The Real Estate Credit Risk Based on the Value Model with Jump and Stochastic Interests

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Abstract

This paper is to assess the real estate credit risk with the help of the value model with jump and stochastic interests. Using the value obeying a stochastic jump-diffusion process, applying the Itô theorem and properties of martingales, combining linear regression in statistics, the value volatility and default probability of credit expiry date of a real estate company are obtained, which can be used as a reference standard for evaluating the credit rank of the company.

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1 Introduction

Since our country has joined WTO in 1997, Financial industry is gradually opening to the outside world completely. Faced with competition of numerous overseas financial institutions, each domestic big bank is bearing great pressure and risk. Especially according to the new rules in the "Barthel Agreement", commercial bank’s capital accumulation rate does not be lower than 8%, but the realistic situation is that many commercial banks have considerable disparity to this standard at present. And because of the impact of the former planned economic system, the current business activities of domestic banks, especially the issuance of credit, is also far from the rational management. On the other hand, In recent years the real estate industry not only has grown into one of pillar industries in the domestic economy, but also is commercial bank’s important source of credit and risk. Because the development of the real estate industry has the direct connection with macro-micro economy, this article establishes a asset model with jump and stochastic interests in the real estate industry, analyzes its volatility rate and carries on the estimation to enterprise’s probability of default, which can be used by the bank as a reference standard for evaluating the credit rank of the company. At present credit risk is one of important bank risk sources, which has attracted extensive attention in the risk management field. Especially in recent years some significant financial events have profound impacts on the financial crisis, the economic risks and even the society and national security. Thus people come to realize that we must adopt more rational, strict and effective risk management model and tool. The international already has quite many risk management models (see [1,3,4]), and the real estate asset models are few(see [5]), but fewer people use the model with jump and stochastic interests(see [8,9,2]) to study real estate assets, however, this kind of model happens to be able to describe the economic phenomena that the event and macro-economy changes lead to assets changes. Therefore, carrying out the related research deeply has theoretical and practical significance.

2 Asset Pricing Model With jump and Stochastic Interests and Probability of Defaults

The Merton model is to view the enterprise total assets as original assets in the option pricing theory, and to use European option pricing Black-Scholes formula to estimate whether or not an enterprise has repayment ability before the expiration date. Following this way, We can get the new asset pricing model. The model hypotheses and establishment are as follows:

(1) An enterprise’s assets structure is total assets = liabilities + owners’
interests (i.e., stock value), all liabilities are regarded as Zero Coupon Bonds
which have expiry data $T$ and face value $D$. Namely the primary assets do not
pay the dividends, an enterprise’s liabilities are regarded as zero coupon bonds
and stocks do not draw dividends in the time from 0 to $T$. Thus, owners’
interests are embodied only in the changes of share price.

(2) The market does not have the frication and does not count the trans-
action cost and tax revenue.

(3) Market arbitrage opportunities do not exist.

(4) Risk-free interest rate is a random variable and follows the expanded
Vasicek process as follows:

$$d\gamma(t) = \left(\theta_\gamma(t) - \alpha_\gamma \gamma(t)\right)dt + \sigma_\gamma dZ_1$$

where $\gamma(0) = \gamma_0 > 0$; non-zero constant $\alpha_\gamma$ is recovery rate of interest rate;
$\theta_\gamma(t)$ is the average level for deterministic function; positive constant $\sigma_\gamma$ is
volatility rate and $Z_1$ is a standard Brownian motion.

(5) Risk-nertral probability of default, interest rate and recovery rate are
dependent of each other.

(6) An enterprise’s total assets value $V_t$ evolves obeying the stochastic dif-
ferential equation with compound Poisson process below:

$$\frac{dV_t}{V_t} = (\gamma(t) - \lambda \kappa)dt + \sigma_V dZ_2 + (\Pi - 1)dY, \quad 0 \leq t \leq T$$

where $\sigma_V$ is volatility rate of company’s assets value; $Z_2$ follows the standard
Brownian motion; $\lambda$ is the jump frequency, also known as jump intensity; $\Pi > 0$
is the jump range follow the Logarithm normal distribution: $\in \Pi \sim N(\mu_z, \sigma_z^2)$,
$\kappa := E[\Pi - 1] = \exp(\mu_z + \frac{\sigma_z^2}{2}) - 1$; $\Pi$ is $i.i.d$ random variables row; $dZ_2, dY, \Pi$
are mutually independent.

