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Influences of Rotation, Magnetic Field, Initial Stress, and Gravity on Rayleigh Waves in a Homogeneous Orthotropic Elastic Half-Space

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Abstract

The aim of this paper is to investigate the influences of rotation, magnetic field, initial stress, and gravity field on Rayleigh waves in a homogeneous orthotropic elastic medium. The government equations is solved by Lame's potential and obtained the frequency equation which determines the velocity of Rayleigh waves, including rotation, initial stress, gravity field, and magnetic field, in a homogeneous orthotropic elastic medium has been investigated. Numerical results analyzing the frequency equation are discussed and presented graphically. The results indicate that the effect of rotation, initial stress, magnetic field, and gravity field on Rayleigh wave velocity are very pronounced. Comparison are made with the results in the absence of rotation, initial stress, magnetic field and gravity field.

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1 Introduction

The problem of interaction between electro-magnetic field, stresses and strains in a magneto-elastic solid is relevant for a number of applications, such as geophysics for understanding the effect of Earth's magnetic field on seismic waves or the damping of acoustic waves in a magnetic field, nuclear clarify from nuclear devices, and etc. The wave propagation in thermo-elastic media is an importance in various fields such as earthquake engineering, soil dynamics, nuclear reactors, high energy particles accelerator, and etc.

In the recent years more attention has given to use the anisotropic material in engineering applications in considerable research activity. Problem of surface waves in an orthotropic elastic medium is very important for the posibility of its extensive application in various branches of science and technology, particularly in optics, earthquake science, acoustics, geophysics and plasma physics. Effect of rotation on a non-homogeneous composite infinite cylinder of orthotropic medium has been illustrated [4]. [5] explained the influence of gravity on Rayleigh waves, assuming that the force of gravity creates a type of initial stress of hydrostatic nature and the medium is incompressible. [1] discussed the influence of gravity field on Rayleigh waves propagation in inhomogeneous elastic medium. On the other hand, [6] discussed the influence of gravity on Rayleigh waves propagation in a homogeneous isotropic elastic solid medium. The influence of magnetic field, initial stress and gravity field in an isotropic material is discussed [7]. Propagation of Rayleigh waves in an elastic half-space of orthotropic material is investigated [2]. [3] investigated Rayleigh waves in a magnetoelastic half-space of orthotropic material under influence of initial stress and gravity field. Magneto thermoelastic plane waves in rotating media in thermoelasticity under GN model is discussed [11]. Thermoelastic waves in a rotating elastic medium without energy dissipation is illustrated [12]. Thermoelastic plane waves in a rotating isotropic medium is investigated [13]. Effect of rotation and relaxation times on plane waves in generalised thermo-elasticity is discussed [14]. [15] investigated time-harmonic sources in a generalized magneto-thermo-viscoelastic continuum with and without energy dissipation.

In this work, the effect of initial stress, magnetic field, rotation, and gravity field on Rayleigh wave propagation in an orthotropic homogeneous elastic solid medium has been discussed using the wave equations which is satisfied by the displacement potentials ϕ and ψ. The frequency equation that determines the velocity of the surface wave have been obtained. The dispersion equations have been obtained, and have been investigated for different cases. In fact, these equation are in agreement with the corresponding classical results when the medium is isotropic. The results obtained are presented graphically.

NOMENCLATURE

$\vec{u}$ is the component of displacement vector,
$\overrightarrow{H_o}$ is the constant primary magnetic field,
$\vec{h}$ is the perturbed magnetic field over
$\vec{j}$ is the electric current density,
μ_e is the magnetic permeability,
ρ is the density of the material,
τ_{ij} is the stress component,
ω_{ij} is the rotation vector,
$\vec{E}$ is the electric intensity,
$\vec{\Omega}$ is the angular velocity,
g is the earth gravity,
p is the initial stress,
t is the time.

2 Formulation of the problem and basic equations

Let us consider that the medium is a perfect electric conductor, we take the linearized Maxwell equations governing the electromagnetic field, taking into account absence of the displacement current (**SI**) as [8]

$$\left.\begin{aligned} curl\,\vec{h} &= \vec{j}\,, \\ curl\,\vec{E} &= -\mu_e \frac{\partial \vec{h}}{\partial t}, \\ div\,\vec{h} &= 0, \\ div\,\vec{E} &= 0 \end{aligned}\right\} \tag{1}$$

where,

$$\vec{h} = curl(\vec{u} \times \overrightarrow{H_o}), \tag{2}$$

$$\vec{H} = \vec{H}_o + \vec{h}.$$

We consider an orthotropic elastic solid under constant primary magnetic field $\overrightarrow{H_o}$ acting on y-axis, gravity field g and an initial compression P along the x direction. We take a set of orthogonal Cartesian axes $oxyz$, the origin o being any point of the boundary and oz pointing normally to the medium and the x-axis being positive along the direction of Rayleigh wave propagation.

At a great distance from the center of the disturbance, the wave is then two-dimensional and is polarized in the (x, z) plane, by which we mean that the component of displacement in the x and z directions; viz., u and w are not zero, but $v = 0$ and u and w are independent of y.

If g is the acceleration due to gravity, then the components of body forces are $X = 0$, $Z = -g$, and the initial stress field due to gravity is hydrostatic, the states of initial stress τ_{ij} are given as [1]

$$\tau_{11} = \tau_{33} = \tau, \qquad \tau_{13} = 0 \tag{3}$$

where, τ is a function of depth. The equilibrium conditions of the initial stress field [1]

$$\frac{\partial \tau}{\partial x} = 0, \quad \frac{\partial \tau}{\partial z} - \rho g = 0, \tag{4}$$

$$\tau_{ij,i} + F_j = \rho \left[\ddot{u} + \left(\overrightarrow{\Omega} \times \overrightarrow{\Omega} \times \overrightarrow{u} \right) + \left(2\overrightarrow{\Omega} \times \overrightarrow{\dot{u}} \right) \right]_j \tag{5}$$

where, $\overrightarrow{\Omega} \times \overrightarrow{\Omega} \times \overrightarrow{u}$ is the centripetal acceleration due to the time varying motion only and $2\overrightarrow{\Omega} \times \overrightarrow{\dot{u}}$ is the Coriolis acceleration. Here $\overrightarrow{u}$ is the dynamic displacement vector measured from steady state deformed possition and supposed to be small, $\overrightarrow{F}$ is the Lorentz's body forces vector. These two term do not appear in the equations for non-rotating media. Introducing Eqs. (1) and (2) into Eq. (5) for the dynamical equations of an elastic medium under initial compression stress P in the x_1-direction, considering Lorentz's body forces F, we obtain (see [5], Chs. 3 and 5)

