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A Mixed Integer Linear Formulation for the Open Shop Earliness-Tardiness Scheduling Problem

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Abstract

This paper addresses the problem of minimizing total earliness/tardiness penalty in an open shop scheduling environment with non-identical parallel machines. This problem is formulated as a mixed integer linear programming model using time-based decision variables. Computational results demonstrate that the proposed mathematical models are efficient in solving open shop problems with 5 stages and 3 jobs.

Keywords: Open shop scheduling; Earliness/Tardiness criterion; Mixed integer programming.

1. INTRODUCTION

The importance of "just-in-time" philosophy in industrial production has persuaded most of researchers to develop theoretical models to capture benefits of it. In some of these models, a inordinate number of research has concentrated on the problem of minimizing earliness–tardiness penalties, wherein both early cost (due to the need for storage) and tardy cost are included. Due to the nondeterministic polynomial time complexity of these problems, most of papers have studied this problem in one-machine scheduling.

The open shop environment could be defined as follows. A set of m machines $M_1, M_2, \ldots, M_m$ must process a set of n jobs $J_1, J_2, \ldots, J_n$. All the processing times are given in advance and the jobs sequence on the machines must be decided by decision maker. Each job can be processed on at most one machine at a time and each machine can process at most one job at a time.
Suppose that completion time of job J_j, i = 1, 2, . . . , n, is denoted by C_j, i.e. the time when the processing of the last operation of this job is finished. Earliness and tardiness of job j are defined by $E_j = max\ (C_j - d_j, 0)$ and $T_j = max\ (d_j - C_j, 0)$ respectively, where d_j is job j due date.

2. Literature Review

Earliness-tardiness scheduling Problem has been studied widely in single-machine environment [1, 2, 3, 4]. A large number of papers also have investigated this problem for the case of having parallel machine, especially when due dates of jobs are the same [5, 6, 7, 8, 9, 10]. However, a few number of researchers have worked on the flow shop scheduling where earliness and tardiness penalties are considered to be included in the objective function. Two machine flow shop with common due dates have been studied by Sung and Min [11]. Branch and bound algorithms have been applied in some papers [12, 13].

According to the above, earliness/tardiness scheduling has not investigated in open shop environments. In this paper, we consider the problem of minimizing summation of weighted earliness/tardiness in open shop with non-identical parallel machines. Earliness/Tardiness penalty is considered to be weighted absolute deviation of completion time from uncommon due dates. This problem could be denoted as $O||\sum(W_jE_j + W_j'T_j)$. A mixed integer linear formulation is proposed for this problem.

The reminder of this paper is organized as follows. The mixed integer linear formulation of the problem is presented in Section 3. Several sets of test problems are solved and reported in Section 4. Section 5 concludes the paper.

3. The proposed time-based mixed integer linear formulation

In this section, the proposed mathematical model is presented. First, we need to define sets, indices, parameters and variables.

Sets and indices:

J: Jobs set; I: Stages set; K_i: Machines set in stage i; t, t': Time indices; j: Job indices; i: Stage indices; k: Machine indices.

Parameters:

n: Number of jobs; S: Number of stages; m_i: Number of parallel machines in stage *i*; P_{ijk}: Processing time job *j* on machine *k* in stage *i*; U_t: Planning horizon; W_j:Tardiness penalty per unit time for job *j*; W_j': Earliness penalty per unit time for job *j*; D_j: Due date job *j*., *M*: A big value which is determined regarding to other parameters of the problem.

