Blood Flow through an Artery Having Radially Non-Symmetric Mild Stenosis

Bijendra Singh¹, Padma Joshi² and B. K. Joshi³

¹. School of Studies in Mathematics
   Vikram University
   Ujjain (M.P.), India
   bijendrasingh@yahoo.com

². Department of Mathematics
   Mahakal Institute of Technology
   Ujjain (M.P.), India
   Joshi.Padma@gmail.com

³. Formerly Professor and Head
   Government Engineering College
   Ujjain (M.P.), India
   bk.joshi684@gmail.com

Abstract

A model of blood flow through an artery has been formulated for improved generalised geometry of multiple stenoses located at equispaced points. We have assumed that the stenosis is mild and radially non-symmetric. For simplicity the graphical analysis is performed for a single loop of stenosis having maximum depression at different points. Graphical analysis demonstrates that increasing values of the parameter \( \alpha \) shows lower variations. The formulation of this model is mathematically more general and includes the results of the previous investigators as a special situation.

Mathematics Subject Classification: 92C35

Keywords: Stenosis, Blood flow, Shape parameter, Non-Newtonian, Rheology
1. Introduction

The leading cause of deaths in developed countries is a cardiovascular disease. The factors which influence the development of this type of disease are not yet exactly answered. Atherosclerosis is a vascular disease which affects arteries by a sub-endothelial build-up of fatty or lipid material rich in cholesterol.

This disease manifested itself as a local degenerative process which hardens the vessel walls and narrowed their lumen. As such the flow of blood to various parts of the body has been restricted. Immediate consequences of such formations can lead to severe pathological conditions. The partial occlusion of a coronary artery can lead to angina pectoris and there will be increased risk of myocardial infarction. The presence of stenosis in the vessels supplying blood to the brain can lead to stroke. This type of occlusion in the vessel carrying blood to the limbs can cause severe pain and loss of the function. So many investigations are performed on the prevention and cure of atherosclerosis. The results of these investigations promised better understanding of the nature of this type of disease. Due to stenosis in the human artery the flow of blood is disturbed and resistance to flow becomes higher than that of normal one. As such fluid mechanical behaviour of an arterial stenosis has drawn considerable attention from various researchers like Young and Tsai [4], Lee and Fung [8], Rodbard [15]. The Rheology of circulation was deeply discussed by Whitmore [12]. The analysis of blood flow through a symmetrically stenosed artery has been studied by Singh et al. [1]. Sanyal and Maji [2] investigated the unsteady blood flow through an indented tube in presence of stenosis. Young [3] observed the effect of time-dependent stenosis on flow of a Newtonian fluid through a tube. Chakravarty and Datta [13] performed rheological study on the effect of mild stenoses on the flow behavior of blood in a stenosed arterial segment. The various geometries of stenosis have been suggested by the researchers. The cosine-shaped geometry was considered and analysed with different parameters by many researchers like Young [3], Kapur [7], Chakravarty [14]. The power-law and casson fluid models with cosine-shaped geometry were discussed by Shukla et al. [5]. A composite shaped geometry of arterial stenosis was suggested and investigated by Joshi et al. [10]. The bell-shaped geometry with different fluids was discussed by Misra and Shit [6], Joshi et al. [11]. In all of the above studies the shape of stenosis was considered to be symmetrical about the axis as well as radius of the flow cylinder. The radially non-symmetric stenosis has been analysed by Sanyal and Maji [2], Srivastava and Saxena [16], Srivastava [17]. The effects of shape of stenosis on the resistance to blood flow through an artery has been investigated by Haldar [9]. Due to the presence of a new parameter the formulation of our model is mathematically more general and includes the model of Haldar [9] as a special case.

2. Formulation of the Mathematical Model

We have considered an artery having mild stenosis. The flow of blood is assumed to be steady, laminar and fully-developed. Blood is taken as a power-law fluid. It is assumed that stenosis is symmetrical about the axis but non-symmetrical with
Blood flow through an artery

respect to radial co-ordinates. The mathematical expression for geometry can be written as,

\[
\frac{R(z)}{R_0} = \begin{cases} 
1 - A \left[ L_0^{s-1} \left( \alpha z - kd - (k-1)L_0 \right) - \left( \alpha z - kd - (k-1)L_0 \right)^s \right] \\
1 
\end{cases} \quad ;k \left( d + L_0 \right) - L_0 \leq \alpha z \leq k \left( d + L_0 \right) 
\]

(1)

where

\[
A = \frac{\delta}{R_0 L_0^s (s-1)} 
\]

and \( \delta \) denotes the maximum height of stenosis at

\[
z = \frac{kd + (k-1)L_0 + L_0/\alpha}{\delta} 
\]

(3)

The schematic diagram of the flow is given by figure 1.