According to the assumption, $V_t$ is recorded as an enterprise’s total assets at
a given time $t$; $D(t, T)$ is recorded as liabilities and $E_t$ is recorded as the value
of stock. Then the following equality holds: $V_t = D(t, T) + E_t, (0 \leq t \leq T)$. At time $T$, if $V_t < D$, an enterprise is unable to pay debts with its assets and
breaks a contract. Then its creditors take over the business and the income of
the enterprise owners namely shareholders is zero. If $V_t > D$, an enterprise can
fulfill obligation of repaying debts and the enterprise owner’s income is $V_T - D$.
Namely,

$$E_T = \max(V_T - D, 0) = \begin{cases} V_T - D, & V_T \geq D \\ 0, & V_T < D \end{cases}$$

In this way, the owners’ income (i.e., stock value) can be regarded as a call-
option which endows people with the right of purchasing this enterprise at the
price $D$ between 0 and $T$. 

In the financial markets, the occurrence of unexpected events can be described for the Poisson process with intensity $\lambda$, which is denoted by $Y_t = P(\lambda)$, the intensity function can be expressed as $P(Y_t = n) = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$, $n = 0, 1, 2, \cdots$. Let reimburse rate be $X_t = \frac{V_t}{D}$, then $X_t$ may be subject to jump-diffusion process:

\[
d\ln X_t = (\gamma(t) - \frac{\sigma_V^2}{2} - \lambda \kappa) dt + \sigma_V dZ_2 + \ln \Pi dY
\]

At this point the reimburse rate $X_t$ of an enterprise’s total assets is viewed as primary assets. According to Itô theorem and Martingale properties, at any time $t(0 < t < T)$, the value of stock namely the price of this call-option is:

\[
E_T = \exp(-E\int_{[0,T]} r(u)du)(1 - F_T^Q(1|X))
\]

\[+ (\omega - \omega_0)F_T^Q((\omega_0 - 1)/\omega_1|X)] + \omega_1 X \sum_{i=0}^{\infty} (\exp(-\lambda T)(\lambda T)^i/i!) \exp[\mu_i + \sigma_i^2/2]
\]

\[\times [N((\ln(X) + \mu_i + \sigma_i^2 - \ln((\omega_0 - 1)/\omega_1))/\sigma_i) - N((\ln(X) + \mu_i + \sigma_i^2)/\sigma_i)]\]

where $N(\cdot)$ follows the standardized normal distribution,

\[
\mu_i = E(\int_{[0,T]} r(u)du) - (\sigma^2/2 + \lambda \kappa)T + i \mu_\pi,
\]

\[
\sigma_i = (\sigma_\pi^2 T + i \sigma_\pi^2 + S^2(T))^{1/2},
\]

\[
S^2(T) = \text{Var}[\int_{[0,T]} r(u)du],
\]

\[
\omega(X) = \omega_0 - \omega_1 X.
\]

And $\omega$ is a loss rate under the condition of breaking a contract, $F_T^Q(\xi|X)$ is an enterprise’s default probability, the expression is:

\[
F_T^Q(\xi|X) = \sum_{i=0}^{\infty} (\exp(-\lambda T)(\lambda T)^i/i!) N((\ln(\xi) - \ln(X) - \mu_i)/\sigma_i).
\]

3 The Estimation of Asset Volatility Rates

Take different value of an enterprise’s total assets at different time, recorded as $V_1, \cdots, V_n$. Let random variable be

\[
X_i = n(V_{i+1} - V_i)^2, (i = 1, 2, \cdots, n - 1), X_n = n(V_1 - V_n)^2.
\]

Then we can prove that $X_i = \sigma_V^2 V_i^2, i = 1, 2, \cdots, n$ for sufficiently large $n$, where $\sigma_V$ is volatility rate of assets. According to the sample points of above two sample spaces, we can make use of regression analysis to estimate the value of the volatility rate $\sigma_V$ in an enterprise’s assets.
Let \( X = (X_1, \cdots, X_n)^T, U = (V_1^2, \cdots V_n^2)^T, \epsilon = (\epsilon_1, \cdots, \epsilon_n)^T, X = \beta U + \epsilon \). Based on the principle of linear regression, \( \beta = L^{-1} U^T X, L = U^T U \) and \( \sigma_V = \sqrt{\beta} \), volatility rate of assets can be determined. Take some real estate development (group) limited liability company A as an example. From the data between the fourth quarter of 2003 and the third quarter of 2004, which were extracted from the balance sheets (specific data see Appendix 2), we get a group of sample values of this enterprise’s total assets:

\[
V_1 = 3,178,469,898.98 \quad V_2 = 3,250,572,247.93
\]

\[
V_3 = 3,280,466,297.80 \quad V_4 = 3,334,384,864.90
\]

Then

\[
X_1 = 2.095 \times 10^{16} \quad X_2 = 3.57462 \times 10^{15}
\]

\[
X_3 = 1.16306 \times 10^{16} \quad X_4 = 9.72429 \times 10^{16}
\]

By calculating, we can see that

\[
X = (2.09500 \times 10^{16}, 3.57462 \times 10^{15}, 1.16306 \times 10^{16}, 9.72429 \times 10^{16})^T;
\]

\[
U = (1.01027 \times 10^{19}, 1.05662 \times 10^{19}, 1.07615 \times 10^{19}, 1.11181 \times 10^{19})^T;
\]

\[
L = U^T U = 4.53131 \times 10^{38}, \beta = L^{-1} U^T X = 0.00321.
\]

Then the volatility rate of asset is \( \sigma_V = \sqrt{\beta} = 0.56680 \).

