Revisiting Olbers’s Paradox in a Cellular Universe

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Abstract

Olbers’s paradox puts some constraints on the current cosmologies, i.e., the sky is not infinitely bright. A way to solve the paradox is to distinguish between the number and the density of the galaxies. A careful analysis has been made of the number of galaxies as a function of distance using the 2MASS Redshift Survey (2MRS) catalog. The obtained results for the finite intensity of light are compared with the intensity of the extra-galactic background at 520 nm.

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1 Introduction

According to Olbers’s paradox, the density of the light from the stars, on the hypothesis of an uniformly populated infinite space, should be infinite [1]. Unfortunately the sky is relatively dark, and in order to solve this paradox, geometrical theories for the large scale structure of our local universe were developed. As an example, some hierarchical models were set up at the beginning of the 20th century [2, 3, 4]. These first geometrical models were later classified as fractals [5, 6]. A first astronomical observation of the spatial distribution
of galaxies was Carpenter’s law

\[ N(R) \approx 316 R^{1.5} \]  

(1)

where \( R \) is the distance in Mpc and \( N \) is the number of galaxies in a sphere of radius \( R \) \[7\]. This law means that the number of galaxies does not grow as \( N \propto R^3 \), as a uniform distribution would suggest. The search for the fractal dimension in the galaxy distribution is now a common target \[8, 9, 10\]. The cellular model, i.e. Voronoi Diagrams, can also be classified as a geometrical theory for the universe but not necessarily as a fractal model due to the natural presence of the voids \[11, 12, 13, 14, 15\]. At the moment of writing, Olbers’s paradox is solved in expanding uniformly cosmological models \[16\] or by introducing the finite size of the stars and the finiteness of the local universe \[17\].

In order to explore how the cellular cosmology explains Olbers’s paradox, this paper reviews some basic formulas useful for counting galaxies, see Section 2, calculates the dimension of the 2MASS Redshift Survey, see Section 3, and solves the paradox, see Section 4.

## 2 Basic formulas

This section reviews Hubble’s law as well as the difference between the number and the number density of the galaxies.

### 2.1 Preliminaries

Starting from \[18\], the suggested correlation between expansion velocity and distance is

\[ V = H_0 D = c z \]  

(2)

where \( H_0 \) is the Hubble constant \[19\] \( H_0 = 100h \) \( \text{km s}^{-1} \) \( \text{Mpc}^{-1} \), with \( h = 1 \) when \( h \) is not specified, \( D \) is the distance in \( \text{Mpc} \), \( c \) is the velocity of light, and \( z \) is the red-shift defined as

\[ z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}} \]  

(3)

with \( \lambda_{\text{obs}} \) and \( \lambda_{\text{em}} \) denoting respectively the wavelengths of the observed and emitted lines as determined from the lab source, the so called Doppler effect. A recent evaluation, see \[20\], quotes

\[ H_0 = (74.3 \pm 2.1) \text{km s}^{-1} \text{Mpc}^{-1} \]  

(4)

and we will use this value in the various numerical codes.
2.2 The spatial distribution of galaxies

We make a distinction between the number of galaxies as a function of the distance \( R \), \( N(R) \), and the number density, \( n(R) \),

\[
n(R) = \frac{N(R)}{\frac{4}{3} \pi R^3} .
\]

The total number of galaxies as a function of distance is

\[
N(R) = \int_0^R n(r) 4 \pi r^2 dr .
\]

The first case to be analyzed is the case in which \( n(r) = n_0 \) with \( n_0 \) constant, which means

\[
N(R) = \frac{4}{3} \pi R^3 n_0 .
\]

The second case to be analyzed is an inverse power law dependence of the number density with the following piece-wise dependence which avoids a pole at \( r = 0 \)

\[
n(r) = \begin{cases} 
  n_0 & \text{if } r \leq R_0 \\
  n_0 \left( \frac{R_0}{r} \right)^\alpha & \text{if } r > R_0
\end{cases} .
\]

The total number of galaxies is

\[
N(R) = \frac{4}{3} \pi R^3 n_0 + 4 \pi n_0 \frac{1}{2} (R^2 - R_0^2) ,
\]

which in the limit \( R \to \infty \) scales

\[
N(R) \propto R^2 .
\]

