Retailer’s Pricing and Ordering Strategy for

Weibull Distribution Deterioration under Trade Credit in Declining Market

Nita H. Shah and Nidhi Raykundaliya

Department of Mathematics, Gujarat University
Ahmedabad-380009, Gujarat, India
nithahshah@gmail.com

Abstract

In this research article, an ordering and pricing policy is formulated for a retailer when the supplier offers a delay in payments to settle the accounts against the retailer’s due. The problem is to study afore said strategy when demand of product is subject to decrease with time. The units in inventory are loosing its efficiency with respect to time. A decision policy is sketched to determine the optimal selling price and the ordering quantity to maximize the retailer’s profit. The numerical examples are given to support the development of the mathematical model. The sensitivity analysis of critical parameter is carried out to observe the changes in the decision variable and profit.

Mathematics Subject Classification: 90B05

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1. Introduction

To attract more retailer’s supplier uses a promotional tool of offering the retailer a trade credit (say) “N” days to settles the account against the purchase made. The trade credit in finance management is termed as “net N” (Brigham (1995)). No interest is charged if the account is settled within this N days. However, if the payment is not made within this credit period, then the interest is charged on the remaining stock in inventory. Goyal (1985) explore the concept of trade credit. He computed interest earned on the sales revenue on unit purchase price. This concept followed by Dave (1985), Shah (1993a, 1993b, 1993c), Aggarwal and Jaggi (1995), Jamal et al. (1997), Hwang et al (1997), Liao et al (2000), Chung and Dye (2001) and their cited references which ignored the fact that the retailer’s selling price is higher than the unit purchase price. They conclude that the retailer earn interest generated revenue by delaying the payment up to last date of offered delay period time. Jamal et al. (2000) and Sarker et al. (2000) incorporated the constant selling price and the difference between the unit
sale price and unit purchase price. Teng et al. (2005) extended the above model by making unit sell price as a decision variable. Teng (2003) established that it is economically advantageous for the retailer to replenish order of smaller size more frequently and take benefits of the permissible trade credit. Chang et al. (2003) extended Teng’s model by considering credit linked order quantity when units in inventory are subject to the constant deterioration.

In above cited references, the demand is taken to be deterministic and constant. However, the demand of the seasonal product is decreases with time. In this study demand is considered to be decreasing function of time and sale price. It is assumed that the unit selling price is higher than unit cost price. The profit is maximized with respect to unit sale price and the cycle time. The units in inventory are subject to deterioration with time i.e. deterioration of unit follow two-parameter Wiebull distribution. Numerically it is established that increase in trade credit lower unit sale price and increase the profit. The sensitivity analysis is carried out to study the variations in the decision variables and objective function.

2. Notations and Assumptions

The mathematical model is developed under the following notations and assumptions.

2.1 Notations:

\( \dot{R}(t) = a(1 - bt)P^{-\eta} \); Where \( a > 0 \) is fixed demand, \( b \) \((0 < b < 1)\) is rate of change of demand and \( \eta > 1 \) is makeup parameter.

\( C \): The unit purchase cost.

\( P \): The unit sale price with \( P > C \) (a decision variable).

\( h \): The inventory holding cost per unit per annum excluding interest charges.

\( A \): The ordering cost per order.

\( M \): The permissible trade credit offered by the supplier to the retailer for settlement of the account against purchases.

\( I_c \): The interest charged per monetary unit in stocks per annum by the supplier.

\( I_e \): The interest earned per monetary unit per year on generated sales revenue.

Note : \( I_c > I_e \)

\( Q \): the order quantity (a decision variable)

\( \theta(t) = \alpha \beta t^{\beta-1} \), the deterioration of units in inventory follows Wiebull distribution.

Where \( \alpha (0 \leq \alpha < 1.) \) denotes the scale parameter and \( \beta(>1) \) denotes the shape parameter. It is assumed that the deterioration of units increases with time \( t (> 0) \).

\( I(t) \): The inventory level at any instant of time \( t, 0 \leq t \leq T \).