Variables:

X_{ijkt}: Equals 1 if machine *k* in stage *i* is processing job j on time *t* and 0 otherwise.
Y_{ijk}: Equals 1 if machine *k* is selected to process job *j* in stage *i* and 0 otherwise.
L_{ijk}: Equals 1 if machine *k* in stage *i* is the last machine which processes job *j* and 0 otherwise.
T_j: Total tardiness job *j*; E_j: Total earliness job *j*.
T_{ijk}: Tardiness job *j* on machine *k* is stage *i*.
E_{ijk}: Earliness job *j* on machine *k* is stage *i*.
C_{ijk}: Completion time job *j* on machine *k* in stage *i*.
Using these definitions, the proposed mathematical model is as follows:

$$MIN(Z) = \sum_{j=1}^{n} \left(W_j T_j + W_j' E_j\right) \tag{1}$$

$$\sum_{j=1}^{n} X_{ijkt} \leq 1 \qquad \forall i \in I, \forall k \in K_i, \forall t \in \{1,2,\dots,U_t\} \tag{2}$$

$$\sum_{i=1}^{S} \sum_{k=1}^{m_i} X_{ijkt} \leq 1 \qquad \forall j \in J, \forall t \in \{1,2,\dots,U_t\} \tag{3}$$

$$\sum_{k=1}^{m_i} \sum_{t=1}^{U_t} X_{ijkt} = P_{ijk} \times Y_{ijk} \qquad \forall j \in J, \forall i \in I \tag{4}$$

$$X_{ijkt} \leq Y_{ijk} \qquad \forall j \in J, \forall i \in I, \forall k \in K_i, \forall t \in \{1,2,\dots,U_t\} \tag{5}$$

$$\sum_{k=1}^{m_i} Y_{ijk} = 1 \qquad \forall j \in J, \forall i \in I \tag{6}$$

$$\sum_{t=1}^{U_t} \left| X_{ijk(t+1)} - X_{ijkt} \right| \leq 2 \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (7)$$

$$C_{ijk} = \frac{\sum_{t=1}^{U_t} t \times X_{ijkt}}{P_{ijk}} + \frac{P_{ijk} Y_{ijk}}{2} \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (8)$$

$$C_{ijk} - D_j = T_{ijk} - E_{ijk} \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (9)$$

$$T_j \geq T_{\mathrm{ijk}} \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (10)$$

$$E_j \geq E_{\mathrm{ijk}} - M\left(1 - L_{ijk}\right) \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (11)$$

$$C_{ijk} \geq C_{i'jk'} - M\left(1 - L_{ijk}\right) \qquad \forall j \in J, \forall i, i' \in I, \forall k, k' \in K_i \qquad (12)$$

$$\sum_{i=1}^{S} \sum_{k=1}^{m_i} L_{ijk} = 1 \qquad \forall j \in J \qquad (13)$$

$$X_{ijkt}, Y_{ijk}, L_{ijk} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall k \in K_i, \forall t \in \{1,2, \dots, U_t\} \qquad (14)$$

Equation (1) presents the objective function which includes weighted earliness and tardiness penalties. Constraint (2) states that on each time t, each machine k in each stage i can only process at most one job. Constraint (3) implies that on each time t, each job j could be under processing by at most one machine. Regarding Constraint (4), processing time job j on machine k in stage i must be equals P_{ijk} if it is assumed that in this stage machine k processes job j ($Y_{ijk} = 1$). Constraint (5) notes that if machine k in stage i processes job j at time t ($X_{ijkt} = 1$), then in this stage, job j must be assigned to machine k ($Y_{ijk} = 1$). Constraint (6) remarks that in each stage i, each job j is assigned exactly to one machine. Using Constraint (7), preemption is avoided. This constraint is nonlinear constraint. However, it could be transformed to the following linear constraints:

$$\sum_{t=1}^{U_t} Z_{ijkt} \leq 2 \qquad \forall j \in J, \forall i \in I, \forall k \in K_i \qquad (7')$$

$$Z_{ijkt} \geq X_{ij(t+1)} - X_{ijt} \qquad \forall i \in I, \forall j \in J, \forall k \in K_i, \forall t \in \{1,2, \dots, U_t\} \qquad (7'')$$

$$Z_{ijkt} \in \{0,1\} \qquad \forall i \in I, \forall j \in J, \forall k \in K_i, \forall t \in \{1,2, \dots, U_t\} \qquad (7''')$$

