Codes Correcting Repeated Burst Errors Blockwise

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Abstract

This paper obtains bounds for linear codes which are capable to correct the errors blockwise which occur during the process of transmission. The kind of errors considered are known as repeated burst errors of length $b$ (fixed), introduced by Dass and Garg (2009). An illustration for such kind of codes has also been provided.

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1. Introduction

In digital data transmission, the search for practical coding techniques on error control has concentrated mainly in two areas viz. error detection and error correction. Another concept - location of errors, which lies midway between these two concepts was introduced by Wolf and Elspas (1963). In this technique the block of received digits is to be regarded as subdivided into mutually exclusive sub-blocks and while decoding it is possible to detect the error and
in addition the receiver is able to specify which particular sub-block contains error. Such codes are referred to as Error-Locating codes (EL-codes). If errors are detected, the receiver requests the re-transmission of the corrupted block of digits and this process is repeated for each incoming block. The use of EL-codes may offer a compromise between short and long block lengths by providing an additional design parameter.

Many kind of errors in coding theory have been dealt with for which codes have been constructed to combat errors. Apart from random errors, one of the widely studied errors is a burst error. It has been observed that in several communication systems, errors occur predominantly in adjacent positions rather than in a random manner. It was in this spirit that the codes correcting single errors and double adjacent errors were developed by Abramson (1959). This idea was generalized to the category of errors called ‘burst errors’. A burst of length $b$ is defined as follows:

**Definition 1.** A burst of length $b$ is a vector whose only non-zero components are among some $b$ consecutive components, the first and the last of which is non-zero.

This definition was given by Fire (1959) and he called such errors as open-loop burst errors. There is another kind of burst error due to Chien and Tang (1965). They defined a burst of length $b$, which shall be called as CT-burst of length $b$, as follows:

**Definition 2.** A CT-burst of length $b$ is a vector whose only non-zero components are confined to some $b$ consecutive positions, the first of which is non-zero.

Channels due to Alexander, Gryb and Nast (1960) fall in the category to control CT bursts. The nature of burst errors differs from channel to channel depending upon the behaviour of channels or the kind of errors which occur during the process of transmission. It was noted by Dass (1980) that in several channels errors occur in the form of a burst but the end digits of the burst do not get corrupted which prompted him to modify the definition of CT burst as follows:

**Definition 3.** A burst of length $b$ (fixed) is an $n$-tuple whose only non-zero components are confined to $b$ consecutive positions, the first of which is non-zero and the number of its starting positions is among the first $n - b + 1$ components.

Also, in very busy communication channels, errors repeat themselves. So is a situation when errors occur in the form of bursts. So we need to consider repeated bursts. Not only repeated bursts emerge as a natural generalization of bursts but the codes developed pertaining to these may play an important role in subjects like biology. The models studied by Srinivas, Jain, Saurav
and Sikdar (2007) fall into this category. They studied the changes in the neuronal network properties during epileptiform activity *in vitro* in planar two-dimensional networks cultured on a multielectrode array, using the *in vitro* model of stroke-induced epilepsy.

An *m*-repeated burst of length *b* (fixed) has been defined by Dass and Garg (2008) as follows:

**Definition 4.** An *m*-repeated burst of length *b* (fixed) is an *n*-tuple whose only non-zero components are confined to *m* distinct sets of *b* consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first *n − mb + 1* components.

In particular, a 2-repeated burst of length *b* (fixed) [refer Dass and Garg (2009)] may be defined as follows:

**Definition 5.** A 2-repeated burst of length *b* (fixed) is an *n*-tuple whose only non-zero components are confined to 2 distinct sets of *b* consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first *n − 2b + 1* components.

As an illustration (021010120000) is a 2-repeated burst of length up to 5 (fixed) whereas (00100200101020) is a 2-repeated burst of length at most 6 (fixed) over GF(3).

Wolf and Elspas (1963) studied binary codes which are capable of detecting and locating a single sub-block containing random errors. A study of such error locating codes in which errors occur in the form of bursts of length *b* (fixed) was made by Dass and Kishanchand (1986). These results were further extended to the block-wise correction of errors which were in the form of bursts of length *b* (fixed). The development of codes correcting repeated burst errors within a sub-block improves the efficiency of the communication channel as it reduces the number of parity-check digits required.

In an earlier paper, the authors [Dass and Arora (2010)] obtained bounds for the number of parity check digits required to locate the errors which are in the form of 2-repeated bursts of length *b* (fixed) and *m*-repeated bursts of length *b* (fixed). In this paper, bounds for the number of parity check digits required in a code to correct such errors have been presented.

