On the Solution of a Special Type of Large Scale Linear Fractional Multiple Objective Programming Problems with Uncertain Data

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Abstract

This paper is concerned with solving large scale linear fractional multiple objective programming (LSLFMOP) problems with chance constraints. Chance constraints involve random parameters in the right-hand sides. These random right-hand sides are considered to be statistically independent random variable. The main features of the proposed solution procedure are based on chance-constrained technique, Charens and Cooper transformation and the Rosen's partitioning procedure. An illustrative numerical example is given to clarify the main results developed in the paper.

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(1) Introduction

Fractional programming has attracted the attention of many researchers in the past. The main reason for interest in fractional programming stems from the fact that programming models could better fit the real problems if we consider optimization of ratio between the physical and/or economic quantities. Literature survey reveals wide applications of fractional programming in different areas ranging from engineering to economics. In real world decision situations, when formulating a LSLFMOP problem, some or all of the parameters of the optimization problem are described by stochastic (or random or probabilistic) variables rather than by deterministic quantities. Most of LSLFMOP problems arising in applications have special structures that can be exploited.

There are many familiar structures for large scale optimization problems such as: (i) the block angular structure, and (ii) angular and dual- angular structure to the constraints, and several kinds of decomposition methods for linear and nonlinear programming problems with those structures have been proposed in [13, 14].

Recently a significant number of studies have indeed been reported on single and multiple objective fractional linear and nonlinear programming problems [1, 2, 3, 4, 5, 7, 8, 9, 11, 15, 16 17, 18, 19, 20, 21, 24, 25, 26].

Ammar [1] discussed Dinkelbach’s global optimization approach for finding the global maximum of the fractional programming problem. Based on this idea, the author gave several characterizations of the solution set of a convex–concave fractional programs.

Caballero and Hernández [2] introduced a new method to estimate the weakly efficient set for the multiobjective linear fractional programming problem. In another work, they [3] introduced a test to establish whether a linear fractional goal programming problem has solutions that verify all goals and, if so, how to find them by solving a linear programming problem. Also, they outline a technique for restoring efficiency based on a minimax philosophy.

A useful and new approach to solve fractional programming with absolute-value functions (FP-A) has been proposed by Chang [4]. Later on, Chang in [5] introduced an approximate approach to reaching as close as possible an optimal solution of the fractional programming problem with absolute value function (FP-A).

Chen [7] considers two popular inventory models: the continuous review and periodic review reorder-point, order-quantity, control systems. Specifically we present two procedures which determine optimal values for the two control parameters (i.e., reorder-point and order-quantity) when the holding-and-shortage costs are non-quasi-convex. The algorithms based on a fractional programming method.
Problems

Dutta et al. [8] developed a method for optimizing multiobjective linear fractional programming problem which yields always an efficient solution.

Gómez et al [9] present a linear fractional goal programming model to a timber harvest scheduling problem in order to obtain a balanced age class distribution of a forest plantation in Cuba.

Husain and Jabeen [11] derived necessary and sufficient optimality conditions for a continuous -time fractional minmax programming problem. In another work, Husain et al. [12] introduced necessary and sufficient optimality conditions for a nondifferentiable fractional minimax programming problem. Moreover, sufficient optimality conditions for a nonlinear multiple objective fractional programming problem involving $\eta$-semidifferentiable type I-preinvex and related functions have been derived by Mishra et al. [16].

Morita et al. [17] presented a probability maximization model of a stochastic linear knapsack problem where the random variables consist of several groups with mutually correlated ones. They proposed a solution algorithm to the equivalent nonlinear fractional programming problem with a simple ranking method. This approach is effectively applicable to one of the portfolio selection problems.

A goal programming (GP) procedure for fuzzy multiobjective linear fractional programming problems has been suggested by Pal et al. [18].

Preda [19] presented necessary and sufficient optimality conditions for a nonlinear fractional multiple objective programming problem involving $\eta$-semidifferentiable functions.

Ravi and Reddy [20] have modeled chemical process plant operations planning in an oil refinery as fuzzy linear fractional multiple goal programming problem. A variant of the fuzzy linear fractional goal programming model of Dutta et al. [8] has been developed and used in that study to solve the problem which has two fuzzy fractional goals and 22 crisp constraints.

Saad and Sharif [21] proposed a solution procedure to solve the chance-constrained integer linear fractional programming problem. Also, Saad and Abou-El-Enien [22] suggested a solution algorithm for integer linear fractional multiple objective programming problems (ILFMOP) with block angular structure of the constraints.