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{21}}{\partial y} + \frac{\partial \tau_{31}}{\partial z} + p \left(\frac{\partial \omega_{12}}{\partial y} - \frac{\partial \omega_{13}}{\partial z} \right) - \rho g \frac{\partial w}{\partial x} + F_x = \rho \left(\frac{\partial^2 u}{\partial t^2} + 2\,\Omega \frac{\partial w}{\partial t} - \Omega^2 u \right), \tag{6}$$

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{32}}{\partial z} + p \frac{\partial \omega_{12}}{\partial x} + F_y = \rho \frac{\partial^2 v}{\partial t^2}, \tag{7}$$

$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} - p \frac{\partial \omega_{12}}{\partial x} + \rho g \frac{\partial u}{\partial x} + F_z = \rho \left(\frac{\partial^2 w}{\partial t^2} - 2\,\Omega \frac{\partial u}{\partial t} - \Omega^2 w \right) \tag{8}$$

where, $\omega_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right)$. Eqs. (6) and (8) in two-dimensions (x, z) reduce to [5].

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} - p \frac{\partial \omega_{13}}{\partial z} - \rho g \frac{\partial w}{\partial x} + F_x = \rho \left(\frac{\partial^2 u}{\partial t^2} + 2\,\Omega \frac{\partial w}{\partial t} - \Omega^2 u \right), \tag{9}$$

$$\frac{\partial \tau_{31}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} - p \frac{\partial \omega_{12}}{\partial x} + \rho g \frac{\partial u}{\partial x} + F_z = \rho \left(\frac{\partial^2 w}{\partial t^2} - 2\,\Omega \frac{\partial u}{\partial t} - \Omega^2 w \right) \tag{10}$$

The stress τ_{ij} in this case can be written as follows

$$\tau_{11} = (c_{11} + p)\frac{\partial u}{\partial x} + (c_{13} + p)\frac{\partial w}{\partial z}, \tag{11}$$

$$\tau_{33} = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z}, \tag{12}$$

$$\tau_{13} = c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \tag{13}$$

where, c_{ij} are the stiffness tensor components in the contraction notation so that $c_{12} = c_{22} = c_{23} = 0$, since the problem is treated in two-dimensions (x, z). Substituting from Eqs. (11-13) in Eqs. (9) and (10) and taking into consideration the above assumption with $c_{44} = \frac{1}{2}(c_{11} - c_{13})$, one may be written as follows

$$(c_{11} + p)\left(2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + c_{13}\left(\frac{\partial^2 w}{\partial x \partial z} - \frac{\partial^2 u}{\partial z^2}\right) - 2\rho g\frac{\partial w}{\partial x} +$$
$$2\mu_e H_o^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) = 2\rho\left(\frac{\partial^2 u}{\partial t^2} + 2\,\Omega\frac{\partial w}{\partial t} - \Omega^2 u\right), \tag{14}$$

$$c_{11}\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial x^2}\right) + (c_{13} + p)\left(\frac{\partial^2 u}{\partial x \partial z} - \frac{\partial^2 w}{\partial x^2}\right) + 2c_{33}\frac{\partial^2 w}{\partial z^2} + 2\rho g\frac{\partial u}{\partial x} +$$
$$2\mu_e H_o^2\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) = 2\rho\left(\frac{\partial^2 w}{\partial t^2} - 2\,\Omega\frac{\partial u}{\partial t} - \Omega^2 w\right). \tag{15}$$

We assume that the elastic displacement potentials ϕ and ψ are given by the relation

$$\vec{u} = \vec{\nabla}\phi + \vec{\nabla} \times \vec{\psi} \tag{16}$$

which may be reduces to.

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \quad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{17}$$

Substituting from Eq. (17) into Eqs. (14) and (15), one may obtain

$$(c_{11} + p + \mu_e H_o^2)\nabla^2\phi - \rho g\frac{\partial \psi}{\partial x} = \rho\left(\frac{\partial^2 \phi}{\partial t^2} + 2\Omega\frac{\partial \psi}{\partial t} - \Omega^2\phi\right), \tag{18}$$

$$(c_{11} + p - c_{13})\nabla^2\psi + 2\rho g\frac{\partial \phi}{\partial x} = 2\rho\left(\frac{\partial^2 \psi}{\partial t^2} + 2\Omega\frac{\partial \phi}{\partial t} - \Omega^2\psi\right) \tag{19}$$

and

$$c_{11}\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}\right) - (c_{13} + p)\nabla^2\psi + 2c_{33}\frac{\partial^2 \psi}{\partial z^2} + 2\rho g\frac{\partial \phi}{\partial x}$$
$$= 2\rho\left(\frac{\partial^2 \psi}{\partial t^2} + 2\Omega\frac{\partial \phi}{\partial t} - \Omega^2\psi\right), \tag{20}$$

$$(c_{11}+\mu_e H_o^2)\frac{\partial^2\phi}{\partial x^2}+\mu_e H_o^2\frac{\partial^2\phi}{\partial z^2}+c_{33}\frac{\partial^2\phi}{\partial z^2}-\rho g\frac{\partial\psi}{\partial x} = \rho\left(\frac{\partial^2\phi}{\partial t^2}+2\Omega\frac{\partial\psi}{\partial t}-\Omega^2\phi\right) \tag{21}$$

where,

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

Since the initial compressive wave has been taken in the x-direction, the velocity of the body waves are different in x and z directions. Thus eqs. (18) and (20) represent the compressive wave along the x and z directions, respectively, and Eqs. (19) and (21) represent the shear wave along those directions respectively.