This paper has been organized as follows. In section 2, a necessary condition for blockwise correction of 2-repeated bursts of length *b* (fixed) has been derived and then a sufficient condition for the existence of such a code has been obtained. An illustration of 2-repeated burst of length 3 (fixed) over GF(2) has also been provided. In section 3, a necessary condition for the blockwise correction of *m*-repeated bursts of length *b* (fixed) as well as a sufficient condition for the same have been given.

In the following, we shall consider a linear code to be a subspace of *n*-tuples over GF(*q*). The block of *n* digits, consisting of *r* check digits and
information digits, is considered to be divided into \( s \) mutually exclusive sub-blocks. Each sub-block contains \( t = n/s \) digits.

2. Codes correcting 2-repeated bursts of length \( b(\text{fixed}) \) within a single sub-block

In this section, we obtain bounds on the number of parity check digits of a code over \( \text{GF}(q) \) that are capable of correcting all 2-repeated bursts of length \( b(\text{fixed}) \) within a single sub-block.

We note that an \((n, k)\) linear EL-code capable of detecting and locating a single sub-block containing an error which is in the form of a 2-repeated burst of length \( b(\text{fixed}) \) must satisfy the following conditions:

(a) The syndrome resulting from the occurrence of a 2-repeated burst of length \( b(\text{fixed}) \) within any one sub-block must be distinct from the all zero syndrome.

(b) The syndrome resulting from the occurrence of any 2-repeated burst of length \( b(\text{fixed}) \) within a single sub-block must be distinct from the syndrome resulting likewise from any 2-repeated burst of length \( b(\text{fixed}) \) within any other sub-block.

Further, an \((n, k)\) linear code capable of correcting an error requires the syndromes of any two vectors to be different irrespective of whether they belong to the same sub-block or different sub-blocks. So, in order to correct 2-repeated bursts of length \( b(\text{fixed}) \) lying within a sub-block the following conditions need to be satisfied:

(c) The syndrome resulting from the occurrence of a 2-repeated burst of length \( b(\text{fixed}) \) must be distinct from the syndrome resulting from any other 2-repeated burst of length \( b(\text{fixed}) \) within the same sub-block.

(d) The syndrome resulting from the occurrence of any 2-repeated burst of length \( b(\text{fixed}) \) within a single sub-block must be distinct from the syndrome resulting likewise from any 2-repeated burst of length \( b(\text{fixed}) \) within any other sub-block.

Remark 1. We observe that condition (b) is the same as condition (d). Also, condition (a) is taken care of by condition (c). From this we infer that correction of errors require more strict conditions than location of errors.

So we need to consider conditions (c) and (d) or equivalently conditions (b) and (c) for the correction of the said type of errors.

We shall derive two results in this section. The first result gives a lower bound on the number of check digits required for the existence of a linear code over \( \text{GF}(q) \) capable of correcting errors that are 2-repeated bursts of length
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$b$(fixed) within a sub-block. In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

**Theorem 1.** The number of parity check digits $r$ in an $(n, k)$ linear code subdivided into $s$ sub-blocks of length $t$ each, that corrects a 2-repeated burst of length $b$(fixed) lying within a single sub-block is at least

$$
\log_q \left\{ 1 + s \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}
$$

(1)

**Proof.** Let there be an $(n, k)$ linear code vector $\text{GF}(q)$ that corrects a 2-repeated burst of length $b$(fixed) within a single corrupted sub-block. The maximum number of distinct syndromes available using $r$ check bits is $q^r$. The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (c) and (d) and then setting this number less than or equal to $q^r$. First we consider a sub-block, say $i$-th sub-block of length $t$.

Since the code is capable of correcting all errors which are 2-repeated bursts of length $b$(fixed) within a single sub-block, any syndrome produced by a 2-repeated burst of length $b$(fixed) in a given sub-block must be different from any such syndrome likewise resulting from another 2-repeated burst of length $b$(fixed) in the same sub-block by condition (c).

Also by condition (d), syndromes produced by 2-repeated bursts of length $b$(fixed) in different sub-blocks must be distinct.

Thus the syndromes produced by 2-repeated bursts of length $b$(fixed), whether in the same sub-block or in different sub-blocks should be distinct.

Since there are $s$ sub-blocks and number of 2-repeated bursts of length $b$(fixed) in a vector of length $t$ [Dass, Garg and Zannetti, 2008] is

$$
\sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)}
$$

(2)

So we must have at least $1 + s \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)}$ distinct syndromes, counting the all zero syndrome.