Singh et al. [23, 24] studied multiparametric sensitivity analysis for programming problems with linear-plus-linear fractional objective function using the concept of maximum volume in the tolerance region.

Smelyanskiy and Skedzielewski [25] describe the computational kernels that are the building blocks of the Interior Point Method (IPM), and they explain the different sources of parallelism in sparse parallel linear solvers, the dominant computation of IPM. Also, they analyze the scalability and performance of two important optimization workloads for solving large scale linear and quadratic programming problems.
Stancu-Minasian and Pop [26] pointed out certain shortcomings in the work of Dutta et al. [8] and give the correct proof of theorem which validates the obtaining of the efficient solutions.

In the present paper, a solution algorithm is suggested for the solution of LSLFMOP problems with chance constraints which has block angular structure of the constraints.

The paper is organized as follows: In the following section, the problem formulation of LSLFMOP with chance constraints (CHLSLFMOP). Chance constraints involve random parameters in the right-hand sides. These random right-hand sides are considered to be statistically independent random variable An algorithm is described in finite steps for solving the problem of concern is proposed in Section 3. For the sake of illustration, a numerical example is provided in Section 4. Finally, the paper is concluded in Section 5.

\( (2) \) Problem Formulation

Consider the following CHLSLFMOP problem with a block angular structure of the constraints as:

Maximize \([f_1(X), f_2(X), \ldots, f_k(X)]\) \hspace{1cm} (1-a) 

subject to

\[
P\left\{ \sum_{j=1}^{q} \sum_{i=1}^{n} a_{ijh_o} x_{ijh_o} \leq v_{h_o} \right\} \geq \alpha_{h_o}, \quad h_o=1,2,\ldots,m_o, \quad (1-b) 
\]

\[
P\left\{ \sum_{j=1}^{n} b_{ijh} x_{ijh} \leq v_{h_j} \right\} \geq \alpha_{h_j}, \quad h_j=m_{j-1}+1, m_{j-1}+2,\ldots,m_j, \quad (1-c) 
\]

\[
X_{ij} \geq 0, \quad i \in N, \quad j=1,2,\ldots,q, \quad q>1 \} \quad (1-d) 
\]

where the \(i^{th}\) objective function can be written as follows:

\[
f_i(X) = \frac{\sum_{j=1}^{q} f_g(X) \sum_{j=1}^{q} C_{ij} X_j + \gamma_i}{\sum_{j=1}^{q} D_{ij} X_j + \beta}, \quad i=1,2,\ldots,k 
\]

(2)

and

\[
\gamma_i, \quad a, \quad b \text{ and } \beta \text{ are constants,} \quad i=1,2,\ldots,k; 
\]

\(k\) : the number of objective functions,

\(q\) : the number of subproblems,

\(m\) : the number of constraints,

\(n\) : the number of variables,

\(n_j\) : the number of variables of the \(j^{th}\) subproblem, \(j=1,2,\ldots,q, \quad q>1, \)

\(m_o\) : the number of the common constraints represented by
Linear fractional multiple objective programming

\[ \sum_{j=1}^{q} \sum_{i=1}^{n} a_{ijh} x_{ijh} \leq V_{h_j}, \]

\( m_j \): the number of independent constraints of the \( j^{\text{th}} \) subproblem represented by

\[ \sum_{i=1}^{n} b_{ijh} x_{ijh} \leq V_{h_j}, j=1,2,\ldots,q, q>1 \]

\( X \): an \( n \)-dimensional column vector of variables,

\( X_j \): an \( n_j \)-dimensional column vector of variables for the \( j^{\text{th}} \) subproblem,

\( j=1,2,\ldots,q, q>1 \),

\( C_{ij} \): an \( n_j \)-dimensional row vector for the \( j^{\text{th}} \) subproblem in the \( i^{\text{th}} \) objective function,

\( D_j \): an \( n_j \)-dimensional row vector for the \( j^{\text{th}} \) subproblem,

\( K = \{1,2,\ldots,k\}, \)

\( N = \{1,2,\ldots,n\}, \)

In addition, \( P \) means probability, \( \alpha_{h_j} \) and \( \alpha_{h_j} \) are a specified probability levels.

For the sake of simplicity, consider that the random parameters, \( V_{h_j} \) and \( V_{h_j} \) are distributed normally and independently of each other with known means \( E\{V_{h_j}\} \) and \( E\{V_{h_j}\} \) and variances \( Var\{V_{h_j}\} \) and \( Var\{V_{h_j}\} \).

Furthermore, we assume that \( \sum_{j=1}^{q} D_j X_j + \beta \) is everywhere positive.