### 4 Real Estate enterprise’s Assets Structure and Verification of the Model

We will inspect the balance sheets of real estate enterprise’s assets include liquid assets (including accounts receivable, goods in stock, cash and accumulated surplus funds), construction in process, the fixed assets (including original value, accumulated depreciation, net value) and intangible and deferred assets net value. Enterprise liabilities include current liabilities (including account payable, other short-term borrowing and short-term liabilities) and medium- and long-term loan. Owner’s interest include capital fund, capital accumulation funds, capital reserve, accumulated surplus reserve and accumulated undistributed profit.

While company A remains as an example, the volatility rate of assets has been obtained, \( \sigma_V \approx 0.57 \). Assume that in september 2003, the company applied for a credit loan to the bank, the life of which is one year (september in 2003 corresponds to the time \( t = 0 \), while september in 2004 corresponds to the due date \( T \), calculated according to the quarter, \( T = 4 \). At that time the
Figure 1: X axis’s data is the value of the sample point U and Y axis’s data is the value of the sample point X. Series I is a smooth curve connected by four points, while series II is a line obtained from the linear regression.
bank loan rate is \( r = 5.58\% \) for one year). The bank can only obtain current financial information of the enterprise when deciding whether to issue credit, that is, we only know the value of \( t = 0 \) and total liabilities of enterprises at that time. The data in company A is that \( V_t|_{t=0} = RMB3,178,469,898.98 \), \( D = RMB1,486,279,804.60 \).

From section 2, we substitute the data into the formula and use approximating calculation. In this way, the probability of defalut can be obtained. Namely,

\[
F_2^Q(l|X) = \sum_{i=0}^{\infty} \frac{\exp(-\lambda T) (\lambda T)^i}{i!} N((\ln(l)-\ln(X)-\mu_t)/\sigma_t) \approx 0.130836 \approx 13.08\%.
\]

This implies that the repayment probability of this enterprise is not small after one year, approximately is 13.08\%. Thus in practical business banks can credit this enterprise. The results which we obtain by the Merton model are that this enterprise’s violation probability is low and banks may credit it. But by the data shown in the enterprise’s balance sheet, one year later namely on september 30,2004, this enterprise’s total assets is greater than the debt. Moreover, the enterprise has the ability of repaying loans. It is rational for a bank to credit an enterprise just according to the results obtained from the Merton model.

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**References**


Appendix: The balance sheet of company A [Unit (RMB)]

the fourth quarter of 2003:
- total assets: 3,178,469,898.98
- total current assets: 2,864,416,334.24
- total fixed assets: 173,646,298.78
- total long-term investment: 130,161,265.39
- total intangible assets and other assets: 10,246,000.57
- total liabilities: 1,486,279,804.60
- total stockholders’ equity: 1,659,958,781.86
- total liabilities and equity: 3,178,469,898.98

the first quarter of 2003:
- total assets: 3,334,388,864.90
- total current assets: 3,021,209,503.82
- total fixed assets: 173,066,751.55
- total long-term investment: 130,030,324.04
- total intangible assets and other assets: 10,082,285.49
- total liabilities: 1,622,408,796.73
- total stockholders’ equity: 1,679,615,112.97
- total liabilities and equity: 3,279,615,112.97

the second quarter of 2003:
- total assets: 3,250,572,247.93
- total current assets: 2,919,621,451.05
- total fixed assets: 173,261,517.21
- total long-term investment: 148,133,794.48
- total intangible assets and other assets: 9,555,485.19
- total liabilities: 1,102,293,709.51
- total stockholders’ equity: 1,663,779,931.72
- total liabilities and equity: 3,250,572,247.93

the third quarter of 2003:
- total assets: 3,280,466,297.80
- total current assets: 2,903,816,900.69
- total fixed assets: 170,340,401.50
- total long-term investment: 196,917,225.60
- total intangible assets and other assets: 9,555,485.19
- total liabilities: 1,579,123,914.97
- total stockholders’ equity: 1,664,182,411.70
- total liabilities and equity: 3,280,466,297.80
Note: Because the original sheet is too long, we only selected some key economical quantities listed in the above economic volume.

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