Therefore we must have

$$
q^r \geq 1 + s \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)}
$$

or

$$
r \geq \log_q \left\{ 1 + s \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}
$$

(3)
Remark 2. For \( s = 1 \), the bound reduces to

\[
\log_q \left\{ \sum_{i=0}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}
\]

which coincides with the necessary condition for the existence of a code correcting 2-repeated bursts of length \( b \) (fixed) [refer Dass, Garg and Zannetti (2008), Theorem 1] for the correction of a 2-repeated burst of length \( b \) (fixed).

In the following result we derive another bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks (1958), also Theorem 4.7 Peterson and Weldon (1972)). This technique not only ensures the existence of such a code but also gives a method for the construction of such a code.

**Theorem 2.** An \( (n, k) \) linear code over \( \mathbb{GF}(q) \) capable of correcting a 2-repeated burst of length \( b \) (fixed), \( 4b < t \), occurring within a single sub-block can always be constructed using \( r \) check digits where \( r \) is the smallest integer satisfying the inequality

\[
q^r > q^{b-1} \left\{ \sum_{i=0}^{3} \binom{t - (i + 1)b + i}{i} (q - 1)^i q^{i(b-1)} \right\}
\]

\[
+ (s - 1) \left\{ [1 + (t - 2b + 1)(q - 1)q^{b-1}] \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}.
\]

(4)

**Proof.** In order to prove the existence of such a code we construct an \( (n-k) \times n \) parity check matrix \( H \) for such a code by a synthesis procedure. For that we first construct a matrix \( H_1 \) from which the requisite parity check matrix \( H \) shall be obtained by reversing the order of the columns of each sub-block.

After adding \( (s-1)t \) columns appropriately corresponding to the first \( (s-1) \) sub-blocks, suppose that we have added the first \( j - 1 \) columns \( h_1, h_2, \ldots, h_{j-1} \) of the \( s \)-th sub-block also, where the first \( b - 1 \) columns \( h_1, h_2, \ldots, h_{b-1} \) may be chosen arbitrarily (non-zero). We now lay down the condition to add the \( j \)-th column \( h_j \) as follows:

According to the condition (b), for the correction of 2-repeated bursts of length \( b \) (fixed) within a single sub-block, the syndrome of any 2-repeated burst of length \( b \) (fixed) within any sub-block must be different from the syndrome resulting from any other 2-repeated burst of length \( b \) (fixed) within the same sub-block. So \( h_j \) can be added provided it is not a linear combination of immediately preceding \( b - 1 \) columns \( h_{j-b+1}, h_{j-b+2} \ldots h_{j-1} \) together with any three distinct sets of \( b \) (fixed) consecutive columns out of the first \( j - b \) columns of the \( s \)-th sub-block.
where either all the coefficients \(\beta_i's\) \(\gamma_i's\) \(\delta_i's\) are zeros or if which-so-ever \(\beta_i's\), \(\gamma_i's\) or \(\delta_i's\) is the last non-zero coefficient say \(\mu_p\) then \(b \leq p \leq j - b; \alpha_i, \beta_i, \gamma_i, \delta_i \in GF(q)\)

The number of ways in which the coefficients \(\alpha_i's\) can be selected is \(q^{b-1}\) and to enumerate the coefficients \(\beta_i's, \gamma_i's, \delta_i's\) is equivalent to enumerate the number of \(3\)-repeated bursts of length \(b\) (fixed) in a vector of length \(j - b\) [refer Dass, Garg and Zannetti (2008)], including the vector of all zeros is

\[
\sum_{i=0}^{3} \binom{j - (i + 1)b + i}{i} (q - 1)^i q^{(b - 1)i}
\]

Therefore, the number possible linear combinations on the R.H.S. of (5) is

\[
q^{b-1} \sum_{i=0}^{3} \binom{j - (i + 1)b + i}{i} (q - 1)^i q^{(b - 1)i}
\]

Now according to condition (d), the syndrome of any \(2\)-repeated burst of length \(b\) (fixed) within a sub-block must be different from the syndrome resulting from any other \(2\)-repeated burst of length \(b\) (fixed) within any other sub-block. In view of this \(h_j\) can be added provided that

\[
h_j \neq (\omega_1 h_{j-b+1} + \omega_2 h_{j-b+2} + \ldots + \omega_{b-1} h_{j-1}) + (\xi_1 h_{i+1} + \xi_2 h_{i+2} + \ldots + \xi_b h_{i+b})
+ \{(\psi_1 h_{i+1} + \psi_2 h_{i+2} + \ldots + \psi_b h_{i+b}) + (\zeta_1 h_{i+1} + \zeta_2 h_{i+2} + \ldots + \zeta_b h_{i+b})\}
\]