The third case generalizes the inverse power law dependence of the number density to the following piece-wise dependence

\[
n(r) = \begin{cases} 
  n_0 & \text{if } r \leq R_0 \\
  n_0 \left( \frac{R_0}{r} \right)^\alpha & \text{if } r > R_0
\end{cases} ,
\]

where \( \alpha \) is the exponent of the power law. The total number of galaxies is

\[
N(R) = \frac{4}{3} \pi R^3 n_0 + 4 \pi n_0 R_0^\alpha \frac{1}{2 - \alpha + 1} (R^{2-\alpha+1} - R_0^{2-\alpha+1}) ,
\]

which in the limit \( R \to \infty \) scales

\[
N(R) = C_{th} R^{2-\alpha+1} ,
\]

where \( C_{th} \) is a constant.
The 2MASS Redshift Survey (2MRS) consists of 44599 galaxies with red-shifts in the interval $0 \leq z \leq 0.09$, see [21]. The Malmquist bias, see [22, 23], was originally applied to the stars and later on to the galaxies by [24]. The observable absolute magnitude as a function of the limiting apparent magnitude, $m_L$, is

$$M_L = m_L - 5 \log_{10} \left( \frac{cz}{H_0} \right) - 25.$$ \hfill (14)

The previous formula predicts, from a theoretical point of view, an upper limit on the maximum absolute magnitude which can be observed in a catalog of galaxies characterized by a given limiting magnitude; Figure 1 presents such a curve and the galaxies of the 2MRS.

The interval covered by the LF for galaxies, $\Delta M$, is defined by

$$\Delta M = M_{\text{max}} - M_{\text{min}} \ ,$$ \hfill (15)

where $M_{\text{max}}$ and $M_{\text{min}}$ are the maximum and minimum absolute magnitude of the LF for the considered catalog. The real observable interval in absolute magnitude, $\Delta M_L$, is

$$\Delta M_L = M_L - M_{\text{min}} \ .$$ \hfill (16)

We can therefore introduce the range of observable absolute maximum magnitude expressed in percentages, $\epsilon_s(z)$, as

$$\epsilon_s(z) = \frac{\Delta M_L}{\Delta M} \times 100\% \ .$$ \hfill (17)

This is a number which represents the completeness of the sample and, given the fact that the limiting magnitude of the 2MRS is $m_L=11.75$, it is possible to
Table 1: Numerical value of the spatial dimension of 2MRS.

<table>
<thead>
<tr>
<th>model</th>
<th>N</th>
<th>C</th>
<th>β</th>
<th>$R_{\text{max}}$ (Mpc)</th>
<th>$z_{\text{max}}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>complete sample 60%</td>
<td>124</td>
<td>3.67</td>
<td>1.62</td>
<td>8.55</td>
<td>0.002</td>
<td>1.38</td>
</tr>
<tr>
<td>Voronoi face</td>
<td>130</td>
<td>7.2</td>
<td>1.62</td>
<td>8.7</td>
<td>0.002</td>
<td>1.38</td>
</tr>
<tr>
<td>Abs mag, $M = -20 \pm 0.5$</td>
<td>111</td>
<td>0.57</td>
<td>1.86</td>
<td>16</td>
<td>0.0037</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Figure 2: The number of galaxies as a function of the distance in Mpc for the 2MRS catalog (dotted line) and observed number of galaxies with the vertical error bars for the first model. The error is computed as the square root of the number of galaxies in the considered bin.

conclude that the 2MRS is complete at 97% for $z \leq 0.00016$. This efficiency, expressed as a percentage, can be considered a version of the Malmquist bias. The number of observed galaxies, as function of the distance $R$, is modeled as

$$N(R) = CR^\beta$$

and three different models are used to find the values of $\beta$ for the 2MRS. The first model analyzes the complete sample at 60%, the second model evaluates the dimension $\beta$ by considering the galaxies with fixed absolute magnitude, and the third model is based on the concept of the face of an irregular Voronoi polyhedron. The three dimensions are presented in Table 1 with the interval of the considered samples $z_{\text{max}}$ or $R_{\text{max}}$, maximum red-shift or maximum distance.

As an example, Figure 2 displays the number of galaxies versus distance for the first model.

The match of the observed exponent in the number of galaxies, see Eq. (18), with the theoretical exponent, see Eq. (13), gives

$$\alpha = 2 - \beta + 1$$

(19)
and Table 1 shows the values of $\alpha$ for the three models here considered. According to the previous calculations, the number of galaxies does not grow as $N(R) \propto R^3$ as predicted by the homogeneous universe but with an exponent $1.38 \leq \beta \leq 1.86$ as measured in the 2MRS. This observed dependence can be compared with the dependence predicted by the cellular universe, $N(R) \propto R^2$, see Eq. (25).

