each duration. This condition gives different possibilities of martensite formation in different heating duration that used.

Figure 7 illustrates the cooling rate values of different heating duration when temperature cools below the starting temperature of martensite formation (M_s). It can be seen there that the starting temperature of martensite formation is achieved in different times with different value of cooling rates due to the different heating durations that used. From this result, it can be known that the rate of the temperature decrease after the pulse is off reduces with increasing of the heating duration. However, as long as the cooling rate remains above the critical cooling rate of martensite formation (1000°C/s) when the temperature is in
the range of martensite formation temperature, the possibility of martensite formation after austenite formation always exist.

In this case, the martensite is formed only for durations of 0.1 s, 0.15 s, and 0.2 s due to the maximum that above the austenization temperature when the steel was heated. Figure 8 illustrates the martensite formation for different heating durations. From these figures, the heating duration of 0.2 s produces the deepest depth of martensite than the other durations. The depth of martensitic formation is only plotted as a function of the axial node due to the discretization effect. In reality, martensite may be forming in between two nodes. The discretization effect happens for heating duration of 0.1s, where martensite is formed between two nodes. That is why it is not appeared in the graph.
The second experiment is carried out varying the laser beam intensity in order to investigate its influences to the temperature and martensite formation inside the steel slab. Figure 9 illustrates the surface temperature of centerline with different laser beam intensities. Firstly, the steel is heated with laser beam’s intensity of 300 W for heating duration of 0.05 s, and as shown in Figure 9, it produces the maximum temperature of 670°C. Then the power of laser beam is increased to 600 W and then to 900 W with the same heating duration, causes increasing of the maximum temperature until 1325°C and 1910°C, respectively. From this result, it is known that the steel slab gets higher internal energy gain from the laser beam irradiation when its intensity increases, which also enhances the heat transport via conduction to deeper depths (axial direction) and eventually increases the austenite formation.

The magnitude of laser beam’s intensity gives a significant effect to the temperature rise. Figure 10 illustrates the temporal variation of time derivative of
surface temperature at the centerline for various laser beam’s intensity. It can be seen there that the laser beam’s intensity gives a significant effect to the temperature profile, which is shown in term of the rate of temperature rise in the beginning of heating period. The temperature rises with rate of around 60000°C/s for laser beam’s intensity of 300 W, and then the rate of temperature raises increase rapidly to around 12000°C/s and 18000°C/s when the laser beam’s intensity is increased to 600 W and 900 W, respectively, which are almost two and three times higher than the first one. Then, when the heating progresses, the rate of temperature rise reduces slightly with different rates, in which a higher laser beam’s intensity produces a magnitude of heating rate that higher than others. It indicates the higher heat transfer via conduction to deeper depths.

However, the rapid increase of temperature can causes a higher possibility of steel to melt due to the high base of temperature that produced in the beginning of heating period. Therefore, in an actual experimentation, the maximum temperature should be controlled with choosing a laser beam’s intensity that appropriate to the condition of treatment to avoid the melting condition. In this case, the melting occurs when the laser beam’s intensity is 900 W. In the melting part, the magnitude of time derivative of temperature drops to zero, that is when the temperature reaches 1500°C, and then increases slightly when the temperature towards to 1525°C. After the temperature exceeds 1525°C, its value continues to increase rapidly until above around 10000°C/s; the opposite also occurs in the cooling period.

Figure 11 illustrates the surface temperature and cooling rate values at centerline for different laser beam’s intensity when temperature cools below the starting temperature of martensite formation (M_s). It can be seen from Figure 10 that after the heating process, the cooling rate of each laser beam’s intensity reduced slightly with different rate. It eventually produces different possibility of martensite formation when the temperature cools toward the starting temperature of martensite formation. It is shown in Figure 11 (right) that the cooling rate reduces with increasing of the laser beam’s intensity.

The martensite only formed for the laser beam’s power of 600W and
900W due to their maximum temperature that exceeds the austenization temperature in heating process. Figure 12 shows the martensite formation of each laser beam’s power. It can be seen there that the same martensite depths like produced in the first experiment are also produced here, but in this case the melting at the depth of 0.00025m occurs when the laser beam’s power of 900 W used.

A part of steel that melted undergoes phase transformation from solid to liquid. In this transformation process, there is an extra energy (thermal energy generation) as a manifestation of laser energy conversion that stored inside the steel when heating and released when cooling. Figure 13 shows the effect of the calculation of thermal energy generation in the heating and cooling cycles respectively, which is compared with a case without it. The greater heating and cooling rates are attained when these absorbed and released energies are taken into account in the fusion region (a temperature range between 1500- 1525 °C). From these two experiments, the martensite and melting are only occurred in the nodes that directly underneath the laser beam irradiation (laser diameter= 0.002 m). This indicates that the localized hardening treatment can be accomplished by using laser as the heat source.
6. Conclusion

A new two-dimensional model with better accuracy has been developed for simulating laser transformation hardening process inside a steel slab. This model has a capability to consider the starting temperature of martensite formation in martensitic transformation, which is a crucial factor that should be taken into account. In addition, the possibility of steel to melt during the heat treatment was also well captured with considering the existing of thermal energy generation. The greater heating and cooling rates are attained when these absorbed and released energies are taken into account in the fusion region.

From the two experiments that have been done using this model can be known that the higher heating duration and laser beam intensity will increase the maximum temperature as well as the martensite formation inside the steel. Laser beam intensity has more significant effects to the temperature change than the heating duration. However, these parameters must be properly controlled to achieve desired depth and to avoid the melting condition during the heating process. In addition, the higher maximum temperature reduces the cooling rate in the temperature range of martensitic formation, which can make percentage of martensite formation become less.

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References


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