In Constraint (8), C_{ijk} is calculated using decision variables $\mathrm{X_{ijkt}}$ and $\mathrm{Y_{ijk}}$. In this constraint, if in stage i, job j is not assigned to machine j ($Y_{ijk} = 0$), we will have $\forall t \in \{1,2, \dots, U_t\}$ $X_{ijkt} = 0$ and $C_{ijk} = 0$. Regarding Constraint (9) and the coefficients of T_{ijk} and E_{ijk} in the objective function, if $C_{ijk} \geq D_j$, we will have $T_{ijk} \geq 0$ and $E_{ijk} = 0$. In case of $C_{ijk} \leq D_j$, T_{ijk} takes 0 and E_{ijk} takes a non negative amount. That total tardiness job j is greater than its tardiness on different machines is formulated by Constraint (10). With regard to Constraint (11), if the last operation of

job j is done by machine j in stage i ($L_{ijk} = 1$), E_j will be greater than E_{ijk}. It should be noted since coefficient of E_{ijk} in the objective function is positive, the mathematical model will set $E_j = E_{ijk}$. Constraint (12) states that if $L_{ijk} = 1$, the completion time job j in stage i on machine k must be greater than other completion of job j $C_{ijk} \geq C_{i'jk'}$. Constraint (13) implies that exactly one machine could be the last machine that processes job j.

4. Computational results

In order to analyze sensitivity of solution time to the problem dimensions, we have generated 100 test problems. These test problems are solved using Cpelx 10.1. Corresponding computational results are reported in Table 1. In this table, S and n are number of stages and number of jobs, respectively. m also presents number of non-identical machines in each stage. for each member *of* $\{(S,n,m)|\ (S,n,m) \in \{2,3,4,5\} \times \{2,3,4,5\} \times \{1,2\}\ \&\ n \leq S)$, *5 instances are produced and solv*ed by the proposed mathematical model. For each job j, w_j and w_j' is chosen in the range of [0.1,1]. Also, P_{ijk} and D_j are chosen in the range of [0,5] and $[0.5\sum_{i=1}^{S}\sum_{j=1}^{n} P_{ij}', 1.5\sum_{i=1}^{S}\sum_{j=1}^{n} P_{ij}']$, respectively. In the last interval we have $P_{ij}' = \sum_{k=1}^{m_i} P_{ijk}/m_i$. Solution time, Average number of constraints (ANOC) and average number of binary variables (ANOB) are presented in Table 1. Figure 1 shows that increasing in ANOB leads to exponentially increasing in solution time.

Table 1. Computational Results

S	n	m	ANOC	ANOB	Solution Time (Sec)
2	2	1	120	88	0
2	2	2	140	96	0
3	2	1	169	132	1
3	2	2	204	144	1
3	3	1	312	288	412
3	3	2	355	306	759
4	2	1	218	176	52
4	2	2	268	192	61
4	3	1	400	384	2562
4	3	2	465	408	3507
4	4	1	632	672	>3600
4	4	2	708	704	>3600
5	2	1	267	220	132
5	2	2	332	240	112
5	3	1	488	480	412

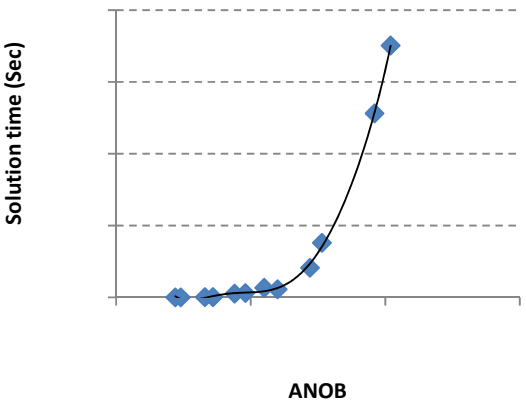

Figure 1. Sensitivity analysis solution time to ANOB

5. Summary

In This paper, we studied the problem of minimizing total earliness/tardiness cost in an open shop scheduling environment with non-identical parallel machines. For this problem, a mixed integer programming model presented. Computational results demonstrated that the proposed mathematical model is capable of solving open shop problems with 5 stages and 3 jobs.

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