Since we consider the propagation of Rayleigh waves in the direction of x only, we restrict our attention only to eqs. (18) and (20), which their solutions take the form

$$\left.\begin{aligned}\phi &= f(z)\exp[i\xi(x-ct)],\\ \psi &= h(z)\exp[i\xi(x-ct)].\end{aligned}\right\} \tag{22}$$

where ζ is the wave number and c is Rayleigh wave velocity

$$\frac{d^{\,2}f}{dz^2}+\xi^2m^2f-\frac{i\,\xi\rho}{\alpha^2}\left(g-2\Omega c\right)h=0, \tag{23}$$

$$\frac{d^{\,2}h}{dz^2}+\xi^2n^2h+\frac{2i\,\xi\rho(g-2\Omega c)}{\beta^2}f=0 \tag{24}$$

where,

$$\left.\begin{aligned}m^2&=\frac{\rho\left(c^2+\Omega/\xi^2\right)}{\alpha^2}-1, & n^2&=\frac{2\rho(c^2+\Omega/\xi^2)-c_{11}+c_{13}+p}{\beta^2},\\ \alpha^2&=c_{11}+p+\mu e H_o^2, & \beta^2&=2c_{33}-c_{11}-c_{13}-p.\end{aligned}\right\} \tag{25}$$

From Eqs. (23) and (24), we get the Eqs. determining ϕ and ψ as follows

$$\left[\left(\frac{d^2}{dz^2}+\lambda_1\xi^2\right)\left(\frac{d^2}{dz^2}+\lambda_2\xi^2\right)\right](f,h)=0 \tag{26}$$

where,

$$\lambda_1^2+\lambda_2^2=m^2+n^2,\ \ \lambda_1^2\lambda_2^2=m^2n^2-2\left(\frac{\rho(g-2\Omega c)}{\xi\alpha\beta}\right)^2. \tag{27}$$

Since the solutions of the above Eqs. must be of exponential nature, we may assume that the solution of Eq. (26) has the form

$$\begin{aligned}f(z) &= A_1\exp[-i\xi\lambda_1 z]+B_1\exp[i\xi\lambda_1 z]\\ &\quad +C_1\exp[-i\xi\lambda_2 z]+D_1\exp[i\xi\lambda_2 z],\end{aligned} \tag{28}$$

where A_1, B_1, C_1 and D_1 are constants.

Since the displacement at $z \longrightarrow \infty$ should be zero for Rayleigh waves. Hence λ_1, λ_2 are taken to be imaginary, and the constants corresponding to B_1 and D_1 must vanish. Therefore, the solutions of Eqs. (18) and (20) are

$$\phi = [A \exp[-i\xi\lambda_1 z] + B \exp[-i\xi\lambda_2 z]] \exp[i\xi(x - ct)], \tag{29}$$

$$\psi = [A' \exp[-i\xi\lambda_1 z] + B' \exp[-i\xi\lambda_2 z]] \exp[i\xi(x - ct)] \tag{30}$$

where, the constants A', B' are related, respectively, with A and B be means of Eq. (23) or Eq. (24). Equating the coefficients of $\exp[-i\xi\lambda_1 z]$ and $\exp[-i\xi\lambda_2 z]$ to zero, we have from Eq. (23)

$$A' = i\xi k_1 A, \qquad B' = i\xi k_2 B$$

where,

$$k_j = \frac{\alpha^2(\lambda_j^2 - m^2)}{\rho\,(g - 2\Omega c)}, \quad j = 1, 2. \tag{31}$$

Hence, we have the final form of the solution as

$$\phi = [A \exp[-i\xi\lambda_1 z] + B \exp[-i\xi\lambda_2 z]] \exp[i\xi(x - ct)], \tag{32}$$

$$\psi = i\xi\,(Ak_1 \exp[-i\xi\lambda_1 z] + Bk_2 \exp[-i\xi\lambda_2 z]) \exp[i\xi(x - ct)]. \tag{34}$$

3 Boundary conditions and discussion

The boundary conditions are that the plane $z = 0$ is free from stresses, i.e.

$$\tau_{13} + \bar{\tau}_{13} = \tau_{33} + \bar{\tau}_{33} = 0 \qquad \text{on } z = 0 \tag{34}$$

where,

$$\tau_{13} = \frac{1}{2}(c_{11} - c_{13})\left(2\frac{\partial^2\phi}{\partial x \partial z} - \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x^2}\right), \tag{35}$$

$$\tau_{33} = c_{13}\frac{\partial^2\phi}{\partial x^2} + c_{33}\frac{\partial^2\phi}{\partial z^2} + (c_{33} - c_{13})\frac{\partial^2\psi}{\partial x \partial z} \tag{36}$$

and the Maxwell's stresses $\bar{\tau}_{ij}$ take the form as follows

$$\bar{\tau}_{ij} = \mu_e[H_i h_j + H_j h_i - (\overrightarrow{H}.\overrightarrow{h})\delta_{ij}], \quad (i,\ j) = 1, 2, 3 \tag{37}$$

which take the forms

$$\bar{\tau}_{13} = 0, \quad \bar{\tau}_{33} = \mu_e H_o^2 \nabla^2\phi \tag{38}$$

then

$$2\frac{\partial^2\phi}{\partial x\partial z}-\frac{\partial^2\psi}{\partial z^2}+\frac{\partial^2\psi}{\partial x^2}=0 \tag{39}$$

$$c_{13}\frac{\partial^2\phi}{\partial x^2}+c_{33}\frac{\partial^2\phi}{\partial z^2}+(c_{33}-c_{13})\frac{\partial^2\psi}{\partial x\partial z}+\mu_e H_o^2(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial z^2})=0, \tag{40}$$

From Eqs. (39) and (40), one may obtain

$$A(2\lambda_1+L_1)+B(2\lambda_2+L_2)=0, \tag{41}$$

$$\begin{aligned}&A(c_{13}+\mu_e H_o^2+(c_{33}+\mu_e H_o^2)\lambda_1^2+(c_{33}-c_{13})\eta_1)\\ &+B(c_{13}+\mu_e H_o^2+(c_{33}+\mu_e H_o^2)\lambda_2^2+(c_{33}-c_{13})\eta_2)=0\end{aligned} \tag{42}$$

where,

$$L_j=i\xi k_j(\lambda_j^2-1), \qquad \eta_j=i\xi k_j\lambda_j. \tag{43}$$

To determine the constants A and B, its necessary that the determinant of the constants coefficients must be vanish, i.e.

$$\begin{vmatrix} c_{13}+\mu_e H_o^2+(c_{33}+\mu_e H_o^2)\lambda_1^2+(c_{33}-c_{13})\eta_1 & 2\lambda_1+L_1 \\ c_{13}+\mu_e H_o^2+(c_{33}+\mu_e H_o^2)\lambda_2^2+(c_{33}-c_{13})\eta_2 & 2\lambda_2+L_2 \end{vmatrix}=0. \tag{44}$$

Eq. (44) determines the Rayleigh surface waves under the influences of the Earth's gravity, rotation, and magnetic field with an initial compression in an orthotropic elastic solid medium.