Fig 1: Geometry of Stenosed Artery

where

- \( R_0 \) : Radius of normal tube
- \( R(z) \) : Radius of stenotic region
- \( L \) : The length of the artery
- \( L_0 \) : The length of the stenosis
- \( d \) : Distance between equispaced points
- \( \delta \) : Maximum height of stenosis \( (\delta \ll R_0) \)
- \( s \) : Parameter determining the shape of stenosis \( (s \geq 2) \)
- \( \alpha \) : A positive integer \( \geq 1 \)
- \( k \) : Number of stenoses that appear in arterial lumen

For power-law fluid, we have
The boundary conditions are,
\[ v = 0 \quad \text{at} \quad r = R(z) \quad , k(d + L_0) - L_0 \leq az \leq k(d + L_0) \] (5)
\[ v = 0 \quad \text{at} \quad r = R_0 \quad , \text{elsewhere} \] (6)

Integrating equation (4) with respect to \( r \) and using boundary condition (5), we can find \( v \).

The flow flux \( Q \) is given by,
\[ Q = \int_{0}^{R(z)} v \cdot 2\pi r \, dr \]

It implies
\[ P(z) = \frac{2\mu Q^n}{R(z)^{\omega+1} \left( \frac{3n + 1}{n\pi} \right)^n} \] (7)

Now integrating equation (7) along the length of artery and using conditions \( p = p_1 \) at \( z = 0 \) and \( p = p_2 \) at \( z = L \).

We obtained
\[ p_1 - p_2 = \left( \frac{3n + 1}{n\pi} Q \right)^n \frac{2\mu}{R_0^{\omega+1}} \int_{0}^{L} \frac{dz}{\left( \frac{R}{R_0} \right)^{3n+1}} \] (8)

Where \( \frac{R}{R_0} \) is given by equation (1).

The resistance to flow \( \lambda \) is defined by
\[ \lambda = \frac{p_1 - p_2}{Q} \] (9)

which on using equation (8) gives
\[ \lambda = \frac{2\mu Q^{(n-1)}}{R_0^{\omega+1} \left( \frac{3n + 1}{n\pi} \right)^n} \int_{0}^{L} \frac{dz}{\left( \frac{R}{R_0} \right)^{3n+1}} \] (10)

For no stenosis
\[ \lambda_N = \frac{2\mu LQ^{(n-1)}}{R_0^{\omega+1} \left( \frac{3n + 1}{n\pi} \right)^n} \] (11)

\[ \bar{\lambda} = \frac{\lambda}{\lambda_N} = \frac{1}{L_0} \int_{0}^{L} \frac{dz}{\left( \frac{R}{R_0} \right)^{3n+1}} \] (12)

The non-dimensional form of resistance to flow, denoted by \( \bar{\lambda} \), is given as
\[ \bar{\lambda} = 1 - \frac{k_{\text{max}} L_0}{\alpha} \left( \frac{d}{L} \right) + \frac{1}{k} \sum_{k=1}^{d_{\text{max}}} \int \frac{dz}{k(d+L_k-L_0)} \left[ 1 - \frac{\delta}{R_0 L_0} s^{\frac{1}{s-1}} \left( L_0^{-\alpha} (\alpha z - k d - (k-1)L_0) - (\alpha z - k d - (k-1)L_0)^{\alpha} \right) \right]^{3n+1} \]  

where \( n \) is power law index.

when \( k = 1 \), the expression (13) reduced to

\[ \bar{\lambda} = 1 - \frac{L_0}{\alpha L} + \frac{d_{\text{max}}}{L} \int \frac{dz}{\alpha} \left[ 1 - \frac{\delta}{R_0 L_0} s^{\frac{1}{s-1}} \left( L_0^{-\alpha} (\alpha z - d) - (\alpha z - d)^{\alpha} \right) \right]^{3n+1} \]  

3. Conclusion

The analytic expression for resistance to flow is given by equation (13). For better understanding of the problem the graphs have been plotted for a single loop of the artery. Comparative graphs are given in the appendix (figure 2 and 3). It can be concluded that increasing values of \( \alpha \) shows the lower variations for different values of \( \frac{\delta}{R_0} \) as well as the length of the stenosis. The analysis coincides with that of Haldar [9] when \( \alpha = k = 1 \).

REFERENCES

APPENDIX

Variations of $\bar{x}$ with $s$ for different values of $\delta/R_0$ and $n=1$

Fig -2
Variations of $\bar{\lambda}$ with $\delta/R_0$ for different values of $L_0$ and $\alpha$

- $L_0 = 0.2, \alpha = 1.0$
- $L_0 = 0.6, \alpha = 1.0$
- $L_0 = 1.0, \alpha = 1.0$
- $L_0 = 0.2, \alpha = 1.1$
- $L_0 = 0.6, \alpha = 1.1$
- $L_0 = 1.0, \alpha = 1.1$
- $L_0 = 0.2, \alpha = 1.2$
- $L_0 = 0.6, \alpha = 1.2$
- $L_0 = 1.0, \alpha = 1.2$

Fig-3

Received: October, 2009