3.1 The cellular universe

The cellular universe can be modeled by the Voronoi diagrams which are characterized by seeds. There are two kinds of seeds, Poissonian and non-Poissonian, which generate the Poissonian Voronoi tessellation (PVT) and the non-Poissonian Voronoi tessellation (NPVT). The Poissonian seeds are generated independently on the $X$, $Y$ and $Z$ axes through a subroutine which returns a pseudo-random real number taken from the uniform distribution between 0 and 1. The non-Poissonian seeds can be generated in an infinite number of different ways: some examples of NPVT are given in [25]. A point with exactly one nearest neighbor is in the interior of a cell, a point with two nearest neighbors is on the face between two cells, a point with three nearest neighbors is on an edge shared by three cells, and a point with four neighbors is a vertex where three cells meet. We now assume that the galaxies are situated on the faces of the PVT. Some of the properties of the PVT may be deduced from approximation arguments introducing the averaged radius of a polyhedron, $\bar{R}$ and the averaged diameter $\bar{D} = 2\bar{R}$. This theoretical quantity has its observational counterpart in the averaged diameter of the voids between galaxies, $\bar{D}^{obs}$. A careful analysis of the effective radius of the voids between galaxies in SDSS DR7 yields $\bar{D}^{obs} = \frac{36.46}{h}$ Mpc, for more details, see [26].

The averaged volume is

$$V = \frac{4}{3} \pi \bar{R}^3 ,$$

(20)

and the averaged number of faces is $n$, which according to [27] is $n = 15.35$. The averaged surface area of a polyhedron, $\bar{S}$, is

$$\bar{S} = 4\pi \bar{R}^2 ,$$

(21)

and the approximate area of a face, $\bar{A}$, is

$$\bar{A} = \frac{4\pi \bar{R}^2}{n} .$$

(22)

The approximate side of a face, $\bar{l}$, is

$$\bar{l} = \sqrt{\frac{4\pi \bar{R}^2}{n}} .$$

(23)
On assuming that our galaxy is at the center of an irregular face of a PVT, the galaxies on the other faces are contained within a distance $D_F$

$$D_F \approx \frac{l}{2}.$$  \hspace{1cm} (24)

On inserting the average radius as given by SDSS DR7 $D_F = 8.34$ Mpc, the predicted number of galaxies for $R \leq D_F$

$$N(R) = CR^2.$$  \hspace{1cm} (25)

A comparison between the simulated cellular structure of the universe up to $\approx 200$ Mpc and the real distribution of galaxies as given by the 2MRS catalog can be seen in Figure 3, more details can be found in [28].

4 Olbers’s paradox

This section solves Olbers’s paradox, reviews the luminosity function for galaxies, and compares theory with observations of the extra-galactic background light.
4.1 The paradox

A standard approach to Olbers’s paradox can be found in [29]: the flux received from a star at distance \( r \), of luminosity \( L_s \) is

\[
f(r) = \frac{L_s}{4\pi r^2}.
\] (26)

In the case of a constant density \( n_s \) of stars, the total intensity, the power per unit area per steradian, is

\[
I = \int_0^\infty n_s \frac{L_s}{4\pi r^2} r^2 dr = \infty .
\] (27)

According to this paradox the sky should be infinitely bright rather than dark. The number of stars is directly proportional to the number of galaxies, which is supposed to be distributed according to Eq. (11)

\[
I = \frac{n_0 R_0 \langle L \rangle}{4\pi} \frac{\alpha}{\alpha - 1} ,
\] (28)

where \( \langle L \rangle \) is the average luminosity of a galaxy,

\[
I = \frac{R_0 j}{4\pi} \frac{\alpha}{\alpha - 1} ,
\] (29)

where \( j = n_0 \langle L \rangle \) is the average luminosity density. In the framework of the cellular cosmology we have a finite intensity from the sky rather than infinite.