4 Particular cases

(i) If the magnetic field and the rotation Ω are neglected, Eq. (44) reduces to

$$\begin{vmatrix} c_{13}+c_{33}\lambda_1^2+(c_{33}-c_{13})\eta_1 & 2\lambda_1+L_1 \\ c_{13}+c_{33}\lambda_2^2+(c_{33}-c_{13})\eta_2 & 2\lambda_2+L_2 \end{vmatrix}=0. \tag{45}$$

which determine the Rayleigh surface waves under the influence of gravity and initial stress and was investigated by Abd-Alla [2].

(ii)If the medium is isotropic, $H_o=0, \Omega=0$ and $\frac{g}{c^2\xi}\ll 1$, then Eq. (44) becomes

$$\begin{aligned}&\left(2-\frac{c^2}{\delta_1^2}\right)^2-4\left[\left(\frac{c^2}{\delta_1^2}-1\right)\left(\frac{c^2}{\delta_2^2}-1\right)\right]^{1/2}+\frac{4g}{c^2(\delta_1^2-\delta_2^2)\xi}\left(1-\frac{c^2}{\delta_2^2}\right)^{1/2}\\ &(\delta_1^2+\delta_2^2-c^2)-\left(1-\frac{c^2}{\delta_1^2}\right)^{1/2}\left(\delta_1^2+\delta_2^2-\frac{\delta_1^2c^2}{\delta_2^2}\right)=0\end{aligned} \tag{46}$$

where,

$$\delta_1^2=\frac{\lambda+2\mu+p}{\rho}, \quad \delta_2^2=\frac{\mu-p/2}{\rho}.$$

Eq. (46) determins the Rayleigh surface waves in an isotropic elastic medium under the effect of gravity and an initial stress.

If the initial stress is neglected, equation (46) reduces to

$$\left(2-\frac{c^2}{c_2^2}\right)^2-4\left[\left(\frac{c^2}{c_1^2}-1\right)\left(\frac{c^2}{c_2^2}-1\right)\right]^{1/2}+\frac{4g}{c^2(c_1^2-c_2^2)\xi}\left[\left(1-\frac{c^2}{c_2^2}\right)^{1/2}\right.$$

$$\left.\times(c_1^2+c_2^2-c^2)-\left(1-\frac{c^2}{c_1^2}\right)^{1/2}\left\{(c_1^2+c_2^2)-\frac{c^2c_1^2}{c_2^2}\right\}\right]=0 \qquad (47)$$

where, $c_1^2=(\lambda+2\mu)/\rho$ and $c_2^2=\mu/\rho$.

The Rayleigh surface waves under the influence of the earth's gravity determined by Eq. (47) which was illustrated by Love [9] by a different approach wich treats the gravity field as a type of body force. If the gravity field and initial stress are neglected, then Eq. (46) tends to

$$\left(2-\frac{c^2}{c_2^2}\right)^2=4\left[\left(1-\frac{c^2}{c_1^2}\right)\left(1-\frac{c^2}{c_2^2}\right)\right]^{1/2} \qquad (48)$$

these equation determines the wave velocity in the elastic medium and agreement with the results obtained by Rayleigh [10].

5 Numerical results and discussions

We wish to investigate the variation of Rayleigh wave velocity in a perfectly conducting medium under effect of rotation, magnetic field, initial stress and gravity field, for computational work, if the medium is orthotropic, we take

For computational work, if the medium is orthotropic, we take

$$c_{11}=2.694, \qquad c_{13}=0.661, \qquad c_{33}=2.363.$$

Figs. (1-3) show the effect of density ρ, magnetic field H_o, initially stressed p, rotation Ω and gravity g, respectively on Rayleigh wave velocity c with respect to initial stress P, wave number ξ, and wave length $2\pi/\zeta$. From Fig. 1, it is obvious that Rayleigh wave velocity decreases with an increasing of initial stress P, also, it is increases with an increasing of the density ρ, magnetic field H_o,rotation Ω and gravity g while it is decreases with an increasing of wave number ξ.

From Fig. 2, it is seen that Rayleigh wave velocity starts decreasing with an increasing of the wave number ξ and then steady with high values of ξ, also, it is clear that c increases with an increasing of magnetic field H_o, initial stress P, and rotation Ω but decreases with the increasing values of density ρ and gravity g.

From Fig. 3, it is clear that Rayleigh wave velocity c increases with an increasing of the wave length $2\pi/\xi$. Finally, it is appear that c increases with an increasing of magnetic field H_o, initial stress P, and rotation Ω but decreases with the increasing of density ρ and gravity g.

6 Conclusions.

From the previous results obtained, it is concludeds that the density, magnetic field, gravity field, rotation and the initial stress have considerable effect on the Rayleigh wave velocity of propagation of Rayleigh wave velocity and attracts the attention of earth scientists in their work. Numerical computations show that the presence initial compressive stress in the medium,magnetic field, the density and the rotation, reduce the Rayleigh wave velocity. It is found that the variation in magnetic field,the density, the rotation and initial stress of the medium directly affects the Rayleigh wave velocity. The results indicate that the effect of rotation, initial stress, magnetic field and the density on Rayleigh wave velocity are very pronounced and very important in multy applications in environment as Engineering, Buildings, Structures, Seismology, Gelogy, Biology, and etc.