\(\omega_i, \xi_i, \psi_i, \zeta_i \in GF(q)\) where either all \(\xi_i's\) are zero or if the \(p\)-th coefficient \(\xi_p\) is the last non-zero coefficients then \(b \leq p \leq j - b; h_{i+1}'s\) and \(h_{i+2}'s\) are the columns corresponding to the remaining \(s - 1\) sub-blocks, not all \(\psi_i's, \zeta_i's\) are zero and if which-so-ever \(\psi_i's\) or \(\zeta_i's\) is the last non-zero coefficient say \(\sigma_p\) then \(b \leq p \leq t\).

The number of ways in which the coefficients \(\omega_i's\) can be selected is \(q^{b-1}\) and to enumerate the coefficients \(\xi_i's\) is equivalent to enumerate the number of bursts of length \(b\) (fixed) in a vector of length \(j - b\).

This number [refer Dass (1980)], including the vector of all zeros is

\[
1 + (j - 2b + 1)(q - 1)q^{b-1}
\]

Therefore, the total number of possible choices for \(\omega_i\) and \(\xi_i\) on the R.H.S. of (8) is

\[
q^{b-1}[1 + (j - 2b + 1)(q - 1)q^{b-1}].
\]

Also the number of linear combinations corresponding to the last two terms on the R.H.S. of (8) is same as the number of \(2\)-repeated bursts of length \(b\) (fixed)
within a sub-block of length $t$, whose number is [refer Dass, Garg and Zannetti (2008)]

$$
\sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)}.
$$

Since there are $(s - 1)$ previously chosen sub-blocks, therefore number of such linear combinations becomes

$$(s - 1) \left\{ \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}$$

(10)

So, the number of linear combinations to which $h_j$ can not be equal to is the product of numbers computed in (9) and (10), i.e.

$$q^{b-1} \left\{ [1 + (j - 2b + 1)(q - 1)q^{b-1}] \cdot (s - 1) \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}$$

(11)

So, for the block-wise correction of the 2-repeated burst of length $b$(fixed) the number of linear combinations to which $h_j$ can not be equal to is the sum of linear combinations computed in (7) and (11) corresponding to (5) and (8).

At worst all these combinations might yield distinct sum.

Therefore $h_j$ can be added to the $s$-th sub-block of $H_1$ provided that

$$q^r > \text{expr.}(7) + \text{expr.}(11)$$

or

$$q^r > q^{b-1} \left\{ \sum_{i=0}^{3} \binom{t - (i + 1)b + i}{i} (q - 1)^i q^{i(b-1)} \\
+ (s - 1) \left\{ [1 + (t - 2b + 1)(q - 1)q^{b-1}] \sum_{i=1}^{2} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\} \right\}.
$$

Replacing $j$ by $t$ in the expression (7) where $t$ is the length of the sub-block which is inequality (4) stated in the theorem.

The required matrix $H$ can be obtained from $H_1$ by reversing the order of the columns in each sub-block.

**Remark 3.** For $s = 1$, the bound reduces to

$$q^{b-1} \left\{ \sum_{i=0}^{3} \binom{t - (i + 1)b + i}{i} (q - 1)^i q^{i(b-1)} \right\}$$

which coincides with the bound [refer Dass, Garg and Zannetti (2008), Theorem 2] for the correction of a 2-repeated burst of length $b$(fixed).
Example 1. For a (26, 11) linear code over GF(2) we construct the following (15, 26) parity check matrix $H$, according to the synthesis procedure given in the proof of Theorem 2 by taking $s = 2$, $t = 13$, $b = 3$.

$$
H = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

The null space of this matrix can be used as a code to correct 2-repeated bursts of length 3(fixed) within a sub-block of length $t=13$. It may be easily verified that:

(i) Syndromes of 2-repeated bursts of length 3(fixed) within any one sub-block are all non-zero.

(ii) The syndrome of a 2-repeated burst of length 3(fixed) within any sub-block is different from the syndrome of a 2-repeated burst of length 3(fixed) within the same sub-block.

(iii) The syndrome of a 2-repeated burst of length 3(fixed) within any sub-block is different from the syndrome of a 2-repeated burst of length 3(fixed) within any other sub-block.