Using the chance constrained programming technique \([6, 10]\), the deterministic version of the CHLSMOP problem (1) can be written as follows:

Maximize \([f_1(X), f_2(X),\ldots,f_k(X)]\)

subject to

\[ \sum_{j=1}^{q} \sum_{i=1}^{n} a_{ijh} x_{ijh} \leq E\{V_{h_j}\} + k_{\alpha_{h_j}} \sqrt{Var\{V_{h_j}\}}, h_j = 1,2,\ldots,m_j, \]

(3-b)

\[ \sum_{j=1}^{q} b_{ijh} x_{ijh} \leq E\{V_{h_j}\} + k_{\alpha_{h_j}} \sqrt{Var\{V_{h_j}\}}, h_j = m_{j-1}+1, m_{j-1}+2,\ldots,m_j, \]

(3-c)

\[ x_{ij} \geq 0, i \in N, j=1,2,\ldots,q, q>1 \} \]  \quad (3-d)

where \( k_{\alpha_{h_j}}, j=0,1,2,\ldots,q, \) is the standard normal value such that \( \Phi(k_{\alpha_{h_j}}) = 1- \alpha \), \( j=0,1,\ldots,q, \) and \( \Phi \) represents the cumulative distribution function of the standard normal distribution.
Problem (3) can be treated using the nonnegative weighted sum approach [13] and will be converted to the following problem with a single-objective function as:

\[
\text{Maximize } \sum_{i=1}^{k} w_i f_i(X) \\
\text{subject to } \\
\sum_{j=1}^{q} \sum_{i=1}^{n} a_{ijh_i} x_{ijh_i} \leq E\{ V_{h_i}\} + k_{\alpha h_i} \sqrt{\text{Var}\{ V_{h_i}\}}, \ h_i=1,2,\ldots,m_o, \quad (4-b) \\
\sum_{j=1}^{q} b_{ijh_j} x_{ijh_j} \leq E\{ V_{h_j}\} + k_{\beta h_j} \sqrt{\text{Var}\{ V_{h_j}\}}, \ h_j=m_j-1+1, m_j-1+2,\ldots,m_j, \quad (4-c) \\
X_{ij} \geq 0, \ i \in N, j=1,2,\ldots,q, q>1. \quad (4-d)
\]

where \( w_j \geq 0, (i=1,2,\ldots,k) \) and \( \sum_{i=1}^{k} w_i = 1 \).

Consequently, using Charnes-Cooper transformation method [6] by making the variable change:

\[
\mu = \frac{1}{\sum_{j=1}^{q} D_j X_j + \beta} \quad (5-a)
\]

with the additional variable changes

\[
Y_j = X_j \mu \quad j=1,2,\ldots,q, q>1 \quad (5-b)
\]

then under these changes, problem (4) is equivalent to the one of solving the following:

\[
\text{Maximize } \left[ \sum_{i=1}^{k} w_i \sum_{j=1}^{q} f_i(Y, \mu) \right] = \left[ \sum_{i=1}^{k} w_i \left( \sum_{j=1}^{q} C_{ij} Y_j + \gamma_i \mu \right) \right] \quad (6-a)
\]

subject to

\[
\sum_{j=1}^{q} A_j Y_j - (E\{ V_{h_i}\} + k_{\alpha h_i} \sqrt{\text{Var}\{ V_{h_i}\}}) \mu \leq 0, \ h_i=1,2,\ldots,m_o, \quad (6-b) \\
\sum_{j=1}^{q} D_j Y_j + \beta \mu = 1 \quad (6-c)
\]

\[
B_j Y_j - (E\{ V_{h_j}\} + k_{\beta h_j} \sqrt{\text{Var}\{ V_{h_j}\}}) \mu \leq 0, \ h_j=m_j-1+1, m_j-1+2,\ldots,m_j \quad (6-d) \\
\mu > 0, \ Y_j \geq 0, \ j=1,2,\ldots,q, q>1 \quad (6-e)
\]

Now, problem (6) has angular and dual-angular structure [14] and can be solved using Rosen's partitioning procedure [14] to find its optimal solution \( Y_j^* \) and \( \mu^* \).
(3) Solution Method

In this section we propose a solution method for solving the CHLSLFMOP problem (1). The proposed algorithm in this paper can be summarized in finite steps as follows:

Solution Algorithm:

Step 1. Formulate CHLSLFMOP problem (1) which have fractional linear objective functions as in Eq. (2).