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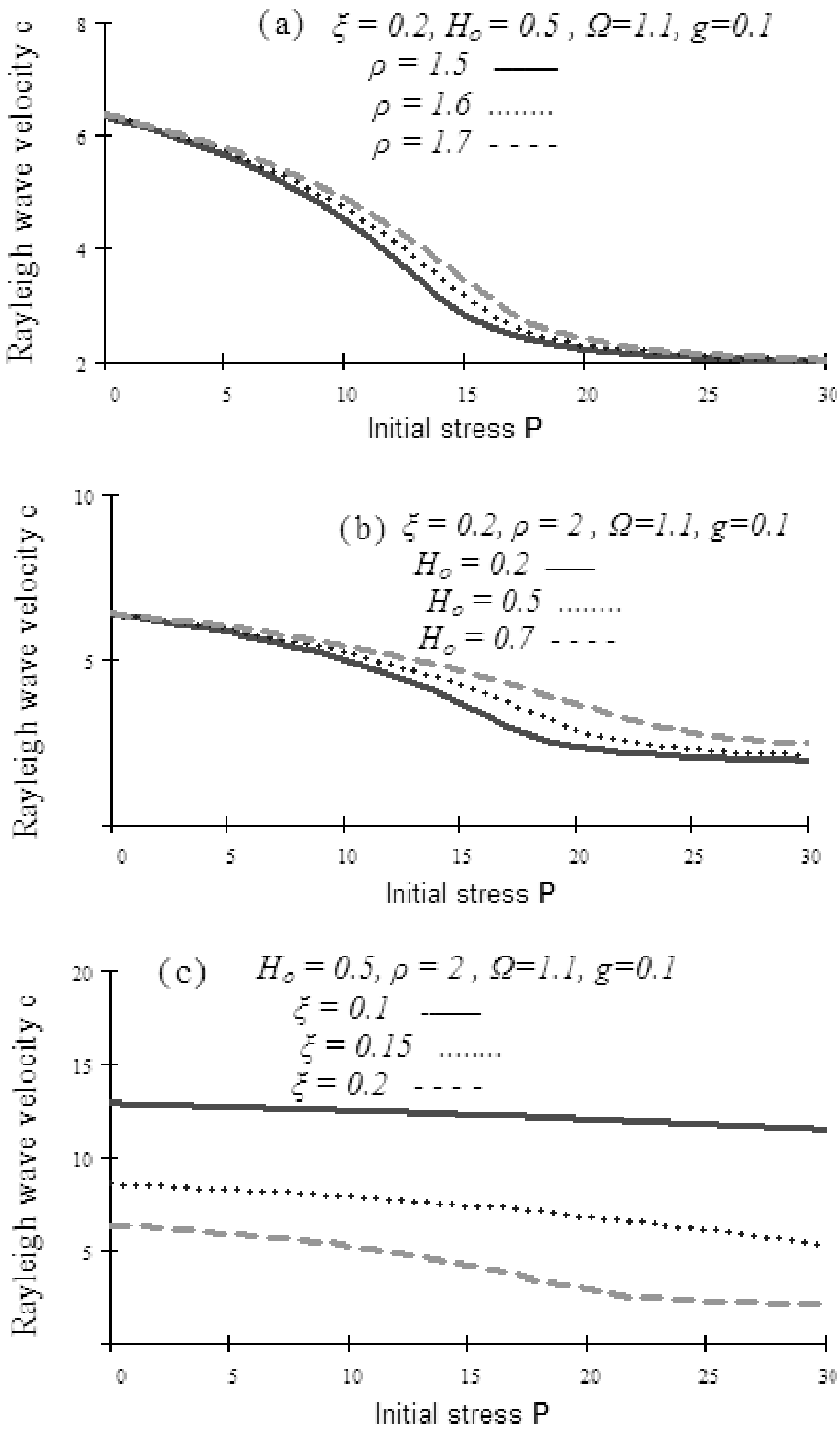

(a) ξ = 0.2, H_o = 0.5 , Ω=1.1, g=0.1
ρ = 1.5
ρ = 1.6
ρ = 1.7
Rayleigh wave velocity c
Initial stress P
(b) ξ = 0.2, ρ = 2 , Ω=1.1, g=0.1
H_o = 0.2
H_o = 0.5
H_o = 0.7
Rayleigh wave velocity c
Initial stress P
(c) H_o = 0.5, ρ = 2 , Ω=1.1, g=0.1
ξ = 0.1
ξ = 0.15
ξ = 0.2
Rayleigh wave velocity c
Initial stress P