\( T \): The cycle time (a decision variable).

\( Z(P,T) \): The total profit per time unit.
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The total profit per unit time of a retailer comprises of: (a) Sales revenue: SR, (b) Purchase cost: PC, (c) inventory holding cost excluding interest charges; IHC, (d) ordering cost: OC, (e) interest charged on unsold items in the stock after the permissible credit period when \( M < T \); \( I_c \), and (f) interest earned on the generated sales revenue during the permissible delay period: \( I_e \).

2.2 Assumptions:

1. The inventory system under consideration deals with the single item only.
2. The planning horizon is infinite.
3. The demand of the product is decreasing function of the time and the sale price.
4. Shortages are allowed and lead-time is zero.
5. The units in inventory are subject to deteriorate with respect to time. The deterioration rate follows Wiebull distribution. The deteriorated units can neither be repaired nor replaced during the cycle time.
6. The retailer generates revenue by selling the product. The generated revenue is deposited in an interest earning account during the allowable credit period. At the end of this period, the retailer settles the account for all the units sold keeping the difference for day-to-day expenses, and starts paying the interest charges on the unsold items in the inventory.

3. Mathematical model

The retailer’s inventory level gradually decrease due to the time dependent and sale price and deterioration of units in inventory. The instantaneous state of inventory level at any instant of time \( t \) during the cycle period \( [0, T] \) can be described by the differential equation:

\[
\frac{dI(t)}{dt} + \theta(t)I(t) = -R(t, P), \quad 0 \leq t \leq T
\]  

with the initial condition \( I(0) = Q \) and the boundary condition \( I(T) = 0 \). Consequently, the solution of (1) is given by

\[
I(t)e^{\alpha t} = Q - \int_0^t R(t, P)e^{\alpha t} dt \]  

Under the assumption that \( \alpha (0 \leq \alpha < 1) \) is vary small, expanding exponential series by neglecting \( \alpha^2 \) and its higher powers, the solution (2) can be written as

\[
I(t) = \left( Q - \int_0^t a(1-bt)P^\theta(1 + \alpha t^\theta)dt \right)(1 - \alpha t^\theta) \]  

Next we compute different cost involved in the total profit per time unit.

A) Sales revenue; SR per time unit is \( SR = \frac{PQ}{T} \) (5)

b) Purchase cost; PC of procuring Q- units per time unit is \( PC = \frac{CQ}{T} \) (6)

c) Inventory holding cost; IHC per time unit is \( IHC = \frac{h}{T} \int_{0}^{T} I(t) dt \) (7)

d) Ordering cost; OC per order is \( OC = \frac{A}{T} \) (8)

Regarding interest charges and earned, (i.e. costs (e) and (f) state in section (2.2) two cases may arise based on the lengths of M and T. Viz \( M \leq T \) or \( M > T \).

**Case 1: \( M \leq T \).**

In this case, the retailer sells \( R(M) \) M units during \([0, M]\] and has CR(M)M to pay the supplier. For the unsold items in the stock, the supplier charges at an interest rate \( I_c \) during the period \([M, T]\]. Hence, the interest charged, \( IC_1 \) per time unit is

\[
IC_1 = \frac{C_I}{T} \int_{M}^{T} I(t) dt
\] (9)

During \([0, M]\], the retailer sells the units and deposits the revenue into the interest earning account at the rate \( I_e \) per monetary unit per annum. Hence, the interest earned, \( IE_1 \) per time unit is

\[
IE_1 = \frac{P_I}{T} \int_{0}^{M} R(t, P) dt \left[ \frac{a}{2} M^2 - \frac{1}{3} b M^3 \right]
\] (10)

Hence, the retailer’s profit per time unit is

\[
Z_1(T, P) = SR - PC - IHC - OC - IC_1 + IE_1
\] (11)

The necessary conditions for \( Z_1(T, P) \) to be optimum is

\[
\frac{\partial Z_1(T, P)}{\partial P} = 0
\] (12)