Step 2. Use the standard normal distribution table [10] to find $k_j$, $j=0,1,2,\ldots,q$.

Step 3. Transform problem (1) to the form of problem (3).

Step 4. Use the nonnegative weighted sum approach [13] to convert problem (3) to problem (4).

Step 5. Use the Charnes-Cooper transformation [6] by making the variable change

$$\mu = \frac{1}{\sum_{j=1}^{q} D_j X_j + \beta}$$

with the additional variable change

$$Y_j = X_j \mu \quad j=1,2,\ldots,q, q>1$$

to rewrite problem (4) in the form of problem (6).

Step 6. (i) Choose $w_i = w_i^* \geq 0, \quad i=1,2,\ldots,k$ and $\sum_{i=1}^{k} w_i^* = 1$.

(ii) Use Rosen's partitioning procedure [14] to obtain an optimal solution $(Y^*, \mu^*)$ from which the optimal solution $X^*$ can be obtained directly, where $X^* = Y^* / \mu^*$.

Step 7. Let $X^*$ be the optimal solution of problem (4) such that:

(i): If $w_i > 0$ for all $i$, then $X^*$ is an efficient solution of problem (1) and go to step 8.

(ii): If $w_i \geq 0$ for all $i$, and $X^*$ is a unique solution of problem (4), then $X^*$ is an efficient solution of problem (1) and then Go to step 8.

(iii): If $w_i \geq 0$ for all $i$, and there are alternative integer solutions of problem (4), use the non-inferiority test [13] to identify which solution is efficient of problem (1), then go to step 8.

Step 8. Stop.

(4) An Illustrative Example

In what follows, we provide a numerical example to illustrate the solution algorithm described in the previous section. For this purpose, let us consider the following CHLSFMOP problem which has the angular structure:

Maximize $(f_1(X), f_2(X))$
subject to
\[ P\{x_1 + x_2 \leq b_0\} \geq 0.7257, \]
\[ P\{x_1 \leq b_1\} \geq 0.5, \]
\[ P\{2x_2 \leq b_2\} \geq 0.4013, \]
\[ x_1 \geq 0, \quad x_2 \geq 0 \]

where
\[ f_1(x) = \frac{x_1 + x_2}{4x_1 + 3x_2 + 3}, \quad \text{and} \quad f_2(x) = \frac{x_1 + 2x_2}{4x_1 + 3x_2 + 3} \]

By using problem (3), we can have
Maximize \((f_1(X), f_2(X))\)
subject to
\[ x_1 + x_2 \leq 5, \]
\[ x_1 \leq 2, \]
\[ 2x_2 \leq 8, \]
\[ x_1 \geq 0, \quad x_2 \geq 0 \]

Assume that
\[ \mu = \frac{1}{4x_1 + 3x_2 + 3} \]
and consider \(w_1^* = 0.5\) and \(w_2^* = 0.5\), and make the variable changes:
\[ y_1 = x_1 \mu \quad \text{and} \quad y_2 = x_2 \mu \]

Thus, the above problem can be rewritten as follows:
Minimize \(F(y_1, y_2) = y_1 + 1.5y_2\)
subject to
\[ 4y_1 + 3y_2 + \mu = 1 \]
\[ y_1 + y_2 - 5\mu \leq 0, \]
\[ y_1 - 2\mu \leq 0, \]
\[ 2y_2 - 8\mu \leq 0, \]
\[ y_1, y_2 \geq 0 \text{ and } \mu > 0 \]

This problem has angular and dual-angular structure. Then by using Rosen's Partitioning procedure \([14]\), the optimal solution \(y_1^* = 0, \quad y_2^* = 0.31, \quad \mu^* = 0.08 \) and \(f^* = 0.46\) can be obtained and thus \(x_1^* = 0, \quad x_2^* = 3.875\)

(5) Concluding Remarks

In this paper we have proposed a solution procedure for solving a special type of CHLSFMOP problems. These problems have block angular structure of the constraints. An illustrative numerical example has been provided to clarify the theory and the proposed algorithm.
However, there are some open points of research which should be explored and studied in the field of fuzzy integer fractional optimization problems. Some of these points are:

(i) A solution algorithm is needed for treating fuzzy bicriterion and multiple objective integer linear and nonlinear fractional programs with fuzzy parameters in the constraints and in the objective functions.

(ii) A solution procedure is required for treating fuzzy large-scale integer linear and nonlinear fractional multiple objective programming problems.

(iii) A solution method should be carried out for solving stochastic large-scale multiple objective integer linear and nonlinear fractional programming problems with block angular structure of the constraints.

References


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