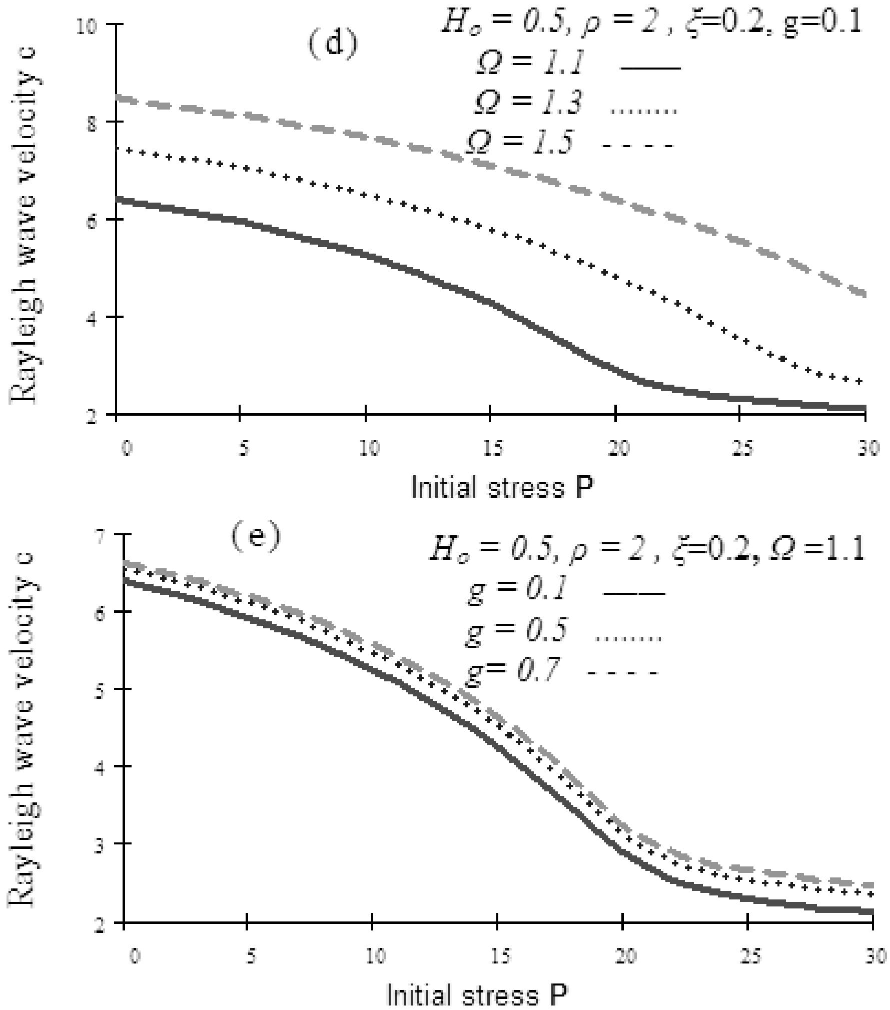
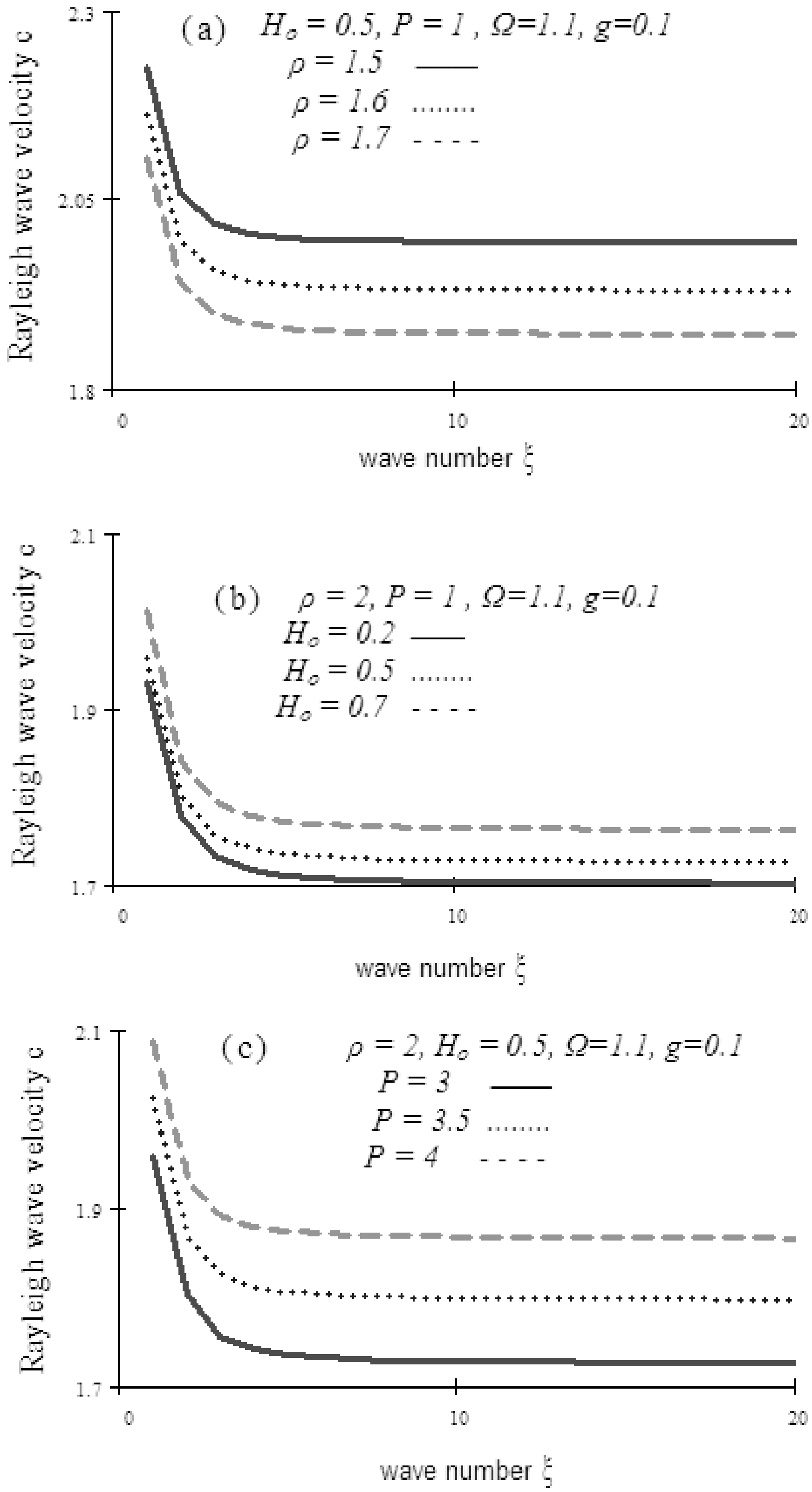
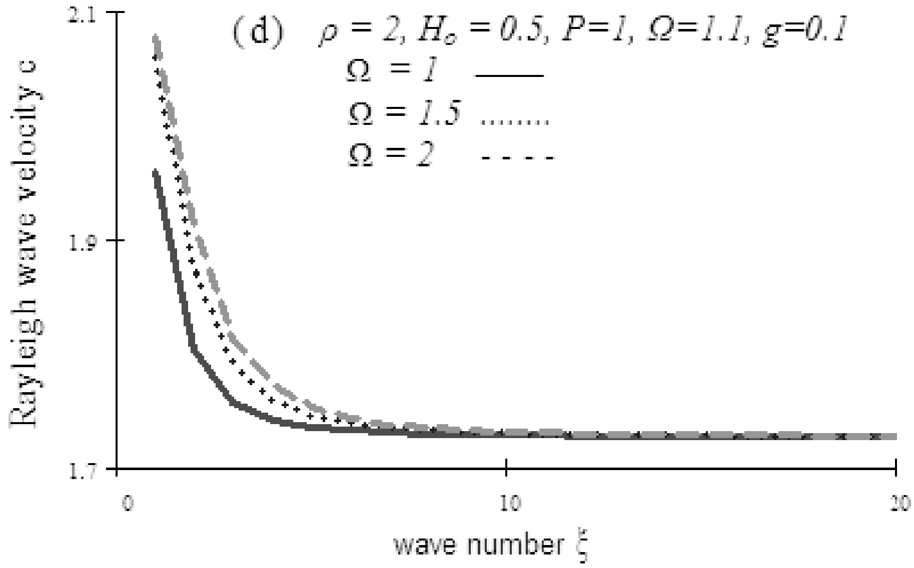