And \( \frac{\partial Z_1(T, P)}{\partial T} = 0 \) (13)

the obtained \( (T, P) = (T_1, P_1) \) (say) maximizes the profit \( Z_1(T, P) \) provided

\[
\frac{\partial^2 Z_1(T, P)}{\partial P^2} < 0 \quad \text{and} \quad \frac{\partial^2 Z_1(T, P)}{\partial T^2} < 0
\]
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and \( \left( \frac{\partial^2 Z_i(T,P)}{\partial P^2} \right) \left( \frac{\partial^2 Z_i(T,P)}{\partial T^2} \right) - \left( \frac{\partial^2 Z_i(T,P)}{\partial T \partial P} \right) > 0 \)

The complexity of the expression in (12)-(14) suggests that it is not easy to get good closed form for the necessary and sufficient conditions. One can solve (12) and (13) for \((T, P)\) by a mathematical software.

**Case2: \( T \leq M \)**

In this scenario, the retailer sells \( R(T)T \) units in all by the end of the cycle time and has \( CR(T)T \) to pay the supplier in full by the end of the credit Period \( M \). Hence the interest charges \( IC_2 = 0 \) and the interest earned per time unit is

\[
IE_2 = \frac{P(t)}{T} \left[ \int_0^T R(t, P)dt + R(T, P)T(M - T) \right] = \frac{aIeP^{-\eta+1}}{T} \left[ \frac{T^2}{2} - \frac{bT^3}{3} + (1 - bT)[T(M - T)] \right]
\]

Hence, the retailer’s profit per time unit is

\[
Z_2(T, P) = SR - PC - OC - IHC + IC_2 + IE_2
\]

The optimum value of \((T, P) = (T_2, P_2)\) (say) can be obtained by solving

\[
\frac{\partial}{\partial P} Z_2(T, P) = 0 \quad (18)
\]

\[
\frac{\partial}{\partial T} Z_2(T, P) = 0 \quad (19)
\]

With the help of suitable numerical software. The obtained \((T_2, P_2)\) maximizes profit \( Z_2(T, P) \) if (14) holds for \((T_2, P_2, Z_2(T, P))\)

At \( T = M \), we have \( Z_1(M, P) = Z_2(M, P) \)

4. **Computational algorithm**

To maximize the profit, the retailer can go through the following steps:

**Step1:** Take parametric values in proper units.

**Step2:** Calculate \((P_1, T_1)\) using eq(12) and (13). If \( M < T_1 \) case 1 is best policy to have maximum profit; otherwise go to step 3.

**Step3:** Compute \((P_2, T_2)\) by solving eq. (18) and (19). If \( M > T_2 \) case 2 gives the maximum profit; else go to step 4.

**Step4:** Compute \( P \) from (12) or (18). \( Z_1(M, P) \) or \( Z_2(M, P) \) is the maximum profit.
Step 5: stop.

5. Numerical Examples

Example 1: For $a = 200000$, $b = 0.2$, $\eta = 2.0$, $h = $ 1.00 / unit / year, $C = $ 20.00 per unit, $A = $ 250 / order, $I_c = 0.12 / $ / year, $I_e = 0.09 / $ / year, $\alpha = 10\%$, $\beta = 1.1$, $M = 30 / 365$ year, the optimal selling price $P_1 = $ 42.59 per unit and cycle time $T_1 = 0.9163$ year. The maximum profit per unit is $1952.5$ and optimum purchase quantity is 95 units. For $P_1 = $ 42.59 / unit and $T_1 = 0.9163$ years.

$$\frac{\partial^2 Z_1}{\partial P^2} = -2.454 \quad \frac{\partial^2 Z_1}{\partial T^2} = -624.51$$ and

$$\left(\frac{\partial^2 Z_1}{\partial P^2}\right)\left(\frac{\partial^2 Z_1}{\partial T^2}\right) - \left(\frac{\partial^2 Z_1}{\partial P \partial T}\right)^2 = 1475.50 > 0.$$ guarantees maximum profit. The 3D-plot drawn in the range [35, 55] for $P$ and [0.5, 1.5] for $T$ exhibits that $Z_1(42.59, 0.9163) = $1952.52 is maximum profit.