(This has been verified through MS-Excel program)

3. Codes correcting $m$-repeated bursts of length $b$(fixed) within a single sub-block

In this section, the results of the previous section have been extended to the case of $m$-repeated burst of length $b$(fixed). An $(n, k)$ linear code over GF($q$) capable of correcting an error which is in the form of an $m$-repeated burst of length $b$(fixed) within a sub-block must satisfy the following conditions:

(e) The syndrome resulting from the occurrence of an $m$-repeated burst of length $b$(fixed) must be distinct from the syndrome resulting from any other $m$-repeated burst of length $b$(fixed) within the same sub-block.
The syndrome resulting from the occurrence of any \( m \)-repeated burst of length \( b \) within a single sub-block must be distinct from the syndrome resulting likewise from any \( m \)-repeated burst of length \( b \) within any other sub-block.

We shall derive two results in this section, the first result gives a lower bound on the number of check digits required for the existence of a linear code over \( \text{GF}(q) \) capable of correcting errors that are \( m \)-repeated bursts of length \( b \) within a sub-block. In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

**Theorem 3.** The number of parity check digits \( r \) in an \((n, k)\) linear code subdivided into \( s \) sub-blocks of length \( t \) each, that corrects errors that are \( m \)-repeated bursts of length \( b \) within a sub-block is at least

\[
\log_q \left\{ 1 + s \sum_{i=1}^{m} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}
\]

**Proof.** The proof of this result is on the similar lines as that of proof of Theorem 1 so we omit the proof.

**Remark 4.** For \( s = 1 \), the bound reduces to

\[
\log_q \left\{ \sum_{i=0}^{m} \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}
\]

which coincides with the necessary condition for the existence of a code correcting \( m \)-repeated bursts of length \( b \) [refer Dass, Garg and Zannetti (2008), Theorem 3].

**Remark 5.** For \( m = 2 \), the result obtained in Theorem 3 coincides with Theorem 1, for the case of correction of 2-repeated bursts of length \( b \) within a sub-block.

**Remark 6.** For \( m = 1 \), the result obtained in Theorem 3 becomes

\[
\log_q \left\{ 1 + s(q - 1)(t - b + 1)q^{b-1} \right\}
\]

which coincides with the result obtained by Dass and Kishanchand (1986) for the correction of a burst of length \( b \) within a sub-block.

In the following result we derive another bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varshomov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks (1958), also Theorem 4.7 Peterson and Weldon (1972)). This technique not only ensures the existence of such a code but also gives a method for the construction of such a code.
Theorem 4. An \((n, k)\) linear code over \(GF(q)\) capable of correcting a \(m\)-repeated burst of length \(b(\text{fixed})\), \(4mb < t\), occurring within a single sub-block can always be constructed using \(r\) check digits where \(r\) is the smallest integer satisfying the inequality

\[
q^r > q^{b-1} \left\{ \sum_{i=0}^{2m-1} \binom{t - (i+1)b + i}{i} (q-1)^i q^{i(b-1)} + (s-1) \right. \\
\left. \cdot \left\{ \sum_{i=0}^{m-1} \binom{t - (i+1)b + i}{i} (q-1)^i q^{i(b-1)} \sum_{i=1}^{m} \binom{t - ib + i}{i} (q-1)^i q^{i(b-1)} \right\} \right\} \quad (13)
\]

Proof. As in Theorem 3 we omit the proof because proof of this result is on the similar lines as that of proof of Theorem 2.

Remark 7. For \(s = 1\), the bound reduces to

\[
q^{b-1} \left[ \sum_{i=0}^{2m-1} \binom{t - (i+1)b + i}{i} (q-1)^i q^{i(b-1)} \right]
\]

which coincides with the sufficient condition for the existence of a code correcting \(m\)-repeated bursts of length \(b(\text{fixed})\) [refer Dass, Garg and Zannetti [2008], Theorem 4].

Remark 8. For \(m = 2\), the result obtained in Theorem 4 coincides with Theorem 2, for the case of correction of 2-repeated bursts of length \(b(\text{fixed})\) within a sub-block.

Remark 9. For \(m = 1\), the result obtained in Theorem 4 becomes

\[
q^{b-1} [1 + q^{b-1}(q-1)\{s(t - b + 1) - b\}]
\]

which coincides with the result obtained in Theorem 4 [refer Dass and Kishanchand (1986)] for the correction of a burst of length \(b(\text{fixed})\) within a sub-block.

References


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