Fig. 1. Variations of Rayleigh wave velocity respect to the initial stress: (a) effect of the density (b)effect of the magnetic field, (c) effect of wave number, (d) effect of angular velocity, (e) effect of gravity


(a) $H_o = 0.5, P = 1, \Omega=1.1, g=0.1$
$\rho = 1.5$ ——
$\rho = 1.6$ ....
$\rho = 1.7$ - - - -
Rayleigh wave velocity c
2.3
2.05
1.8
0
10
20
wave number ξ
(b) $\rho = 2, P = 1, \Omega=1.1, g=0.1$
$H_o = 0.2$ ——
$H_o = 0.5$ ....
$H_o = 0.7$ - - - -
Rayleigh wave velocity c
2.1
1.9
1.7
0
10
20
wave number ξ
(c) $\rho = 2, H_o = 0.5, \Omega=1.1, g=0.1$
$P = 3$ ——
$P = 3.5$ ....
$P = 4$ - - - -
Rayleigh wave velocity c
2.1
1.9
1.7
0
10
20
wave number ξ



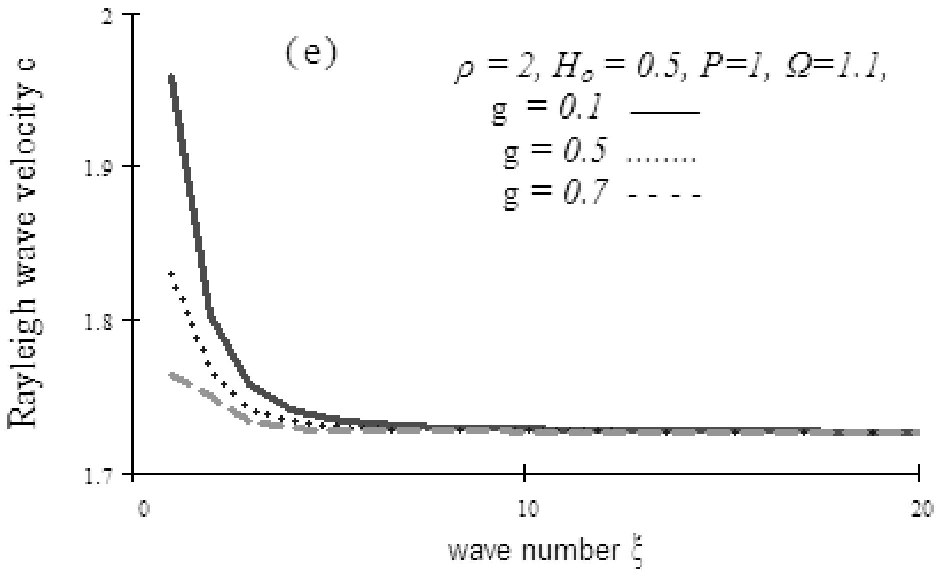
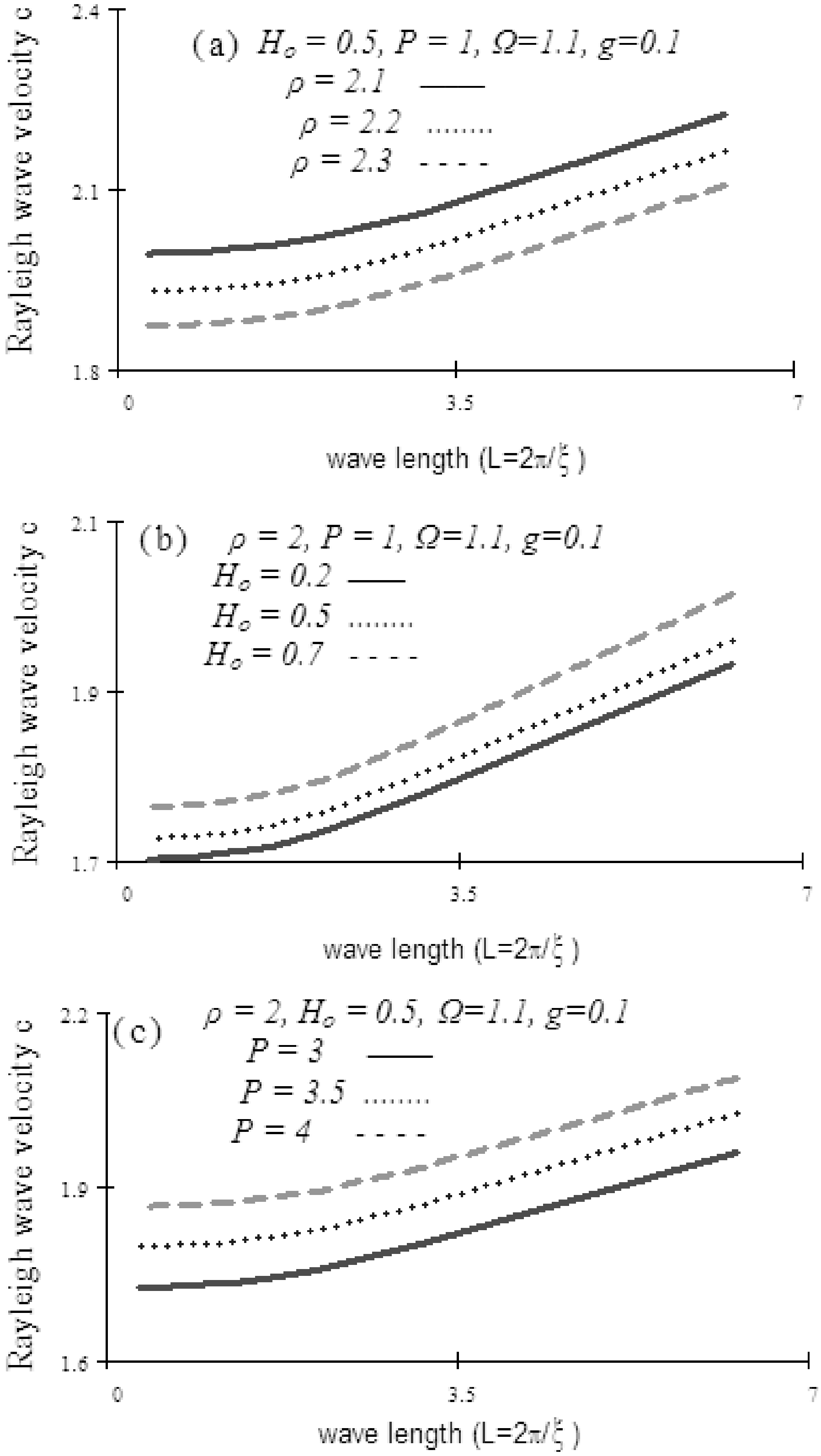