Example 2: Consider $[a, b, \eta, A, C, h, I_c, M, \alpha, \beta] = [2000000, 0, 1.1, 6, 50, 21, 1, 54, 10\%, 60 / 365, 0.2, 1.1]$ in proper units. The optimal solution is $P_2 = $ 55.78 per unit and cycle time $T_2 = 0.1427$ years. The maximum profit per unit is $Z_2(P_2, T_2) = $ 112379 and optimum purchase quantity $Q$ is 460 units. For $P_2 = $ 55.78 per unit and $T_2 = 0.1427$ years

$$\frac{\partial^2 Z_2}{\partial P^2} = -34.99 \quad \frac{\partial^2 Z_2}{\partial T^2} = -94141.82$$ and

$$\left(\frac{\partial^2 Z_2}{\partial P^2}\right)\left(\frac{\partial^2 Z_2}{\partial T^2}\right) - \left(\frac{\partial^2 Z_2}{\partial P \partial T}\right)^2 = 3266920.9 > 0.$$ Guarantees maximum profit. The 3D-plot (Fig. 2) drawn in the range [45, 90] for $P$ and [0.05, 1.5] for $T$ explores that obtained profit $Z_2(55.78, 0.1427) = $112379 is maximum.

Using the data of example 1, the sensitivity analysis is carried out by changing values of $M, \alpha, \beta, b$ from -40%, -20%, 20%, 40%. The variation in cycle time, selling price, purchase units and total profit per time unit are exhibited in table 1.
Table 1: sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T</th>
<th>P</th>
<th>Q</th>
<th>K₁</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-40</td>
<td>0.38</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
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<td>0.19</td>
<td>0.18</td>
<td>0.56</td>
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<tr>
<td></td>
<td>20</td>
<td>-0.19</td>
<td>-0.21</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>-0.40</td>
<td>-0.39</td>
<td>0.10</td>
</tr>
<tr>
<td>α</td>
<td>-40</td>
<td>-6.54</td>
<td>-0.42</td>
<td>-6.18</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>-3.45</td>
<td>-0.23</td>
<td>-2.90</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>3.93</td>
<td>0.23</td>
<td>4.91</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>8.44</td>
<td>0.51</td>
<td>9.65</td>
</tr>
<tr>
<td>β</td>
<td>-40</td>
<td>-3.93</td>
<td>-0.37</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>-20</td>
<td>-1.73</td>
<td>-0.16</td>
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<tr>
<td></td>
<td>20</td>
<td>1.43</td>
<td>0.11</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>2.55</td>
<td>0.21</td>
<td>1.05</td>
</tr>
<tr>
<td>b</td>
<td>-40</td>
<td>23.85</td>
<td>-0.63</td>
<td>25.26</td>
</tr>
<tr>
<td></td>
<td>-20</td>
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<td>0.70</td>
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<tr>
<td></td>
<td>40</td>
<td>-13.61</td>
<td>-0.98</td>
<td>-13.68</td>
</tr>
</tbody>
</table>

It is observed that increases in delay period decreases cycle time and selling price While increases in procurement quantity not significantly and profit significantly. Increase in deterioration rate ‘α’ increases selling price and profit. The variation in shape parameter significantly increases cycle time and decreases profit. The decrease in profit is due to more deterioration unit with respect to time. The increase in declining demand rate decrease cycle time, purchase quantity and profit significantly.

6. Conclusions

The optimal ordering and pricing policies are explored for a retailer when units in inventory are subject to deterioration with time and demand of a product is declining in the market under trade credit offered by the supplier to settle the accounts against the purchase made. The profit is maximizes. It is established that the retailer should replenish smaller order and take advantage of permissible delay payments more frequently. The model can be generalized for stochastic demand, random input, allowing shortages etc.
References


Fig: 1 $M \leq T$
Fig-2 $M > T$

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