4.2 The luminosity function

The Schechter function, introduced by [30], provides a useful fit for the LF of galaxies

\[
\Phi(L) dL = \left(\frac{\Phi^*}{L^*}\right) \left(\frac{L}{L^*}\right)^{\alpha_s} \exp\left(-\frac{L}{L^*}\right) dL ,
\] (30)

here \( \alpha_s \) sets the slope for low values of \( L \), \( L^* \) is the characteristic luminosity, and \( \Phi^* \) is a normalization. The equivalent distribution in absolute magnitude is

\[
\Phi(M) dM = (0.4 \ln 10) \Phi^* 10^{0.4(\alpha_s + 1)(M^* - M)} \exp(-10^{0.4(M^* - M)}) dM ,
\] (31)

where \( M^* \) is the characteristic magnitude as derived from the data. The scaling with \( h \) is \( M^* - 5 \log_{10} h \) and \( \Phi^* h^3 \left[ Mpc^{-3} \right] \). The average value of luminosity per unit volume, \( j \), is

\[
j = L^* \Phi^* \Gamma(\alpha_s + 2) \frac{W}{Mpc^3} .
\] (32)
The relation connecting the absolute magnitude, $M$, of a galaxy with its luminosity is
\[ \frac{L}{L_\odot} = 10^{0.4(M_\odot - M)} , \tag{33} \]
where $M_\odot$ is the reference magnitude of the sun in the bandpass under consideration. As an example, in the $r^*$ band (6230 Å or 0.623 nm) of the Sloan Digital Sky Survey (SDSS) $M_\odot = 4.62$, $M^* = -20.83$, $\alpha_s = -1.2$, $\Phi^* = 0.014$, and $L_\odot = 3.84 \times 10^{36}$ W Mpc$^{-3}$, see [31], which means
\[ j = 9.48 \times 10^{34} WMpc^{-3} . \tag{34} \]

4.3 The extra-galactic background light

The extra-galactic background light (EBL) is a measurable astronomical quantity which represents the contribution from external galaxies. The presence of the galactic starlight, the zodiacal light, and the air-glow complicates the astronomical measurements. The study of the EBL started in the 1970s and some reviews has been written, see [32, 33]. A recent astronomical measurement quotes an intensity of $I(EBL)(520 \text{ nm}) \leq 12.0 \text{ nW m}^{-2} \text{ sr}^{-1}$, see [34]. Conversely, from a theoretical point of view, inserting the average luminosity, as given by Eq. (34) into the theoretical intensity, as given by Eq. (29), we have
\[ I(EBL)(520\text{ nm}) = C \frac{R_{0,1} \alpha}{(\alpha - 1) \pi} \frac{nW}{m^2 \text{ sr}} , \tag{35} \]
where $R_{0,1}$ is $R_0$ expressed in units of Mpc and $C = 0.0249$. The previous equation allows finding a theoretical expression for $\alpha$
\[ \alpha = -3.141 \frac{I(EBL)(520\text{ nm})}{C R_{0,1} - 3.141 I(EBL)(520\text{ nm})} . \tag{36} \]

As a practical example, inserting into the previous formula $R_{0,1} = 1$ and $I(EBL)(520 \text{ nm}) = 12$, we obtain $\alpha = 1.000661$.

A finite value for the EBL’s intensity is also found in the framework of the expanding universe, see the final equation (5.167) in [35], which is
\[ I(EBL)(440\text{ nm}) = \nu_0 i_{\nu_0} = 3 \times 10^{-6} \frac{\text{erg}}{\text{cm}^2 \text{ s sr}} = 3 \frac{nW}{m^2 \text{ sr}} , \tag{37} \]
where $\nu_0$ is the frequency in the blue band centered at $\approx 440$ nm and $i_{\nu_0}$ is the energy flux per unit area and solid angle at the frequency $\nu_0$. 
5 Conclusions

The possibility of having a dark sky, i.e. of solving Olbers’s paradox, is connected with the number density of galaxies. An inverse power law behavior for the number density as given by Eq. (11) solves the paradox. The astronomical task is to find the dimension of the spatial distribution of galaxies and in our case an analysis of the 2MRS catalog gives a dimension $1.62 \leq \beta \leq 1.86$ against the $\beta = 1.5$ of Carpenter’s law [7]. Other authors indicate that $\beta \approx 2.2$, see [36, 37]. This means that special efforts should be given to measure the spatial dependence of the number of galaxies in a cosmology-independent way. We briefly recall that in the classical situation, if $\beta > 2$, the paradox is still there. The finite level of luminosity here found is connected with the extra-galactic background light which is fixed in a certain range of nW m$^{-2}$ sr$^{-1}$ for the optical band [34].

References


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