Fig. 2. Variations of Rayleigh wave velocity respect to the wave number: (a) effect of the density (b)effect of the magnetic field, (c) effect of initial stress, (d) effect of angular velocity, (e) effect of gravity


(a) $H_o = 0.5, P = 1, \Omega=1.1, g=0.1$
$\rho = 2.1$
$\rho = 2.2$
$\rho = 2.3$
Rayleigh wave velocity c
2.4
2.1
1.8
0
3.5
7
wave length (L=2π/ξ)
(b) $\rho = 2, P = 1, \Omega=1.1, g=0.1$
$H_o = 0.2$
$H_o = 0.5$
$H_o = 0.7$
Rayleigh wave velocity c
2.1
1.9
1.7
0
3.5
7
wave length (L=2π/ξ)
(c) $\rho = 2, H_o = 0.5, \Omega=1.1, g=0.1$
$P = 3$
$P = 3.5$
$P = 4$
Rayleigh wave velocity c
2.2
1.9
1.6
0
3.5
7
wave length (L=2π/ξ)

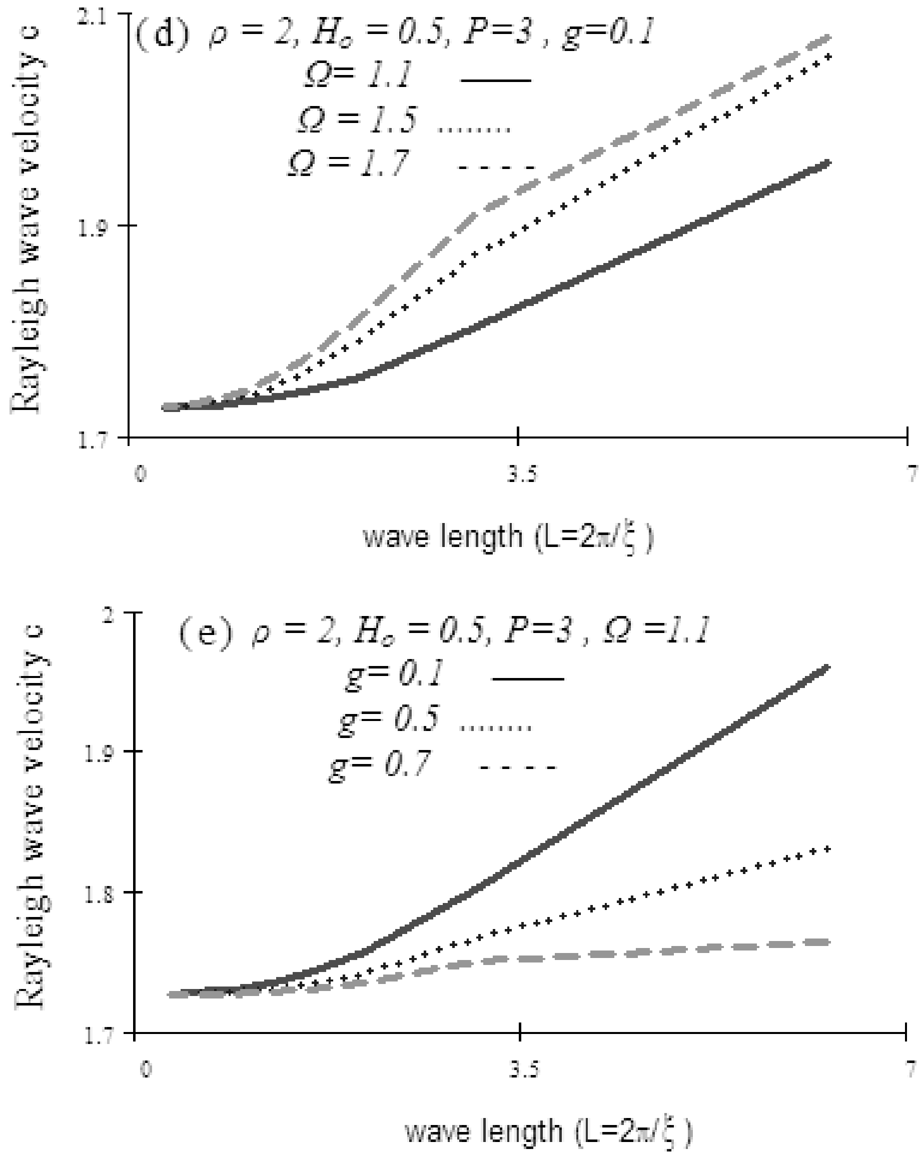


Fig. 3. Variations of Rayleigh wave velocity respect to the wave length: (a) effect of the density (b)effect of the magnetic field, (c) effect of initial stress, (d) effect of angular velocity, (e) effect of gravity